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Global Asymptotic and Exponential Stability of Tri-Cell Networks with Different Time Delays

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Abstract. In this paper, a bidirectional ring network with three cells and different time delays is presented. To propose this model which is a good extension of three-unit neural networks, coupled cell network theory and neural network theory are applied. In this model, every cell has self-connections without delay but different time delays are assumed in other connections. A suitable Lyapunov function is presented for this model which helps to get sufficient conditions to guarantee asymptotic and exponential stability of the model. Also, these conditions are independent of time delays. Finally, analytical results are confirmed by numerical examples which are stated.

Keywords. Asymptotic stability, Exponential stability, Nonlinear systems, Cell network.

MSC. 93D20; 93D99; 93C10; 92B99.

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1 Introduction

Ordinary differential equations systems are very useful to model natural phenomena in many areas for many years. Also, time delay is a widespread phenomenon which appears in the most models. Therefore, the study of delay differential equations systems is important in scientific and practical problems, for example in investigating delay neural networks, [9, 10, 16, 27, 29] and references therein.

Artificial neural network (ANN) as an information processing system is introduced by Hopfield in 1984. Then, Marcus and Westervelt proposed a neural network with time delay in 1989. These delays are due to the finite switching speed of amplifiers in electronic neural networks, or due to finite signal propagation time in biological networks. During the last 30 years, many kinds of delayed neural networks have been used in different fields, such as associative memory [9, 25, 26], pattern recognition [13], optimization [1], signal processing [2] and so on. Furthermore, study on the dynamics of delayed neural networks has attracted the attention of many researchers, [1, 2, 9, 10, 11, 12, 13, 14, 15, 16, 18, 25, 26, 27, 28, 29].

Delay neural networks are large-scale nonlinear dynamical systems with complex behaviors, such that their dynamics are richer and more complicated than neural networks without delay. In 1987, because of exhaustive analysis of large-scale delayed neural networks, Bobcock and Westervelt suggested studying carefully the simple neural networks from the viewpoint of dynamical behaviors such as stability, periodic solutions and etc. Then, they may be carried these results on (over to) large systems. Furthermore, we should be mentioned that among neural networks, ring networks have been found widely in some structures such as neocortex, chemistry, electrical engineering. In fact, these systems are discussed to understand the behaviors of recurrent networks.

One of works on ring networks is the study of Wei and Li [20] which they considered a ring with three neurons and studied the global existence of periodic solutions in the network. They presented the following system with three different time delays

$$\begin{cases} \dot{x}_1(t) = -\mu x_1(t) + a_1 \tanh(x_3(t-\tau_1)), \\ \dot{x}_2(t) = -\mu x_2(t) + a_2 \tanh(x_1(t-\tau_2)), \\ \dot{x}_3(t) = -\mu x_3(t) + a_3 \tanh(x_2(t-\tau_3)). \end{cases} \tag{1}$$

Although, the activation function in this model was not in the general form, they used the function "tanh", and every neuron had only a connection with one of other neurons but the existence of different time delays was the advantage of this model.

After that, Wei and Velarde [21] developed the model of [20] as follows

$$\begin{cases} \dot{x}_1(t) = -\mu_1 x_1(t) + f_1(x_3(t-\tau_3)), \\ \dot{x}_2(t) = -\mu_2 x_2(t) + f_2(x_1(t-\tau_1)), \\ \dot{x}_3(t) = -\mu_3 x_3(t) + f_3(x_2(t-\tau_2)). \end{cases} \tag{2}$$

In this model, the activation functions have been considered in general form and the internal decay rates were assumed different. Wei and Velarde [21] investigated only the influence of different time delays on the stability and bifurcation of system (2). Notice that the connections between neurons in system (2) were not bidirectional.

After that, Song et al. [17] considered the following bidirectional associative memory neural network

$$\begin{cases} \dot{x}_1(t) = -\mu_1 x_1(t) + c_{21} f_1(x_2(t-\tau_2)) + c_{31} f_1(x_3(t-\tau_2)), \\ \dot{x}_2(t) = -\mu_2 x_2(t) + c_{12} f_2(x_1(t-\tau_1)), \\ \dot{x}_3(t) = -\mu_3 x_3(t) + c_{13} f_3(x_1(t-\tau_1)), \end{cases} \tag{3}$$

where f_k's denoted the activation functions with general form and the real constants $c_{l1}(l = 2, 3)$ and $c_{1k}(k = 2, 3)$ denoted the connected weights through the neurons in two layers. This system was not a ring network since there was not any connection between x_2 and x_3 but the study of this model helps us to propose our model. One year later, Yan [22] considered a delayed tri-neuron network by extending of system (3),

$$\begin{cases} \dot{x}_1(t) = -\mu x_1(t) + f(x_1(t)) + f_{12}(x_2(t-\tau)) + f_{13}(x_3(t-\tau)), \\ \dot{x}_2(t) = -\mu x_2(t) + f(x_2(t)) + f_{21}(x_1(t-\tau)) + f_{23}(x_3(t-\tau)), \\ \dot{x}_3(t) = -\mu x_3(t) + f(x_3(t)) + f_{31}(x_1(t-\tau)) + f_{32}(x_2(t-\tau)), \end{cases} \tag{4}$$

where f_{ij}'s were activation functions in general form and μ was same μ_k's in systems (2) and (3). It should be mentioned that this network is a bidirectional ring network with different activation functions in general form but the time delay between connections are the same. The positive point of this model was that they assumed self-connections in their model which had not been investigated before that time. The model of Yan was similar to the model which we would like to propose, but we interested in studying a model with different time delays, internal decay rates and connected weights. Finally, we studied the work of Zou et al. [29] which modeled a bidirectional three-unit ring network by the following system of delay differential equations in a parameter space consisting of two different delays,

$$\begin{cases} \dot{x}_1(t) = -\mu x_1(t) + a\tanh(x_2(t-\tau_2)) + a\tanh(x_3(t-\tau_1)), \\ \dot{x}_2(t) = -\mu x_2(t) + a\tanh(x_3(t-\tau_2)) + a\tanh(x_1(t-\tau_1)), \\ \dot{x}_3(t) = -\mu x_3(t) + a\tanh(x_1(t-\tau_2)) + a\tanh(x_2(t-\tau_1)), \end{cases} \tag{5}$$

where $a(\neq 0)$ is the connection strength, and $\tanh(x)$ is the activation function.

In this paper, we combine the idea of models (4) and (5), then we can extend them to a general ring network for three neurons. In fact, these studies motivate us to propose a bidirectional neural network with self connection and arbitrary activation functions. Moreover, it should be noted that none of them did not study the asymptotically stability and exponential stability of equilibrium of their models. Therefore, the aim of our work is to investigate asymptotically stability and exponential stability in our network. For proposing our model, we use coupled cell network theory as a new theory which is formalized in [4, 5, 6, 7, 19]. In this theory, every system (neuron or other systems) is called a cell. In fact, a coupled cell network is a directed graph with vertices as cells and directed edges as connections between cells. We should be point out that there are not many studies on coupled cell systems with discrete delays. Because of the generality of coupled cell network theory, we are interested in studying these systems. In [3], we study the dynamics of two-cell systems with discrete delays. In this study, we extend the above models by applying the coupled cell network theory. Indeed, we present *a new tri-cell network with bidirectional connections and self-connections with different delays in transferring data between cells*. In our network, each cell can be assumed as a neuron or another thing (for example in signaling networks; protein, gene or etc.). We would like to investigate global asymptotic and exponential stability of this tri-cell network.

We develop the above neural networks as follows by using some ideas of [14]

$$\dot{y} = -Dy + If(y) + Bg(y(t-\tau_1)) + Ch(y(t-\tau_2)) \tag{6}$$

such that $y(t) = (y_1, y_2, y_3)^T$, $D = diag\{d_1, d_2, d_3\}$ with $d_i > 0,\ i = 1, 2, 3$, and

$$B := \begin{bmatrix} 0 & 0 & b_{13} \\ b_{21} & 0 & 0 \\ 0 & b_{32} & 0 \end{bmatrix}, \quad C := \begin{bmatrix} 0 & c_{12} & 0 \\ 0 & 0 & c_{23} \\ c_{31} & 0 & 0 \end{bmatrix}, \quad f(y) = \begin{pmatrix} f_1(y_1) \\ f_2(y_2) \\ f_3(y_3) \end{pmatrix},$$

$$g(y(t-\tau_1)) = \begin{pmatrix} g_1(y(t-\tau_1)) \\ g_2(y(t-\tau_1)) \\ g_3(y(t-\tau_1)) \end{pmatrix}, \quad h(y(t-\tau_2)) = \begin{pmatrix} h_1(y(t-\tau_2)) \\ h_2(y(t-\tau_2)) \\ h_3(y(t-\tau_2)) \end{pmatrix},$$

where f is assumed a second order function, $f(0) = \frac{\partial f}{\partial y}(0) = 0$ and $g(0) = h(0) = 0$. Therefore, origin is an equilibrium point of (6). Our model is bidirectional with two loops where one direction is with time delay τ_1 and the other direction is with different time delay τ_2. Also, our network is with instantaneous self-connection and arbitrary activation C^1-functions. We would like to mention that to facilitate the design of neural networks, it is necessary and important that the neural networks with general activation functions are studied.

In this study, our main aim is to obtain sufficient conditions that guarantee global asymptotic and exponential stability of system (6). To the best of our knowledge, there has not been any work so far considering the globally asymptotic and exponential stability of system (6) which is very important in theories and applications and also is a very challenging problem. Motivated by the above considerations, in the next section, we will study the effect of the time delay parameter on asymptotic and exponential stability of system (6) under weak conditions which are independent of time delays. Finally, we show that our results agree with numerical simulations.

2 Main Results

Consider system (6) with the network which is illustrated in Figure 1.

In the process of stability analysis of origin as an equilibrium point in system (6), we use the following notations

(H_1)

$$\begin{aligned} y &= \eta_1, & f(y) &= \eta_2, & g(y(t)) &= \eta_3, \\ g(y(t-\tau_1)) &= \eta_4, & h(y(t)) &= \eta_5, & h(y(t-\tau_2)) &= \eta_6. \end{aligned}$$

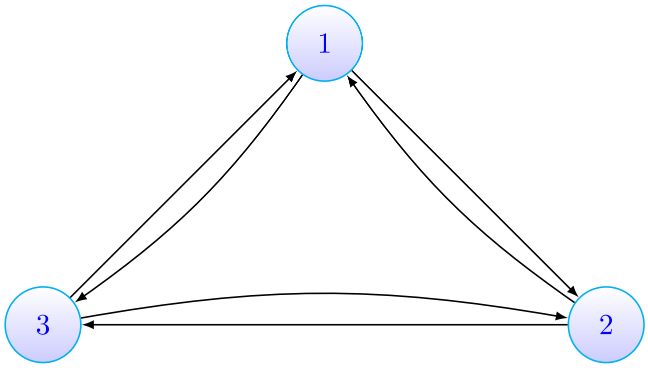

Figure 1: Bidirectional tri- cell network

where $\eta_i \in R^3$, $i = 1, \cdots, 6$.
Also, we assume that

(H_2) for given $\epsilon > 0$, if $s \in N_\epsilon(0)$ then $D^+ f_j(s) > 0, \quad j = 1, 2, 3,$

where D^+ shows the upper right Dini derivative and

$$D^+ f_j(s) = \limsup_{\triangle t \to 0^+} \frac{f_j(s + \triangle t) - f_j(s)}{\triangle t}.$$

(H_3) functions g and h are decreasing on intervals $[t - \tau_1, t]$ and $[t - \tau_2, t]$, respectively.

(H_4) $\frac{\eta_{2i}}{\eta_{1i}} > \frac{1}{2}$, for $i = 1, 2, 3$.

In this system, we would like to obtain sufficient conditions of global asymptotic stability and exponential stability of origin. For this aim, we define the following function

$$V(t) = 2\sum_{i=1}^{3} \int_0^{y_i(t)} f_i(s)ds + \sum_{i=1}^{3} \int_{t-\tau_1}^{t} g_i^2(y(s))ds + \sum_{i=1}^{3} \int_{t-\tau_2}^{t} h_i^2(y(s))ds. \tag{7}$$

In the following lemma, we show that $V(t)$ has the properties of Lyapunov function.

Lemma 1. Consider (7), thus

1. $V(t) \geq 0$ for all t,
2. $V(t) = 0$ for a special t if and only if $y(t) = 0$.

Proof. At first, we prove that $V(t) \geq 0$, for all $t \in R$. It is clear that $g_i^2(y(s)) \geq 0$, $h_i^2(y(s)) \geq 0$. We know that if $f \geq g$, then $\int_a^b f(s)ds \geq \int_a^b g(s)ds$ provided that $a \leq b$. Moreover, $t - \tau_1 < t$, $t - \tau_2 < t$ and thus $\sum_{i=1}^{3} \int_{t-\tau_1}^{t} g_i^2(y(s))ds \geq 0$, $\sum_{i=1}^{3} \int_{t-\tau_2}^{t} h_i^2(y(s))ds \geq 0$. It is thus enough to show that

$$2\sum_{i=1}^{3} \int_0^{y_i(t)} f_i(s)ds \geq 0. \tag{8}$$

According to (H_2), f_j's, $j = 1, 2, 3$, are increasing for $s \in N_\epsilon(0)$:

$$D^+ f_j(s) > 0 \Rightarrow \limsup_{\triangle t \to 0^+} \frac{f_j(s + \triangle t) - f_j(s)}{\triangle t} > 0 \Rightarrow f_j(s + \triangle t) > f_j(s). \tag{9}$$

Therefore, for every $s > 0$, we conclude that $f_j(s) > f_j(0) = 0$. In addition, we know that $y_i(t) \geq 0$ and then $\int_0^{y_i(t)} f_i(s)ds \geq 0$. This part of proof is complete.
It is not hard to show that if $y(t) = 0$ for a special t , then $V(t) = 0$. The origin is an equilibrium point of system (6), so $g(0) = h(0) = 0$. Also $\int_a^a f_i(s)ds = 0$. Hence, if $y(t) = 0$ then

$$V(t) = 2\sum_{i=1}^{3}\int_0^0 f_i(s)ds + \sum_{i=1}^{3}\int_{t-\tau_1}^{t} g_i^2(0)ds + \sum_{i=1}^{3}\int_{t-\tau_2}^{t} h_i{}^2(0)ds = 0. \tag{10}$$

□

Now, by aid of this Lyapunov function we can investigate globally asymptotically stability of origin.

Theorem 1. The Origin, as an equilibrium point, at system (6) is globally asymptotically stable if $\eta_2^T(\eta_2 + B\eta_4 + C\eta_6) < 0$.

Proof. By differentiating of both sides of (7) and using the notation(H_1), we have

$$\begin{aligned}
\dot{V}(t) &= 2\sum_{i=1}^{3} f_i(y_i(t))\dot{y}_i(t) + \sum_{i=1}^{3}(g_i^2(y(t)) - g_i^2(y(t-\tau_1))) + \sum_{i=1}^{3}(h_i^2(y(t)) - h_i^2(y(t-\tau_2))) \\
&= 2f^T(y(t))\dot{y}(t) + g^T(y(t))g(y(t)) - g^T(y(t-\tau_1))g(y(t-\tau_1)) \\
&\quad + h^T(y(t))h(y(t)) - h^T(y(t-\tau_2))h(y(t-\tau_2)) \\
&= 2\eta_2{}^T(-Dy + If(y) + Bg(y(t-\tau_1)) + Ch(y(t-\tau_2))) + \eta_3{}^T\eta_3 \\
&\quad - \eta_4{}^T\eta_4 + \eta_5{}^T\eta_5 - \eta_6{}^T\eta_6 \\
&= 2\eta_2{}^T(-D\eta_1 + I\eta_2 + B\eta_4 + C\eta_6) + \eta_3{}^T\eta_3 - \eta_4{}^T\eta_4 + \eta_5{}^T\eta_5 - \eta_6{}^T\eta_6 \\
&= -2\eta_2{}^T D\eta_1 + 2\eta_2{}^T(\eta_2 + B\eta_4 + C\eta_6) + \eta_3{}^T\eta_3 - \eta_4{}^T\eta_4 + \eta_5{}^T\eta_5 - \eta_6{}^T\eta_6.
\end{aligned}$$

According to condition (H_3), functions g and h are decreasing. Thus

$$\begin{cases} \eta_3{}^T\eta_3 - \eta_4{}^T\eta_4 \leq 0, \\ \eta_5{}^T\eta_5 - \eta_6{}^T\eta_6 \leq 0. \end{cases}$$

Indeed, it suffices $\eta_2^T(\eta_2 + B\eta_4 + C\eta_6) < 0$ then $\dot{V}(t) < 0$, since $d_i > 0$ for $i = 1, 2, 3$. □

In the classic analysis, the existence of a positive Lyapunov function helps us to investigate exponential stability of a system. Now, we claim that system (6) at origin is exponential stability. We prove it as follows by using Lyapunov function (7).

Theorem 2. The origin, as an equilibrium point, at system (6) is exponential stable if $\eta_2^T(\eta_2 + B\eta_4 + C\eta_6) < 0$ and $d_i > 1$, $i = 1, 2, 3$.

Proof. According to condition (H_3), continuous functions g_i's, h_i's are decreasing on $[t-\tau_1, t]$ and $[t-\tau_2, t]$. Hence, we can define the following terms for $i = 1, 2, 3$

$$M^*_{g_i} := \max_{s\in[t-\tau_1,t]} |g_i(y(s))|, \quad m^*_{g_i} := \min_{s\in[t-\tau_1,t]} |g_i(y(s))|$$
$$M^*_{h_i} := \max_{s\in[t-\tau_2,t]} |h_i(y(s))|, \quad m^*_{h_i} := \min_{s\in[t-\tau_2,t]} |h_i(y(s))|.$$

Thus

$$\begin{aligned}\int_{t-\tau_1}^{t} g_i^2(y(s))ds &\le M^{*2}_{g_i}\tau_1\\ &\le \frac{M^{*2}_{g_i}}{m^{*2}_{g_i}} {g_i}^2(y(t))\tau_1,\\ \text{or } &\le \frac{M^{*2}_{g_i}}{m^{*2}_{g_i}} {g_i}^2(y(t-\tau_1))\tau_1,\end{aligned}$$

and

$$\begin{aligned}\int_{t-\tau_2}^{t} h_i^2(y(s))ds &\le M^{*2}_{h_i}\tau_2\\ &\le \frac{M^{*2}_{h_i}}{m^{*2}_{h_i}} {h_i}^2(y(t))\tau_2,\\ \text{or } &\le \frac{M^{*2}_{h_i}}{m^{*2}_{h_i}} {h_i}^2(y(t-\tau_2))\tau_2.\end{aligned}$$

As a result

$$\begin{aligned}\sum_{i=1}^{3}\int_{t-\tau_1}^{t} g_i^2(y(s))ds &\le \sum_{i=1}^{3}\frac{M^{*2}_{g_i}}{m^{*2}_{g_i}} {g_i}^2(y(t))\tau_1\\ &\le \beta_g \eta_3^T \eta_3 \tau_1\\ \text{or } &\le \beta_g \eta_4^T \eta_4 \tau_1\end{aligned}$$

and

$$\begin{aligned}\sum_{i=1}^{3}\int_{t-\tau_2}^{t} h_i^2(y(s))ds &\le \sum_{i=1}^{3}\frac{M^{*2}_{h_i}}{m^{*2}_{h_i}} {h_i}^2(y(t))\tau_2\\ &\le \beta_h \eta_5^T \eta_5 \tau_2\\ \text{or } &\le \beta_h \eta_6^T \eta_6 \tau_2\end{aligned}$$

where $\beta_g = \max_i\{\frac{M^{*2}_{g_i}}{m^{*2}_{g_i}}\}$, $\beta_h = \max_i\{\frac{M^{*2}_{h_i}}{m^{*2}_{h_i}}\}$. Also, according to the condition (H_2), the function f_i is increasing on the interval $[0, y_i(t)]$ and $\max_{s\in[0,y_i(t)]} f_i(s) = f_i(y_i(t))$. It is easy to see that $\int_0^{y_i(t)} f_i(s)ds \le f_i(y_i(t))y_i(t)$. By substituting the above equations in the Lyapunov function $V(t)$, Eq. (7), we have

$$
\begin{aligned}
V(t) &= 2\sum_{i=1}^{3}\int_{0}^{y_i(t)} f_i(s)ds + \sum_{i=1}^{3}\int_{t-\tau_1}^{t} {g_i}^2(y(s))ds + \sum_{i=1}^{3}\int_{t-\tau_2}^{t} {h_i}^2(y(s))ds \\
&\leq 2\sum_{i=1}^{3} f_i(y_i(t))y_i(t) + \beta_g \eta_3^T\eta_3\tau_1 + \beta_h\eta_5^T\eta_5\tau_2 \\
&\leq 2\eta_2^T\eta_1 + \beta_g\eta_3^T\eta_3\tau_1 + \beta_h\eta_5^T\eta_5\tau_2.
\end{aligned}
$$

Also, it is not hard to compute

$$
\begin{aligned}
V(t) &\leq 2\sum_{i=1}^{3} f_i(y_i(t))y_i(t) + \beta_g\eta_4^T\eta_4\tau_1 + \beta_h\eta_6^T\eta_6\tau_2 \\
&\leq 2\eta_2^T\eta_1 + \beta_g\eta_4^T\eta_4\tau_1 + \beta_h\eta_6^T\eta_6\tau_2.
\end{aligned}
$$

Moreover,

$$
\dot{V}(t) \leq -2\eta_2^T D\eta_1 + 2\eta_2^T\eta_2 + 2\eta_2^T B\eta_4 + 2\eta_2^T C\eta_6 + \eta_3^T\eta_3 - \eta_4^T\eta_4 + \eta_5^T\eta_5 - \eta_6^T\eta_6.
$$

We know $D = diag\{d_i\}_{i=1}^3$. If $d_i > 1$ then $-2\eta_2^T D\eta_1 \leq -2\eta_2^T I\eta_1$. Furthermore, if

$$
\eta_2^T(\eta_2 + B\eta_4 + C\eta_6) < 0
$$

and condition (H_3) are satisfied, then

$$
\dot{V}(t) \leq -2\eta_2^T D\eta_1 \leq -2\eta_2^T I\eta_1. \tag{11}
$$

We assume that the specific t_0 such that $\eta_2^T\eta_1$ is sufficiently large for all $t > t_0$ and $\eta_5^T\eta_5 < \eta_2^T\eta_1$, $\eta_3^T\eta_3 < \eta_2^T\eta_1$. Hence $V(t) \leq (2 + \beta_g\tau_1 + \beta_h\tau_2)\eta_2^T\eta_1$. Since $\alpha = 2 + \beta_g\tau_1 + \beta_h\tau_2 > 0$, we have

$$
\frac{V(t)}{\alpha} \leq \eta_2^T\eta_1. \tag{12}
$$

By using (11) and (12) we have $\dot{V}(t) \leq \dfrac{-2}{\alpha}V(t)$, therefore

$$
\ln V(t) - \ln V(t_0) \leq \frac{-2}{\alpha}(t - t_0) \;\Rightarrow\; V(t) \leq e^{\frac{-2(t-t_0)}{\alpha}}V(t_0). \tag{13}
$$

By the definition of $V(t)$, we also have

$$
\begin{aligned}
V(t) &\geq 2\sum_{i=1}^{3}\int_{0}^{y_i(t)} f_i(s)dt \\
&\geq 2\sum_{i=1}^{3} f_i(y_i(t))y_i(t).
\end{aligned}
$$

Define $\xi := \frac{2f_i(y_i(t))}{y_i(t)}$, it is clear that $2f_i(y_i(t)) = \xi y_i(t)$ for $i = 1, 2, 3$. By using (H_4), $\xi > 1$. Therefore

$$
V(t) \geq \xi\sum_{i=1}^{3} y_i(t)y_i(t) \tag{14}
$$

$$
\geq \xi||y(t)||^2. \tag{15}
$$

Then, by applying (13) and (14), we have

$$||y(t)||^2 \leq (V(t_0)\xi^{-1})e^{\frac{-2}{\alpha}(t-t_0)} \tag{16}$$
$$\leq (V(t_0))e^{\frac{-2}{\alpha}(t-t_0)} \qquad \forall t \geq t_0 \tag{17}$$

and the proof of exponential stability of system (6) is complete. □

Theorems 1 and 2 show that global asymptotic and exponential stability of system (6) do not dependent on delays.

3 Numerical Simulation

Now, in order to confirm our theoretical results in the previous section, we consider the following tri-cell network with two time delays

$$\begin{cases} \dot{y}_1 = -6y_1 + (y_1 + 0.3y_1^3) - 2(y_3(t-\tau_1)) - 2(y_2(t-\tau_2)), \\ \dot{y}_2 = -7y_2 + (y_2 + 0.3y_2^3) - 3(y_1(t-\tau_1)) - 2(y_3(t-\tau_2)), \\ \dot{y}_3 = -7y_3 + (y_3 + 0.3y_3^3) - 2(y_2(t-\tau_1)) - 3(y_1(t-\tau_2)). \end{cases} \tag{18}$$

First, we consider (18) with $\tau_1 = 3$, $\tau_2 = 7$ and initial value $(1.5, 0.5, 2.5)$. In this case, the settling time is larger than 60. By Theorems 1 and 2, we expect global asymptotic and exponential stability of the origin, see figures 2, 3 and 4. Also, by these theorems, the stability of the origin is independent on delays. Second, we consider (18) with $\tau_1 = 0.3$, $\tau_2 = 0.7$ and initial value $(1.5, 0.5, 2.5)$, see figures 5, 6 and 7. Noted that the settling time is almost 4. In these two cases, solutions are stable but it should be noted that if time delays are large, then the settling times for solutions are so large. Thus, we change the constant value of C matrix and we check the settling time for the following system when $\tau_1 = 3$, $\tau_2 = 7$

$$\begin{cases} \dot{y}_1 = -6y_1 + (y_1 + 0.3y_1^3) - 2(y_3(t-\tau_1)) - 0.08(y_2(t-\tau_2)) \\ \dot{y}_2 = -7y_2 + (y_2 + 0.3y_2^3) - 3(y_1(t-\tau_1)) - 0.5(y_3(t-\tau_2)) \\ \dot{y}_3 = -7y_3 + (y_3 + 0.3y_3^3) - 2(y_2(t-\tau_1)) - 0.1(y_1(t-\tau_2)). \end{cases} \tag{19}$$

See figures 8, 9 and 10. The settling time is almost 15.

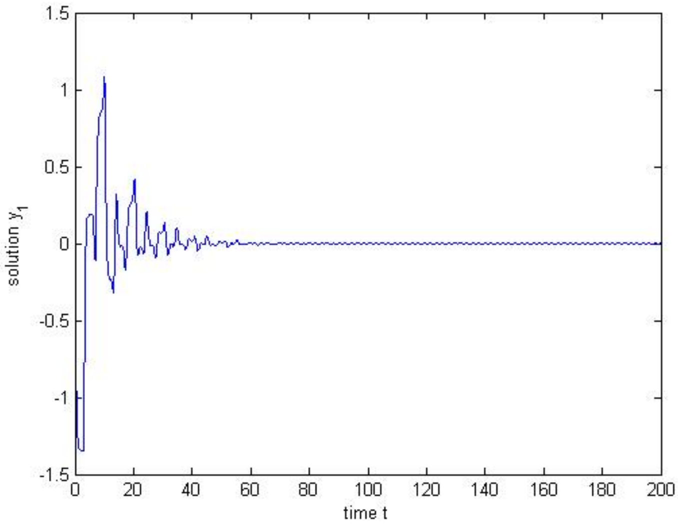

Figure 2: Solution y_1 of system 18 with $\tau_1 = 3$, $\tau_2 = 7$ and initial value $(1.5, 0.5, 2.5)$.

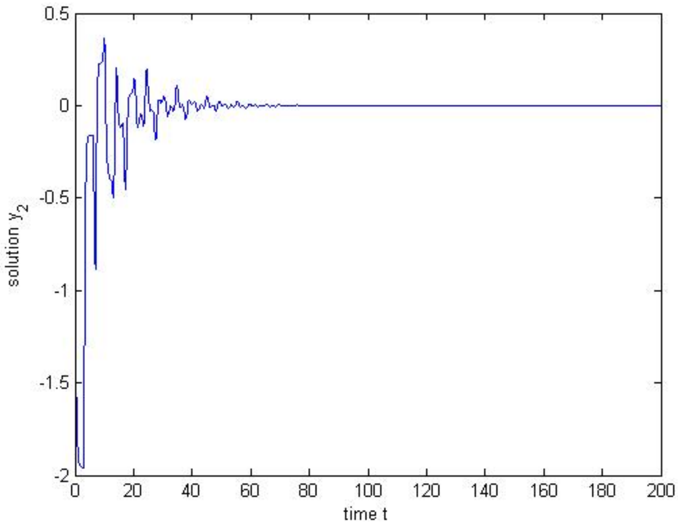

Figure 3: Solution y_2 of system 18 with $\tau_1 = 3$, $\tau_2 = 7$ and initial value $(1.5, 0.5, 2.5)$.

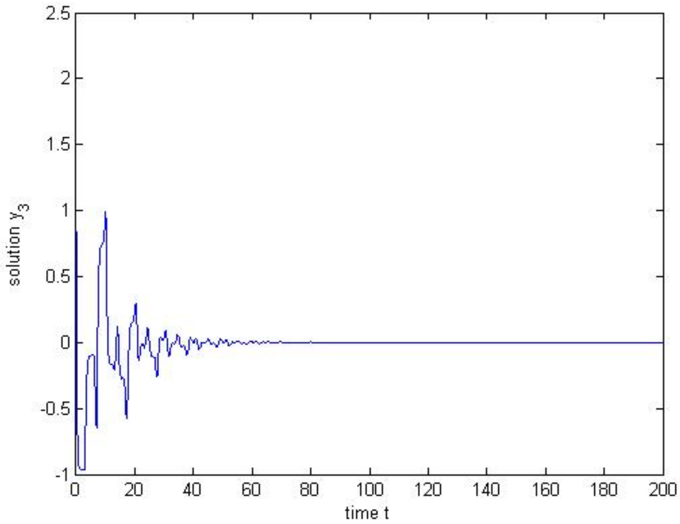

Figure 4: Solution y_3 of system 18 with $\tau_1 = 3$, $\tau_2 = 7$ and initial value $(1.5, 0.5, 2.5)$.

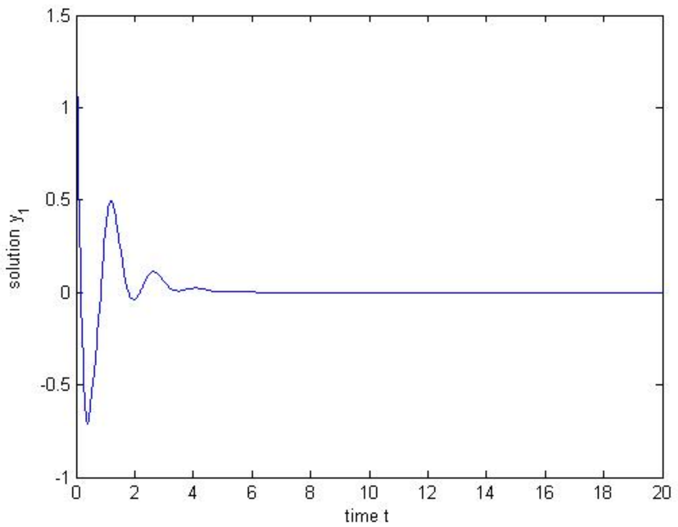

Figure 5: Solution y_1 of system 18 with$\tau_1 = 0.3$, $\tau_2 = 0.7$ and initial value $(1.5, 0.5, 2.5)$.

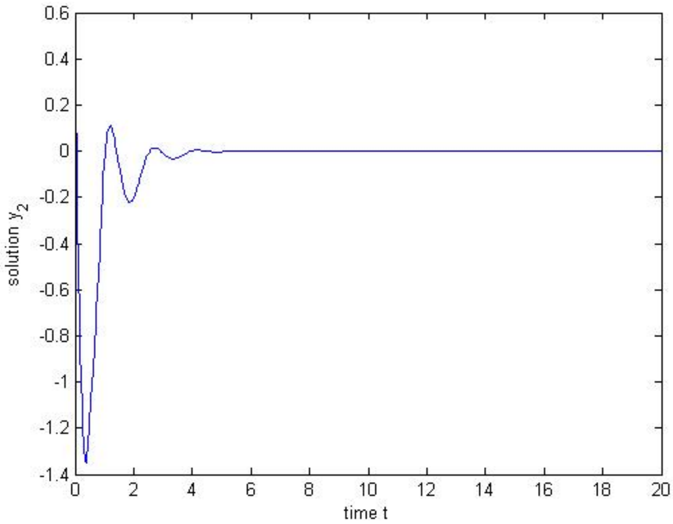

Figure 6: Solution y_2 of system 18 with$\tau_1 = 0.3$, $\tau_2 = 0.7$ and initial value $(1.5, 0.5, 2.5)$.

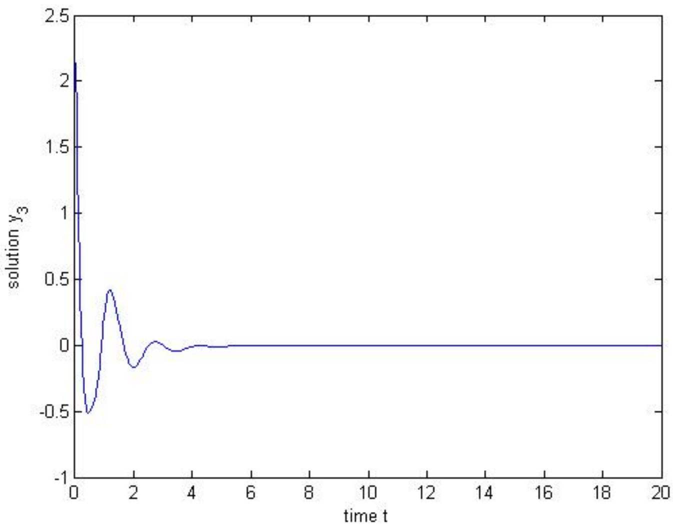

Figure 7: Solution y_3 of system 18 with$\tau_1 = 0.3$, $\tau_2 = 0.7$ and initial value $(1.5, 0.5, 2.5)$.

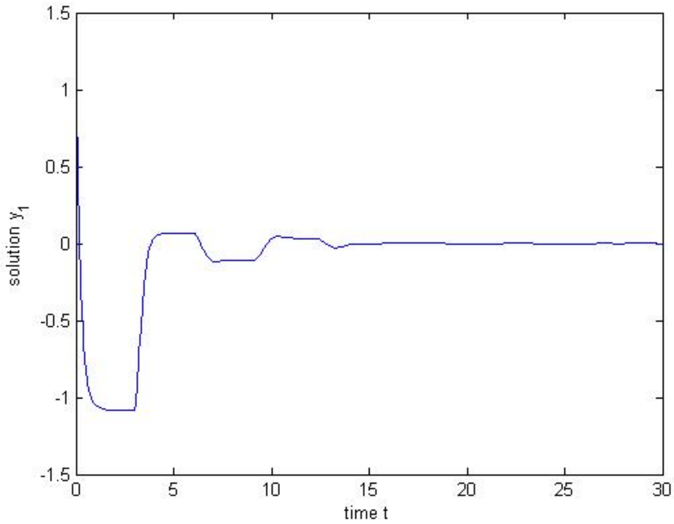

Figure 8: Solution y_1 of system 19 with $\tau_1 = 3$, $\tau_2 = 7$ and initial value $(1.5, 0.5, 2.5)$.

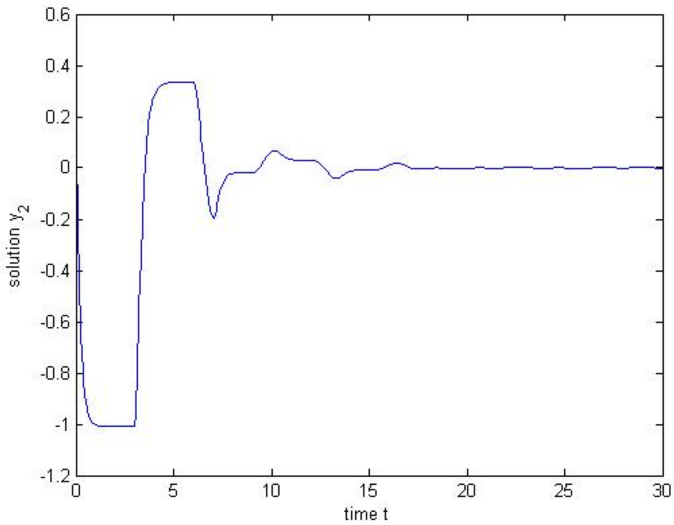

Figure 9: Solution y_2 of system 19 with $\tau_1 = 3$, $\tau_2 = 7$ and initial value $(1.5, 0.5, 2.5)$.

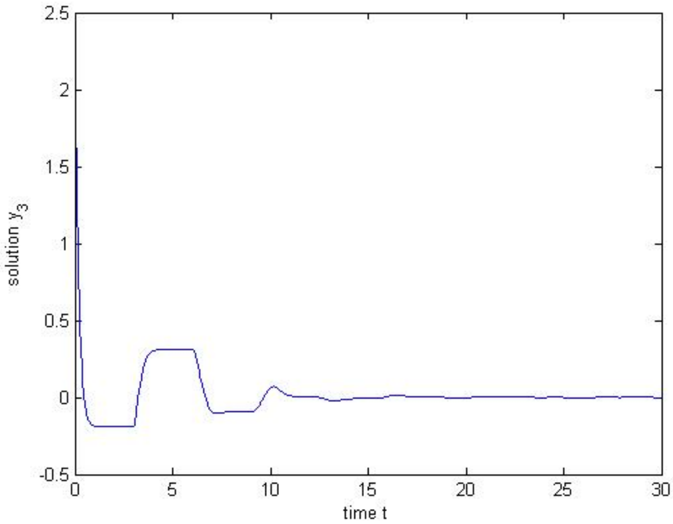

Figure 10: Solution y_3 of system 19 with $\tau_1 = 3$, $\tau_2 = 7$ and initial value $(1.5, 0.5, 2.5)$.

4 Conclusion

In this paper, we studied a tri-cell network with activation function may be neither bounded nor differentiable in connection with itself and activation functions in connection with other cells considered decreasing. We obtained independent conditions of delay for global asymptotic and exponential stability of the network. Moreover, we verified our analytical results by numerical examples.

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پایداری مجانبی و نمایی سراسری شبکه‌های سه سلولی با تاخیرهای زمانی متفارت

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چکیده

در این مقاله، یک شبکه حلقوی دوطرفه با ۳ سلول و تاخیرهای زمانی متفاوت ارایه شده است. برای ارایه این مدل که تعمیمی مناسب از شبکه‌های عصبی ۳ نرونی است، نظریه شبکه سلولی درگیر و شبکه عصبی اعمال می‌شود. در این مدل، هر سلول دارای خود اتصالی‌های بدون تاخیر است و بقیه اتصالات با تاخیر فرض شده است. یک تابع لیاپونف مناسب برای مدل ارائه شده که در کسب شرایط کافی برای تضمین پایداری نمایی و مجانبی مدل به ما کمک می‌کند. همچنین این شرایط مستقل از تاخیرهای زمانی است. سرانجام نتایج تحلیلی با مثال‌های عددی که بیان می‌شوند تایید شده‌اند.

کلمات کلیدی

پایداری مجانبی، پایداری نمایی، دستگاههای غیرخطی، شبکه سلولی.