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Robust Control Synchronization on Multi-Story Structure under Earthquake Loads and Random Forces using H_∞ Algorithm

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Abstract. In this paper, the concept of synchronization control along with robust H_∞ control are considered to evaluate the seismic response control on multi-story structures. To show the accuracy of the novel algorithm, a five-story structure is evaluated under the EL-Centro earthquake load. In order to find the performance of the novel algorithm, random and uncertainty processes corresponding to Riccati equation is solved under a specific dynamic. Time history graphs corresponding to maximum displacement and floors force control are presented and evaluated. Despite the existence of random process and uncertainty in structure, stability and optimal performances are shown.

Keywords. Synchronization, Random process and uncertainty, Robust H_∞ control, EL-Centro earthquake load, Riccati equation.

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1 Introduction

Since severe dynamic loads destroy constructional structures, researchers are trying to reduce dynamic load destructions caused by winds and earthquakes. Seismic monitoring technologies have significantly reduced the response of structures to dynamic loads. Passive, semi-active and active control may be used to control the specific structures. For semi-active and active controls, sensors collect data from structures during dynamic loading. The corresponding information is submitted to the controller so that after processing, appropriate control force could be determined by a control algorithm. These control signals are sent to the operator to control structure vibrations. The purpose of control algorithms is to determine the optimal power control and reduce the structure vibration response [1, 12]. One of the good measures may be reduction in vibration response which minimizes the internal forces of the structure. The control algorithm shall be adjusted so that the relative displacement between the degrees of freedom reaches its minimum value. Here, the emphasis is on coordinating the related drifts of adjacent story buildings by synchronized controllers. The application of synchronization approach to robotics started from motion coordination of driven wheels in control of mobile robots. Borenstein and Koren [11], in 1987 and Feng et al. in 1993 applied a cross-coupling controller to synchronize motions of two driven wheels for a differential mobile robot in trajectory tracking. Sun and Mills proposed an adaptive synchronization controller to coordinate motions of multiple manipulators in assembly tasks [3]. Rodriguez-Angeles and Nijmeijer reported their work on mutual synchronization of robot manipulators via estimated state feedback [14]. Sun et al. developed an orientation control scheme by synchronizing two driven wheels of a differential mobile robot. Sun and Wang [15] proposed the use of a cross coupling-based synchronization control strategy to address the problem of multi-robot control in time-varying formations. In 2009, Chung and Slotine presented a synchronization tracking control law for cooperation of multi-robot systems and oscillation synchronization in robotic manipulation and locomotion ([4]).

Motion synchronization in systems that uses several agents with different coordinates but for the same goal was used in 2013 [6]. In 2009, Sun Dong used the synchronization method to trace the path to maintain the association variable time. In 2012, Francisco Palacios by a mathematical model calculated the total response attach for the seismic system. In addition, in 2014, Quanmin Zhu in a case study by synchronization control algorithms studied control for mechanical arms. Mesbahi and Malek used synchronization control for specific structures with H_2/LQG algorithms [9]. In these kinds of applications, to achieve a common goal, a total system stability is needed, i.e., all correct routing for creating stability and coordination for each of the subsystems must be considered together. The dynamics of mathematical models which are used to design the controller does not correspond with the actual dynamics of structure, and this discrepancy is due to a defect in the structure, changes in structure mass, fatigue impact on structure materials and the incorrect placement of sensors and operators in a convenient location. This discrepancy can be stated as unmodeled dynamics or as uncertainty in the modeling. The main objective of H_∞ controller is controlling and regulating of adjusted outputs so that the input disturbances and model uncertainties do not have any effect on the system performance. H_∞ controller decreases the worst system response to disturbances and unknown inputs and acts in minimizing the responses and control forces in the worst conditions, and causes the least displacement and relative displacement (drift) of floors in structure [8].

2 Synchronizer controller

In large scale systems, coordinating between different affairs is a great help in time saving [13]. In the man's history, for the purposes of security, safety, accuracy, nicety, efficiency and communication, synchronization between different agents has been used. In this paper, the emphasis is on synchronization between drifts of the two adjacent floors and the related forces in this regard.

Suppose a multi-agent system with n agents (n -floors) involved together. Thus, a synchronizer is needed for all agents. The purpose of synchronizer control is to synchronize the whole drifts so that the floors maintain a special kinematics relationship such that regulating all the floors for maintaining the kinematics relationship which can be done as a guide and the positioning of floors in the boundary line (or curve) of a multi-story structure.

2.1 Coupled position error

Suppose that $S(p, t)$ is a function of time and location variables. In such function, p is state vector and t is time variable. Boundary of $S(p, t)$ is shown by $\partial S(p, t) = 0$. Now, let $x_i(t)$ and $x_i^d(t)$ stand for state and desirable value of state variables in the i th agent. $x_i^d(t)$ has $\partial S(x_i^d, t) = 0$ feature. Thus, error of the state variable in the i th agent is stated as:

$$e_i(t) = x_i(t) - x_i^d(t) \quad i = 1, \dots, n. \quad (1)$$

The purposes of synchronizer controller for all the agents are (See [14]):

1. Convergence of state variables to the desired values $x_i^d(t)$
2. $e_i \rightarrow 0$ when $t \rightarrow \infty$, $i = 1, \dots, n$, to maintain the relations on the desirable curve
3. $\partial S(x_i, t) = 0$, $i = 1, \dots, n$

2.2 Synchronized position error

In an optimistic case, for the five-story building under earthquake, the error of state variables in each moment is equal to the amount of displacement in each floor, say x_1, x_2, x_3, x_4 and x_5 . Thus, the objective of control is to tend these values to zero as $t \rightarrow t_{max}$ (Figure 1). Let us define the vector of displacement

$$e = (x_1, x_2, x_3, x_4, x_5)^T. \quad (2)$$

Assume that $\varepsilon_{5 \times 1}$ is a vector and each component stays on the final desired curve as $t \rightarrow t_{max}$, i.e., in the boundary curve one must satisfy $\Gamma(x_i^d, t_{max}) = 0$. By this approach, the relevant displacement (drift) between the floors, ε is considered as synchronizing error (Figure 1). Relevant displacement (drift) between the floors is determined as follows:

$$\varepsilon = (x_1, x_2 - x_1, x_3 - x_2, x_4 - x_3, x_5 - x_4)^T \quad (3)$$

Reduction of relevant displacements between the floors will lead to a reduction of the internal forces of structure. Note that the coupled position error will be defined by (4):

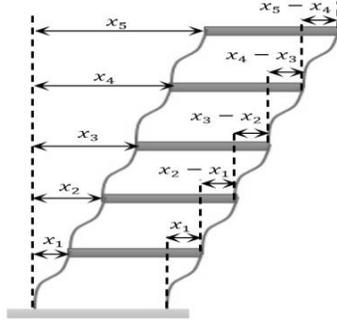


Figure 1: Displacement and relevant displacement (drift) of the floors

$$E_c = (I + \alpha T) e, \quad (4)$$

where T satisfies by

$$\varepsilon = T e, \quad (5)$$

and without loss of generality one may assume:

$$T = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In this example α is an identical matrix. This means that $(I + \alpha T)$ is an invertible matrix and as e converges to zero the error E_c converges to zero too and vice versa. With respect to the definitions, as $t \rightarrow t_{max}$ coupled position error E_c , e and ε can converge to zero simultaneously [6].

In the following section, the concept of synchronization control with acceleration feedback, in combination of both (i) H_∞ control and (ii) robust H_∞ control are considered to evaluate the seismic response control for multi-story structures in Section 7.

3 Dynamical structure model in the state space

Motion equation of one structure under dynamic loads as a second order differential equation is [10]:

$$M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = B_0 u(t) + TW(t), \quad (6)$$

where q , M , K , C , $u(t)$, $W(t)$, B_0 and T are respectively displacement vector, mass matrix, stiffness and damping matrices, control force vector, disturbance vector (stimulus comes from earthquake), force control position and vector of position external disturbance. On the other hand, defining the displacement and velocity as state-space variables yields,

$X(t) = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix}$, $\dot{q}(t)$ is a velocity vector, state equations include:

$$\dot{X}(t) = AX(t) + B_1W(t) + B_2u(t) \quad (7)$$

Where $A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}$ is state matrix, $B_2 = \begin{bmatrix} 0 \\ M^{-1}B_0 \end{bmatrix}$ is the internal stimulus location matrix, $B_1 = \begin{bmatrix} 0 \\ M^{-1}T \end{bmatrix}$ is external stimulus location matrix to the system [4].

Consider the following systems:

$$\begin{aligned} \dot{X}(t) &= AX(t) + B_1W(t) + B_2u(t) \\ z(t) &= C_1X(t) + D_{11}W(t) + D_{12}u(t) \\ y(t) &= C_2X(t) + D_{21}W(t) + D_{22}u(t) \end{aligned} \quad (8)$$

In different times t , outputs $z(t)$ are system evaluation parameters while, $y(t)$ are sensor outputs. Here, C_1, D_{11} and D_{12} are different matrices corresponding with $z(t)$ outputs and C_2, D_{21} and D_{22} corresponds with sensor outputs $y(t)$. Up to here, an n -story building under wind or earthquake load inputs with corresponding outputs from system evaluation parameters and sensors responses is considered in (8) as same as in [7].

Displacement, velocity and acceleration can be controlled using one of the algorithms H_2/LQG , H_∞ and robust H_∞ . Mesbahi and Malek [9] used H_2/LQG algorithm successfully, in order to reduce displacement around 80 and 50% under Bam and El-Centro earthquakes, respectively. Both H_∞ and H_∞ robust algorithms are considered in Sections 4 and 6.

4 H_∞ control algorithm

H_∞ algorithm is a scale model for the worst response during loading (or external disturbances) [8]. In the sense of infinity norm for a general function G , we define:

$$\|G\|_\infty = \text{Sup}_{\omega \in \mathbb{R}} \bar{\sigma}(G(j\omega)) \quad (9)$$

where, $j\omega$ is in general a complex value for the frequency.

Now, the H_∞ control algorithm [8] can be used to control the system (8), (Figure 2). The main purpose of H_∞ controller is to minimize the effect of input disturbance on regulated outputs. The H_∞ controller guarantees stability and system functioning against external disturbances with bounded infinity norm [8]. This system may be considered as a closed-loop system where P stands for the n -story building model, S is the controller that computes the control feedback u . Consider the measured outputs y where it is imported to the system model P . In which w is a disturbance input and z is an evaluation output (Figure 2).

For $G(P, S)$ as a function of P , the n -story building model and S , the feedback calculator, system (7) reduces to the matrix form:

$$\begin{bmatrix} \dot{X} \\ Z \\ y \end{bmatrix} = P \begin{bmatrix} X \\ W \\ u \end{bmatrix} \quad (10)$$

where

$$z = G(P, S)W, \quad (11)$$

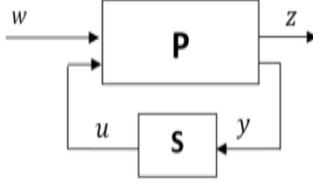


Figure 2: Closed-loop system and controller.

then the objective of H_∞ control is to find a stabilizer controller to minimize the effect of W on z . H_∞ algorithm computes the maximum efficiency of the system in the sense of infinity norm:

$$\|G(P, S)\|_\infty = \sup_{W \neq 0} \frac{\|z\|_2}{\|W\|_2} = \sup \bar{\sigma}(G(P, S)(j\omega)) \quad (12)$$

where $\bar{\sigma}$ is a maximum of a special value for transfer function G and $j\omega$ is in general a complex value for the frequency. Therefore, the following sub-controller is designed and noted by S_{sub} and in the sense of infinity norm satisfies the following inequality [8]:

$$\|G(P, S_{sub})\|_\infty < \gamma, \quad \gamma > 0 \quad (13)$$

5 Riccati equation

After constructing the related Riccati equations, with the use of matrix (10) one can write down the following properties (See [8]):

- (i) There exist $X_\infty \in \mathbb{R}^{n \times n}$ which satisfies the following Riccati equation:

$$X_\infty A + A^T X_\infty + X_\infty (\gamma^{-2} B_1 B_1^T - B_2 B_2^T) X_\infty + C_1 C_1^T = 0$$

- (ii) There exist $Y_\infty \in \mathbb{R}^{n \times n}$ which satisfies the following Riccati equation:

$$A Y_\infty + Y_\infty A^T + Y_\infty (\gamma^{-2} C_1^T C_1 - C_2^T C_2) Y_\infty + B_1 B_1^T = 0$$

- (iii) $\rho(X_\infty, Y_\infty) < \gamma^2$, ρ represents the maximum eigenvalue of the matrix $G(P, S_{sub})$.

Then, the sub-optimal H_∞ control problem has a unique solution and will be defined by:

$$K_{sub} = \begin{bmatrix} A_\infty & -Z_\infty L_\infty \\ F_\infty & 0 \end{bmatrix} \quad (14)$$

where for $F_\infty \in \mathbb{R}^{q \times n}$, $L_\infty \in \mathbb{R}^{n \times p}$ and $Z_\infty \in \mathbb{R}^{n \times n}$ yields

$$\begin{aligned} A_\infty &= A + \gamma^{-2} B_1 B_1^T X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2 \\ F_\infty &= -B_2 X_\infty, L_\infty = -Y_\infty C_2^T \\ Z_\infty &= (I - \gamma^2 Y_\infty X_\infty)^{-1} \end{aligned}$$

The above Riccati equations are solved for the unknowns A_∞ , F_∞ and Z_∞ . Thus, the dynamic controllers are:

$$A_c=A_\infty, \quad B_c= -Z_\infty L_\infty, \quad C_c=F_\infty, \quad D_c= 0 \quad (15)$$

Now, one may look for the drifts to vanishes while both coupled errors and synchronization errors go to zero as $t \rightarrow t_{max}$. In this way, minimization of the drifts between each of the adjacent floors is the synchronized control goal.

Now, by the standard codes of 2800-Iran [10], inelastic drift of each floor is calculated as follows:

$$\delta_p = C_d \delta_e \quad (16)$$

where δ_e is an elastic drift of each floor under earthquake and C_d is a parameter that depends on the seismic system in the structure. For the general structure one may have:

$$(\delta_p) \leq 0.015h_j \quad (17)$$

where h_j is the height difference between the (j) -th floor and $(j - 1)$ -th floor.

Up to now, one may combine the ideas from sections 2, 3, 4 and 5 in order to find both H_∞ controllers along with synchronization goals, (Figures 6, 7 and 8).

In order to combine the robust H_∞ control algorithm with synchronization ideas given in Section 2, consider Section 6. This section discusses the existence of the unmodel dynamic forces that come from real life uncertainty parameters.

6 Robust H_∞ control algorithm

In the real world, mathematical models which are used to design some controllers do not match with the real dynamic of the structures. For this reason, designers may assume to have unmodel dynamic parameters that come from uncertainties. This unmodel dynamics and uncertainties may be considered as some faults in the structures' design, imprecision in identifications, misplacing of sensors or actuators. Therefore, it is essential to design the controller in a way to be robust to these unmodel dynamics and uncertainties. In H_∞ design, these unmodel dynamics and uncertainties can be considered as the disturbance input of the model. This helps the designers to design the controller robust for these types of inputs. State space model with unmodel dynamic is illustrated as follows:

$$\begin{aligned} \dot{X}(t) &= AX(t) + B_1W(t) + B_2u(t) + B_f f(t) \\ z(t) &= C_1X(t) + D_{11}W(t) + D_{12}u(t) + D_{1f} f(t) \\ y(t) &= C_2X(t) + D_{21}W(t) + D_{22}u(t) + D_{2f} f(t) + v \end{aligned} \quad (18)$$

where $f(t)$ is the unmodel dynamic of the model.

Then, this unmodel dynamic can be considered as an unknown input, $f(t)$, to the model. Considering $f(t)$ as the random dynamic force, then the disturbance inputs change as $\overline{W} = [W^T \quad f^T]^T$. Similarly $\overline{B}_1 = [B_1 \quad B_f]$, $\overline{D}_{11} = [D_{11} \quad D_{1f}]$ and $\overline{D}_{21} = [D_{21} \quad D_{2f}]$ are defined, in accordance with Figure 3 and System dynamics (10) then, the definitive system (8) becomes definitive-random represented as follows:

$$\begin{aligned} \dot{X}(t) &= AX(t) + \overline{B}_1\overline{W}(t) + B_2\overline{u}(t) \\ \overline{z}(t) &= C_1X(t) + \overline{D}_{11}\overline{W}(t) + D_{12}\overline{u}(t) \\ \overline{y}(t) &= C_2X(t) + \overline{D}_{21}\overline{W}(t) + D_{22}\overline{u}(t) + v \end{aligned} \quad (19)$$

$$\bar{z} = G(\mathbf{P}, \bar{\mathbf{S}}) \bar{W} \quad (20)$$

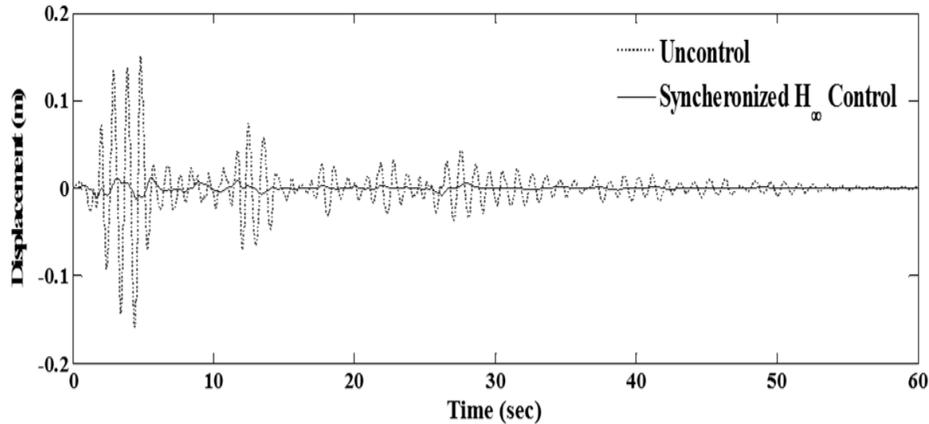


Figure 3: Closed loop system with unmodel dynamics and control sensors

The goal is to minimize the norm H_∞ with controller $\bar{\mathbf{S}}$ and Conversion function $G(\mathbf{P}, \bar{\mathbf{S}})$ is of \bar{W} to \bar{z} . Now, assuming $\|f(t)\| \leq \beta$, that β is the fixed amount, this function controller was achieved by using (15)-(20) and steps (i), (ii) and (iii) of the Riccati equation. This then will give the designed robust control synchronization for external disturbances and dynamic random process.

7 N-Story structure model

In this context, Kajima Shizuoka's structure, a five-story building, is considered. Acceleration sensors are installed in the first, third and fifth floors. It has semi-active hydraulic operators. The first story height is $4.2m$ and height of each of the remaining stories is $3.6m$ (Figure 4). The motion equation of this building can be presented by (6) that is a second order differential equation.

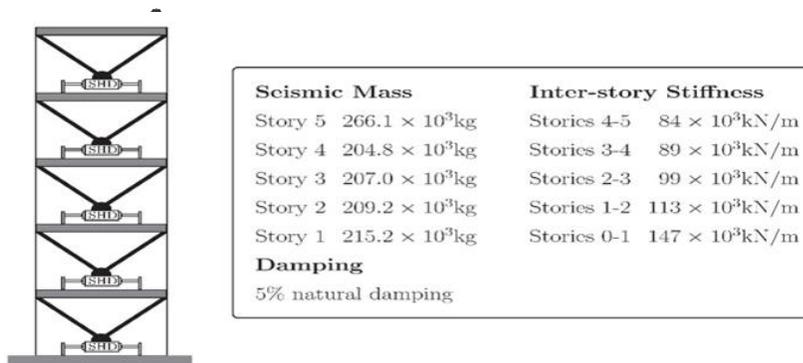


Figure 4: A five-story structure model similar to Kajima Shizuoka building [9, 10]

In this case, $q(t) \in \mathbb{R}^5$ is displacement vector, $u(t) \in \mathbb{R}^5$ is control force vector, and $w(t) \in \mathbb{R}$ is disturbance vector (caused by earthquakes or wind load). B_0 and T are control force place and external disturbance place-matrices respectively, M , B_0 , T , C and K are defined as follows:

$$\begin{aligned}
 M = 10^3 \times & \begin{bmatrix} 215.2 & 0 & 0 & 0 & 0 \\ 0 & 209.2 & 0 & 0 & 0 \\ 0 & 0 & 207.0 & 0 & 0 \\ 0 & 0 & 0 & 204.8 & 0 \\ 0 & 0 & 0 & 0 & 266.1 \end{bmatrix}, & B_0 = & \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
 T = -M & \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, & C = 10^3 \times & \begin{bmatrix} 650.4 & -231.1 & 0 & 0 & 0 \\ -231.1 & 548.9 & -202.5 & 0 & 0 \\ 0 & -202.5 & 498.6 & -182.0 & 0 \\ 0 & 0 & -182.0 & 466.7 & -171.8 \\ 0 & 0 & 0 & -171.8 & 318.5 \end{bmatrix} \\
 K = 10^6 \times & \begin{bmatrix} 260 & -113 & 0 & 0 & 0 \\ -113 & 212 & -99 & 0 & 0 \\ 0 & -99 & 188 & -89 & 0 \\ 0 & 0 & -89 & 173 & -84 \\ 0 & 0 & 0 & -84 & 84 \end{bmatrix}
 \end{aligned} \tag{21}$$

The natural damping is considered as 5% and the natural frequencies of the structure are as follows:

$$\omega_i = \{42.5423, 36.4257, 28.2253, 17.7417, 6.3343\} \text{ rad/sec}, \tag{22}$$

where the mass in kilograms (kg), damping coefficient in Ns/m and roughness factor is N/m . In order to compute control forces in the modeling, only acceleration of first, third and fifth story are used as the partial-state feedback.

To apply H_∞ controller in the form of a synchronizer control in this construction model, it is necessary to determine errors of state variables from expected values and introduce it in evaluation outputs.

Mathematical design of a hybrid model consisting and synchronize control aims to regulate both displacements and drifts between adjacent floors. The above structure is modelled in MATLAB software in a stimulated environment for various earthquakes (Figure 5). In this model, first of all, the record of earthquake $W(t)$ was imported into the model and then, vibrating responses of structures transferred to the controller by accelerating sensors, $y(t)$. Eventually, controller forces appropriate controlling force $u(t)$ to the structure which causes exorbitant vibrations in the structures.

Furthermore, in the system of (19), it has been supposed that the random dynamic process is defined by an unknown input function:

$$f(t) = 0.02\sin(10t), \quad B_f = [1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 0]^T \tag{23}$$

D_{1f} and D_{2f} similarly adjusted with the defined outputs; then, by forming, B_1 , D_{11} and D_{21} and given this random dynamic process, a robust H_∞ controller is designed.

8 Numerical simulation

To assess the designed control algorithm, the model of the structure was analyzed under El-Centro (1940) in two states of with and without H_∞ controller. The results of anal-

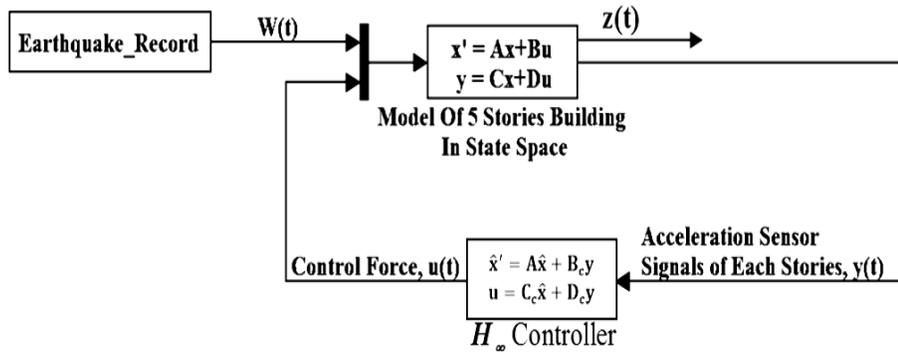


Figure 5: Simulated closed loop model by MATLAB package

ysis for output regulation and synchronizing in historical diagrams of structure response are illustrated. Figures 6-8 show displacement output, acceleration, relative displacement (drift) of the 5th floor of above structure in two, synchronized control H_∞ and uncontrolled state under El-Centro earthquake.

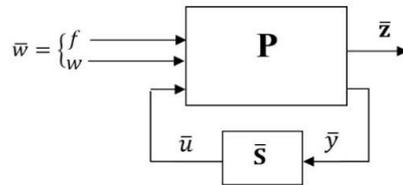


Figure 6: Time history graphs of displacement for the fifth floor, for uncontrolled and synchronized H_∞ control algorithm in El-Centro earthquake (1940)

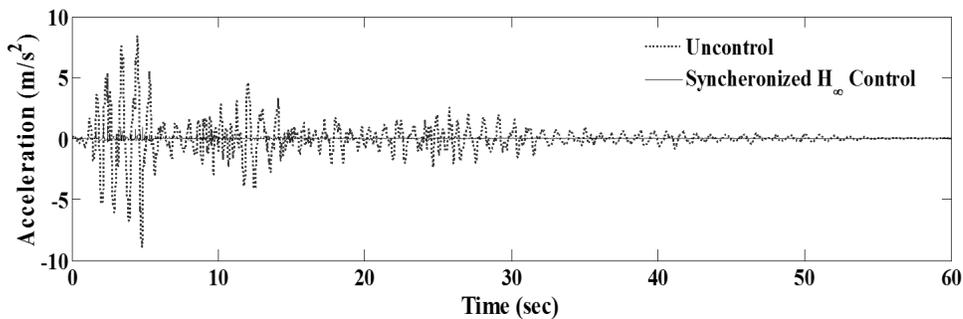


Figure 7: Time history graphs of acceleration for the fifth floor for uncontrolled and synchronized H_∞ control algorithm in El-Centro earthquake (1940)

According to Figures 6-8, it is clear that the proposed control algorithm is well able to prevent excessive vibrations of structure and this reduces the internal forces of structure and also structural damage to the building during an earthquake. On the other hand, the objective of the synchronizer control in addition to output regulation of structure

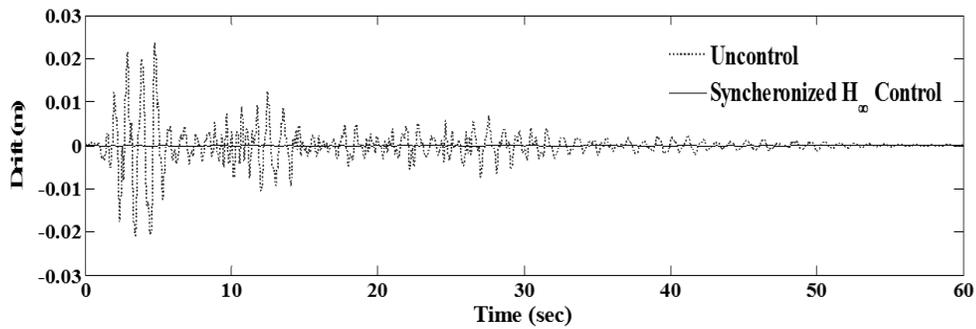


Figure 8: Time history graphs of drift for the fifth floor for uncontrolled and synchronized H_∞ control algorithm in El-Centro earthquake (1940)

are provided by converging \mathbf{e}, E to zero, while $t \rightarrow \infty$ in Figures 6-8 under the El-Centro earthquake. In Figures 9 and 10, the relative acceleration and displacement of the 5th floor which is modelled by two controllers, robust synchronized H_∞ controller and H_∞ controller are shown. As it is seen in Figure 9 and 10, robust synchronization H_∞ controller could totally adjust the unknown dynamic effect. While, in response to the control system with synchronized H_∞ controller, the effect of a random dynamic process could be totally seen. However, this effect is very small when compared to the response of the H_∞ control, but it can be eliminated by correct design of a control algorithm.

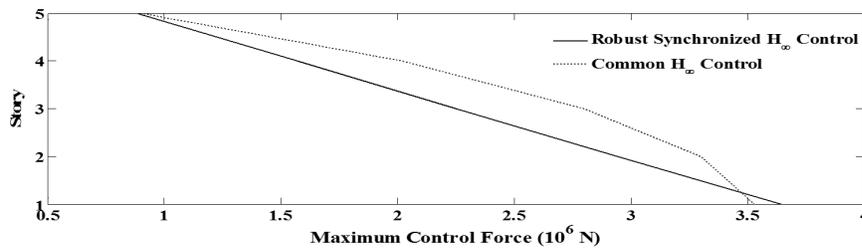


Figure 9: Time history graphs of acceleration for the fifth floor for H_∞ controller and robust synchronized H_∞ controller algorithm in El-Centro earthquake (1940)

To reap the benefits of controllers, the maximum relative deformation floor with robust synchronized H_∞ control, common H_∞ control algorithms and uncontrolled system are calculated. The results are shown in Figure 11. By using data in Figure 11, the maximum control force that is required for each floor with robust synchronization H_∞ control and common H_∞ control algorithms are depicted in Figure 12, on a scale from 417.3 to 1.

Figure 13 shows the superiority of robust synchronized H_∞ controller to the H_∞ control. Figure 12 shows maximum values of the controlling force for each floors, while Figure 11 shows maximum values of floors drift, this comparison shows that synchronization results in smaller regulated outputs (i.e. floors drift) without large changes in maximum control force of each floor, but as it can be seen in Figure 13 which is time history of applying the control force on the fifth floor (similar to other stories), synchronization changes the actuator behaviour when applying the control force, and these changes resulted in smoother applied force and less peaks of applied force for the robust

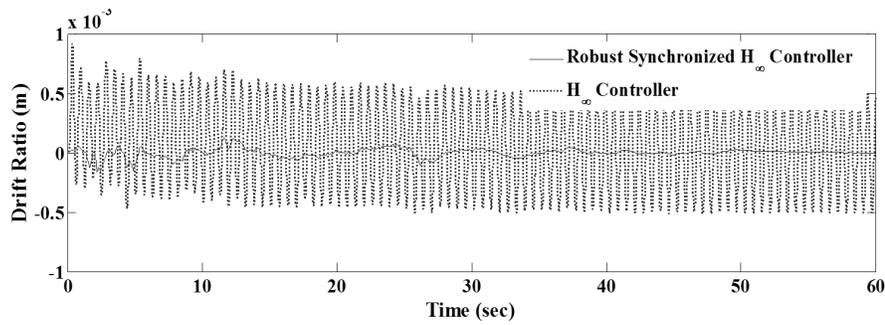


Figure 10: Time history graphs of drift for the fifth floor for H_∞ controller and robust synchronized H_∞ controller algorithm in El-Centro earthquake (1940)

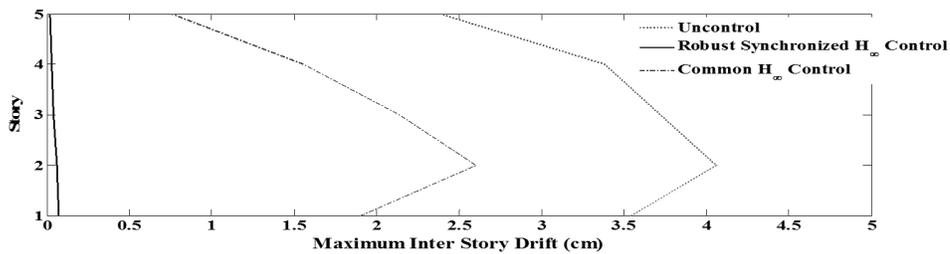


Figure 11: The maximum relative displacement floors with robust synchronization H_∞ control, common H_∞ control and uncontrol under EL-Centro earthquake

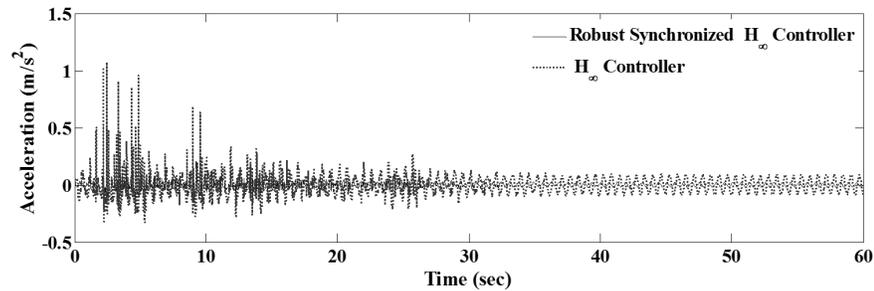


Figure 12: Maximum force for controlling floors in robust synchronization H_∞ control and common H_∞ control algorithms under EL-Centro earthquake

synchronized H_∞ control as compared to H_∞ control and this is the main difference between both strategies. This means that synchronization by eliminating the excessive effort of the actuators, results in smaller regulated outputs which satisfied the aims of synchronization.

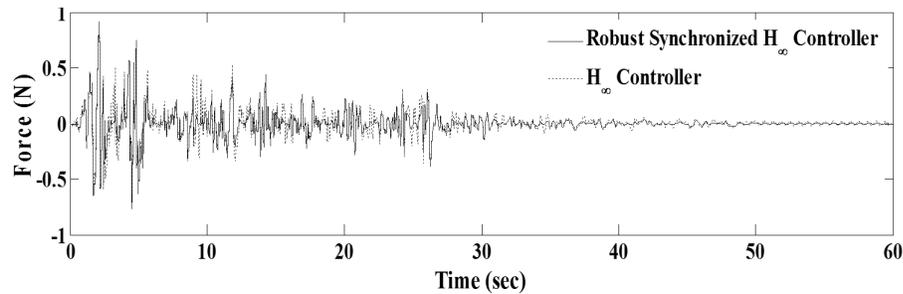


Figure 13: Compare time history of controlling force on the fifth floor with a robust synchronized H_∞ controller with H_∞ controller under EL-Centro earthquake

9 Conclusion

In this article, simulations for coordinating control of the drifts which happen in structures under earthquake were conducted. A synchronizer control algorithm with H_∞ controller for a five-story building was designed and its function were evaluated. Numerical computations that were set with the error of state variable and a synchronization error in the output were provided as objectives of the synchronizer control system. In addition, by regulating these two errors, the related drifts and acceleration values for each adjacent floor were reduced. Subsequently, maximum force of each floor and internal forces of structure also decreased. On the other hand, H_∞ algorithm provided this possibility for the designer to maintain the stability of the system in desirable level despite of random dynamic forces in the system. Generally, it can be concluded that with proper synchronizing of output regulation, the structure could be judged safe against damage. Moreover financial and physical damages caused by earthquake could be prevented. Numerical results showed the superiority of the robust synchronized H_∞ control algorithm compare over the other algorithms.

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هماهنگ سازی الگوریتم کنترل مقاوم H_∞ برای سازه های چند طبقه تحت بار زلزله و نیروهای تصادفی

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چکیده

در این مقاله مفهوم کنترل هماهنگ سازی با ترکیبی از کنترل مقاوم H_∞ فرمول بندی ریاضی شده است. برای ارزیابی کنترل پاسخ لرزه ای در سازه های چند طبقه، با گذشت زمان هماهنگ سازی بین جابجایی نسبی در طبقات همجوار و جابجایی کلی طبقات محاسبه گردیده است. به منظور بررسی صحت الگوریتم جدید، یک سازه پنج طبقه تحت زلزله السنترو (۱۹۴۰) مورد ارزیابی قرار گرفته است. برای بدست آوردن عملگر الگوریتم جدید تحت دینامیک تصادفی با وجود عدم قطعیت، فرآیند فوق با استفاده از معادله ریکاتی حل گردیده است. برای زمان های مختلف، بیشینه تغییر مکان و نیروهای کنترلی طبقات مورد ارزیابی قرار گرفته است. با استفاده از الگوریتم هماهنگ سازی مقاوم H_∞ ، با وجود فرآیند تصادفی و عدم قطعیت در سازه مورد نظر، پایداری به خوبی حفظ گردیده است. این تحقیق بیانگر برتری استفاده از هماهنگ ساز مقاوم H_∞ نسبت به دیگر الگوریتم های کنترلی می باشد.

کلمات کلیدی

هماهنگ سازی، فرآیند تصادفی و عدم قطعیت، کنترل مقاوم H_∞ ، رکورد زلزله السنترو، معادله ریکاتی.