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Universal Approximator Property of the Space of Hyperbolic Tangent Functions

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Abstract. In this paper, first the space of hyperbolic tangent functions is introduced and then the universal approximator property of this space is proved. In fact, by using this space, any nonlinear continuous function can be uniformly approximated with any degree of accuracy. Also, as an application, this space of functions is utilized to design feedback control for a nonlinear dynamical system.

Keywords. Hyperbolic tangent functions, Universal approximator, Stabilizer control.

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1 Introduction

In recent years, the concept of asymptotic stability has been highly considered in the nonlinear control theory. Several researchers have applied the control Lyapunov functions [11, 12], Smooth feedback controls [1, 6], continuous feedback controls [5, 7], receding horizon or moving horizon control [4, 10] and fuzzy control [2, 15] techniques to design controller. Moreover, Zhang et al [16, 20, 17, 19, 21] have presented an approach based on the fuzzy rule base to achieve a tangent hyperbolic system form of a control system. Some applications of this approach can be seen in [3, 9, 18, 22].
But, in [13] a different approach is presented and the tangent hyperbolic systems are given without use of fuzzy concepts and the model parameters are obtained by solving a nonlinear programming problem, while parameters identification in the works of Zhang et al is usually based on the neural networks.
In this paper, the universal approximator property of the space of hyperbolic tangent (HT) functions is proved. Also, as an application, this space is utilized to design feedback control for a nonlinear dynamical system.

2 The space of hyperbolic tangent functions

In this section,the space of HT functions is introduced. For any $x = (x_1, x_2, \cdots, x_n) \in \Omega \subset \mathbb{R}^n$, define the following generalized variables

$$\begin{cases} \overline{x}_{\alpha_i+k_i} = x_i - d_{\alpha_i+k_i}, \\ k_i = 1, 2, \cdots, w_i; \; i = 1, 2, \cdots, n, \end{cases} \tag{1}$$

where $\alpha_i = \sum_{j=0}^{i-1} w_j$, $w_0 = 0$, $m = \sum_{i=1}^{n} w_i$ and d_j for $j = 1, 2, \cdots, m$ are given constants. We define the space of TH functions as follows:

$$HT(\Omega) = \{f_m(x, a, b, k) = \sum_{i=1}^{m} \frac{a_i \; e^{k_i \overline{x}_i} \; + b_i \; e^{-k_i \overline{x}_i}}{e^{k_i \overline{x}_i} \; + \; e^{-k_i \overline{x}_i}} : \tag{2}$$
$$a_i \in \mathbb{R}, b_i \in \mathbb{R}, \; k_i \in \mathbb{R}, \; x \in \Omega, \; m \geq n\},$$

where $\overline{x}$ is defined by (1). Assume that

$$p_i = \frac{a_i + b_i}{2}, \; q_i = \frac{a_i - b_i}{2}, \quad i = 1, 2, \cdots, m. \tag{3}$$

By (3), the space of TH functions (2) can be written as follows:

$$HT(\Omega) = \{f_m(x, p, q, k) = p + q.tanh(k\overline{x}) : \; p \in \mathbb{R}, \; q \in \mathbb{R}^m, k \in \mathbb{R}^m, x \in \Omega, m \geq n\}, \tag{4}$$

where $p = \sum_{i=1}^{m} p_i$, $q = (q_1, q_2, \cdots, q_m)$, $k = (k_1, k_2, \cdots, k_m), \overline{x} = (\overline{x}_1, \cdots, \overline{x}_m)$ and

$$tanh(k\overline{x}) = (tanh(k_1\overline{x}_1), tanh(k_2\overline{x}_2), \cdots, tanh(k_m\overline{x}_m))^T.$$

If we assume that $b_i = -a_i$ and $d_j = 0$ for $i, j = 1, 2, \cdots, m$, then by (2), we achieve the following subspace of $HT(\Omega)$

$$HT_0(\Omega) = \{f_n(x, a, k) = \sum_{i=1}^{n} a_i(\frac{e^{k_i x_i} \ - \ e^{-k_i x_i}}{e^{k_i x_i} \ + e^{-k_i x_i}}) : \ a_i \in \mathbb{R}, k_i \in \mathbb{R}, \ i = 1, 2, \cdots, n\}.$$

Also, we can write it as follows

$$HT_0(\Omega) = \{f_n(x, q, k) = q \ tanh(kx) : \ q \in \mathbb{R}^n, \ k \in \mathbb{R}^n\},$$

where $q = (a_1, a_2, \cdots, a_n)$, $k = (k_1, k_2, \cdots, k_n)$ and $n = m$.

3 Universal approximator property

We will show that $HT(\Omega)$ is dense in the space of real continuous functions, where $\Omega \in \mathbb{R}^n$ is a compact set. In the other word, by applying space of HT functions, we can uniformly approximate any nonlinear continuous function with any degree of accuracy, i.e. the space of HT functions is a universal approximator.

Definition 1. Let $Z(\Omega)$ be a subspace of $C(\Omega)$. We say that $Z(\Omega)$ separates the points on Ω, if for every $x, y \in \Omega$ where $x \neq y$, there exists $f(.) \in Z(\Omega)$ such that $f(x) \neq f(y)$.

Definition 2. Let $Z(\Omega)$ be a subspace of $C(\Omega)$. The set of functions $Z(\Omega)$ is called an algebra if $Z(\Omega)$ is closed under addition, multiplication and scalar multiplication.

Definition 3. Let $Z(\Omega)$ be a subspace of $C(\Omega)$. We say that $Z(\Omega)$ vanishes at no point of Ω if for each $x \in \Omega$, there is $f(.) \in Z(\Omega)$ such that $f(x) \neq 0$.

Lemma 1. [14] (Stone-Weierstrass theorem): Let Ω be a compact set and $Z(\Omega)$ be a subspace of $C(\Omega)$. If Z is an algebra, separates on Ω and vanishes at no point of Ω, then the uniform closure of $Z(\Omega)$ consists of all real continuous functions on Ω; i.e., $Z(\Omega)$ is dense in $C(\Omega)$.

Theorem 1. For any given real continuous function $g(.)$ on the compact set $\Omega \subset \mathbb{R}^n$ and arbitrary $\epsilon > 0$, there exists an $f_m(., a, b, k) \in HT(\Omega)$ such that

$$\sup_{x \in \Omega} |g(x) - f_m(x, a, b, k)| < \epsilon.$$

Proof. By Lemma 1, it suffices to show that $HT(\Omega)$ is an algebra, separates points of Ω and vanishes at no point of Ω. At first, we show that $HT(\Omega)$ is algebra. Let $f_{m_1}(., a, b, k)$ and $f_{m_2}(., \hat{a}, \hat{b}, \hat{k}) \in HT(\Omega)$. We can write them as

$$f_{m_1}(x, a, b, k) = \sum_{i_1=1}^{m_1} (\frac{a_{i_1} \ e^{k_{i_1} \overline{x}_{i_1}} \ + \ b_{i_1} \ e^{-k_{i_1} \overline{x}_{i_1}}}{e^{k_{i_1} \overline{x}_{i_1}} \ + e^{-k_{i_1} \overline{x}_{i_1}}}),$$

and

$$f_{m_2}(x,\hat{a},\hat{b},\hat{k}) = \sum_{i_2=1}^{m_2}\left(\frac{\hat{a}_{i_2}\ e^{\hat{k}_{i_2}\overline{x}_{i_2}}\ +\ \hat{b}_{i_2}\ e^{-\hat{k}_{i_2}\overline{x}_{i_2}}}{e^{\hat{k}_{i_2}\overline{x}_{i_2}}\ +e^{-\hat{k}_{i_2}\overline{x}_{i_2}}}\right),$$

respectively. We have

$$f_{m_1}(x,a,b,k) + f_{m_2}(x,\hat{a},\hat{b},\hat{k}) = \sum_{z=1}^{m_1+m_2}\left(\frac{c_z\ e^{k'_z\overline{x}_z}\ +\ c'_z\ e^{-k'_z\overline{x}_z}}{e^{k'_z\overline{x}_z}\ +e^{-k'_z\overline{x}_z}}\right), \tag{5}$$

where

$$c = (a_1,\cdots,a_{m_1},\hat{a}_1,\cdots,\hat{a}_{m_2}),\quad c' = (b_1,\cdots,b_{m_1},\hat{b}_1,\cdots,\hat{b}_{m_2}),$$

and

$$k' = (k_1,\cdots,k_{m_1},\hat{k}_1,\cdots,\hat{k}_{m_2}).$$

Also, c_z, c'_z and k'_z are the components of vectors c, c' and k' respectively. It is easy to see that the function defined by (5) belongs to $HT(\Omega)$ i.e., $f_{m_1} + f_{m_2} \in HT(\Omega)$. By the same way, we can get

$$\begin{aligned}
&f_{m_1}(x,a,b,k) \times f_{m_2}(x,\hat{a},\hat{b},\hat{k}) = \\
&\sum_{i_1=1}^{m_1}\sum_{i_2=1}^{m_2}\left(\frac{(a^*_{i_1}+\hat{a}^*_{i_2})\ e^{k_{i_1}\overline{x}_{i_1}}e^{\hat{k}_{i_2}\overline{x}_{i_2}}\ +\ (a^*_{i_1}+\hat{b}^*_{i_2})e^{k_{i_1}\overline{x}_{i_1}}e^{-\hat{k}_{i_2}\overline{x}_{i_2}}}{(e^{k_{i_1}\overline{x}_{i_1}}\ +e^{-k_{i_1}\overline{x}_{i_1}})(e^{\hat{k}_{i_2}\overline{x}_{i_2}}\ +e^{-\hat{k}_{i_2}\overline{x}_{i_2}})}\right) \\
&+\sum_{i_1=1}^{m_1}\sum_{i_2=1}^{m_2}\left(\frac{(b^*_{i_1}+\hat{a}^*_{i_2})\ e^{-k_{i_1}\overline{x}_{i_1}}e^{k'_{i_2}\overline{x}_{i_2}}\ +\ (b^*_{i_1}+\hat{b}^*_{i_2})e^{-k_{i_1}\overline{x}_{i_1}}e^{-\hat{k}_{i_2}\overline{x}_{i_2}}}{(e^{k_{i_1}\overline{x}_{i_1}}\ +e^{-k_{i_1}\overline{x}_{i_1}})(e^{\hat{k}_{i_2}\overline{x}_{i_2}}\ +e^{-\hat{k}i_2\overline{x}_{i_2}})}\right) \\
&=\sum_{i_1=1}^{m_1}\sum_{i_2=1}^{m_2}\left(\frac{a^*_{i_1}\ e^{k_{i_1}\overline{x}_{i_1}}\ +\ b^*_{i_1}\ e^{-k_{i_1}\overline{x}_{i_1}}}{e^{k_{i_1}\overline{x}_{i_1}}\ +e^{-k_{i_1}\overline{x}_{i_1}}}+\frac{\hat{a}^*_{i_2}\ e^{\hat{k}_{i_2}\overline{x}_{i_2}}\ +\ \hat{b}^*_{i_2}\ e^{-\hat{k}_{i_2}\overline{x}_{i_2}}}{e^{\hat{k}_{i_2}\overline{x}_{i_2}}\ +e^{-\hat{k}_{i_2}\overline{x}_{i_2}}}\right) \\
&=\sum_{z=1}^{m_1+m_2}\left(\frac{c_z\ e^{k_z x_z}\ +\ c'_z\ e^{-k_z x_z}}{e^{k_z x_z}\ +e^{-k_z x_z}}\right),
\end{aligned} \tag{6}$$

where $a^*_i, b^*_i, \hat{b}^*_i$ and $\hat{a}^*_i$ satisfy the following equations:

$$\begin{cases}
a^*_{i_1}+\hat{a}^*_{i_2} = a_{i_1}\ \hat{a}_{i_2}, \\
a^*_{i_1}+\hat{b}^*_{i_2} = a_{i_1}\ \hat{b}_{i_2}, \\
b^*_{i_1}+\hat{a}^*_{i_2} = b_{i_1}\ \hat{a}_{i_2}, \\
b^*_{i_1}+\hat{b}^*_{i_2} = b_{i_1}\ \hat{b}_{i_2},
\end{cases}$$

and $c = (a^*_1,\cdots,a^*_{m_1},\hat{a}^*_1,\cdots,\hat{a}^*_{m_2})$, $c' = (b^*_1,\cdots,b^*_{m_1},\hat{b}^*_1,\cdots,\hat{b}^*_{m_2})$ and $k_z = (k_1,\cdots,k_{m_1},k'_1,\cdots,k'_{m_2})$. The function defined by (6) belongs to $HT(\Omega)$ and hence $f_{m_1} \times f_{m_2} \in HT(\Omega)$. Finally, for any constant $c \in \mathbb{R}$, we have

$$cf_{m_1}(x,a,b,k) = c\sum_{i=1}^{m}\left(\frac{a_i\ e^{k_i\overline{x}_i}\ +\ b_i\ e^{-k_i\overline{x}_i}}{e^{k_i\overline{x}_i}\ +e^{-k_i\overline{x}_i}}\right) = \sum_{i=1}^{m}\left(\frac{a^*_i\ e^{k_i\overline{x}_i}\ +\ b^*_i\ e^{-k_i\overline{x}_i}}{e^{k_i\overline{x}_i}\ +e^{-k_i\overline{x}_i}}\right).$$

where $a^*_i = ca_i$ and $b^*_i = cb_i$ for $i = 1,2,\cdots,m$. Therefore, $HT(\Omega)$ is an algebra.
Now, we show that $HT(\Omega)$ separates the points of Ω. Consider $x^{(\circ)}$ and $y^{(\circ)} \in \Omega$ with $x^{(\circ)} \neq y^{(\circ)}$ where $x^{(\circ)} = (x^{(\circ)}_1, x^{(\circ)}_2,\cdots,x^{(\circ)}_n)^T$ and $y^{(\circ)} = (y^{(\circ)}_1, y^{(\circ)}_2,\cdots,y^{(\circ)}_n)^T$, then there exists index l

such that $x_l^{(\circ)} \neq y_l^{(\circ)}$. Without loss of generality, suppose that $x_l^{(\circ)} > y_l^{(\circ)}$. Define m=n. So, for any $x \in \Omega$, the generalized vector corresponding to relation (1) is $\overline{x} = x$. We consider two cases:

Case I : $x_l^{(\circ)} \neq -y_l^{(\circ)}$.

We define

$$k_i = \begin{cases} x_l^{(\circ)} - y_l^{(\circ)}, & i = l \\ 1, & i \neq l, \end{cases} \tag{7}$$

and

$$\begin{cases} a_i = b_i = 0, & i = 1, 2, \cdots, n, \ i \neq l \\ a_l = e^{-k_l x_l}, \ b_l = 1. \end{cases} \tag{8}$$

By (8) and (2), function $f(.) \in HT(\Omega)$ can be defined as follows

$$\begin{aligned} f(x, a, b, k) &= \sum_{i=1}^{m} \left(\frac{a_i \ e^{k_i \overline{x}_i} \ + \ b_i \ e^{-k_i \overline{x}_i}}{e^{k_i \overline{x}_i} \ + e^{-k_i \overline{x}_i}} \right) \\ &= \frac{1 + e^{-(x_l^{(\circ)} - y_l^{(\circ)}) x_l}}{e^{(x_l^{(\circ)} - y_l^{(\circ)}) x_l} + e^{-(x_l^{(\circ)} - y_l^{(\circ)}) x_l}}, \ x \in \Omega. \end{aligned}$$

Hence

$$\begin{aligned} & f(x^{(\circ)}) - f(y^{(\circ)}) = \\ & \frac{e^{(x_l^{(\circ)} - y_l^{(\circ)}) y_l^{(\circ)}} + e^{-(x_l^{(\circ)} - y_l^{(\circ)}) y_l^{(\circ)}} - e^{(x_l^{(\circ)} - y_l^{(\circ)}) x_l^{(\circ)}} - e^{-(x_l^{(\circ)} - y_l^{(\circ)}) x_l^{(\circ)}}}{(e^{(x_l^{(\circ)} - y_l^{(\circ)}) x_l^{(\circ)}} + e^{-(x_l^{(\circ)} - y_l^{(\circ)}) x_l^{(\circ)}})(e^{(x_l^{(\circ)} - y_l^{(\circ)}) y_l^{(\circ)}} + e^{-(x_l^{(\circ)} - y_l^{(\circ)}) y_l^{(\circ)}})}. \end{aligned} \tag{9}$$

We show that $f(x^\circ) \neq f(y^\circ)$. By contradiction, assume that $f(x^\circ) = f(y^\circ)$. Therefore from (9), we have

$$e^{(x_l^{(\circ)} - y_l^{(\circ)}) x_l^{(\circ)}} + e^{-(x_l^{(\circ)} - y_l^{(\circ)}) x_l^{(\circ)}} = e^{(x_l^{(\circ)} - y_l^{(\circ)}) y_l^{(\circ)}} + e^{-(x_l^{(\circ)} - y_l^{(\circ)}) y_l^{(\circ)}}.$$

So

$$cosh(x_l^{(\circ)} - y_l^{(\circ)}) x_l^{(\circ)} = cosh(x_l^{(\circ)} - y_l^{(\circ)}) y_l^{(\circ)}. \tag{10}$$

Since $x_l^{(\circ)} > y_l^{(\circ)}$ and $x_l^{(\circ)} \neq -y_l^{(\circ)}$, the relation (10) is impossible. Hence we have $f(x^\circ) \neq f(y^\circ)$.

Case II: $x_l^{(\circ)} = -y_l^{(\circ)}$.

We choose k_i, $(for \ i = 1, 2, \cdots, n)$ as (7), and a_i and b_i as follows

$$\begin{cases} a_i = b_i = 0, & i = 1, 2, \cdots, n, \ i \neq l \\ a_l = 0, b_l = 1, \end{cases} \tag{11}$$

By (11) and (2), we construct $f(.) \in HT(\Omega)$ as follows

$$f(x) = \frac{e^{-(x_l^{(\circ)} - y_l^{(\circ)}) x_l}}{e^{(x_l^{(\circ)} - y_l^{(\circ)}) x_l} + e^{-(x_l^{(\circ)} - y_l^{(\circ)}) x_l}}, \ x \in \Omega.$$

Hence

$$f(x^{(\circ)}) - f(y^{(\circ)}) = \frac{e^{-2(x_l^{(\circ)})^2}}{e^{2(x_l^{(\circ)})^2} + e^{-2(x_l^{(\circ)})^2}} - \frac{e^{2(x_l^{(\circ)})^2}}{e^{2(x_l^{(\circ)})^2} + e^{-2(x_l^{(\circ)})^2}}$$
$$= -tanh(2(x_l^{(\circ)})^2).$$

Since $x_l^{(\circ)} > y_l^{(\circ)}$ and $x_l^{(\circ)} = -y_l^{(\circ)}$, so $x_l^{(\circ)} \neq 0$. Hence $tanh(2(x_l^{(\circ)})) \neq 0$. Thus $f(x^{(\circ)}) \neq f(y^{(\circ)})$.

From cases (I) and (II), we conclude that $HT(\Omega)$ separates the points of Ω.

Finally, we prove that $HT(\Omega)$ vanishes at no point of Ω. We select $f \in HT(\Omega)$ with $a_i = b_i = k_i = 1$ and $m = n$, i.e.

$$f(x) = \sum_{i=1}^{m} \frac{a_i \ e^{k_i \overline{x}_i} \ + \ b_i \ e^{-k_i \overline{x}_i}}{e^{k_i \overline{x}_i} \ + e^{-k_i \overline{x}_i}} = m \neq 0, \ \ x \in \Omega.$$

On the other hand, from (4), $HT(\Omega)$ is a subspace of $C(\Omega)$. Hence, by Lemma (1) the proof is completed. □

4 Application of HT functions to design stabilizer control

Consider the following system of differential equations:

$$\begin{cases} \dot{x}(t) = f(x), \\ x(0) = \alpha, \ \ t \geq 0. \end{cases} \quad (12)$$

This system is called globally asymptotically stable if for any arbitrary $\alpha \in \mathbb{R}^n$, the state of system (12) satisfies the relation $\lim_{t \to \infty} x(t) = 0$. Let $x = 0$ be an equilibrium point of the system (12) and $V : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that

$$\begin{cases} V(0) = 0 \ and \ V(x) > 0, \ \ \forall x \neq 0, \\ |x| \to \infty \Rightarrow V(x) \to \infty, \\ \dot{V}(x) < \ \ 0, \ \ \forall x \neq 0. \end{cases}$$

Then, the origin is globally asymptotically stable (to more details see [8]).

In this section we show that HT functions can be utilized to design stabilizer control for non-linear dynamical systems. For this goal, we consider the following control system

$$\begin{cases} \dot{x}_1 = -sin(x_1) - 0.1x_2, \\ \dot{x}_2 = tan(x_1) - sin(x_2) + u. \end{cases} \quad (13)$$

The treatment of this nonlinear system for $u = 0$ and initial values $(x_1(0), x_2(0)) = (3, 1)$ is given in Figure 1. It can be seen that this system is not stable at the equilibrium point $(0, 0)$ and the trajectories x_1 and x_2 are divergent. Now the following approximations, in the neighbourhood of origin, can be suggested

$$x_2 \simeq tanh(x_2),\ sin(x_1) \simeq tanh(x_1),\ tan(x_1) \simeq tanh(x_1),\ sin(x_2) \simeq tanh(x_2).$$

By above approximations, the nonlinear control system (13) can be approximated in the neighbourhood of origin as follows

$$\begin{cases} \dot{x}_1 = -tanh(x_1) - 0.1tanh(x_2), \\ \dot{x}_2 = tanh(x_1) - tanh(x_2) + u. \end{cases} \tag{14}$$

This system can be written as the following matrix form

$$\dot{x} = Atanh(x) + Bu,$$

where $x = [x_1, x_2]^T, tanh(x) = [tanh(x_1),\ tanh(x_2)]^T$ and

$$A = \begin{bmatrix} -1 & -0.1 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

We can consider a feedback control as

$$u = Htanh(x) = h_1 tanh(x_1) + h_2 tanh(x_2), \tag{15}$$

where $H = [h_1, h_2]$. By attention to the Theorem 2 in [13], we find vector H such that it satisfies the following matrix system

$$P(A + BH) + (A + BH)^T P + I = 0, \tag{16}$$

where $P = diag(p_1^2, p_2^2)$ is a positive definite matrix. Having solved system (16) with respect to the components of matrix P and vector H, we get

$$p_1 = 0.7071, \quad p_2 = 0.2723, \quad h_1 = -0.3255, \quad h_2 = -5.7451.$$

It is easy to see that function

$$V(x) = 2\big(p_1^2\ ln(cosh(x_1)) + p_2^2\ ln(cosh(x_2))\big)$$

is a Lyapunov function for system (14) which satisfies $V(x) \geq 0, V(0) = 0$, $\lim_{\|x\| \to \infty} V(X) = \infty$ and

$$\dot{V}(x) = \frac{d}{dt}V(x) = \dot{x}\frac{dV}{dx} = -tanh^2(x_1) - tanh^2(x_2) < 0, x \neq 0.$$

Hence, after applying feedback control (15), system (14) is globally asymptotically stable. The treatment of trajectories x_1 and x_2 of system (14), for initial values $(x_1(0), x_2(0)) = (3, 1)$, is illustrated in Figures 2 and 3. It can be seen that the trajectories converge to the origin (i.e. the equilibrium point of (14)). Now, we utilize the obtained control to stabilize the main nonlinear system (13). The trajectories are shown in Figures 4 and 5 which show that the obtained feedback control is an asymptotically stabilizer control.

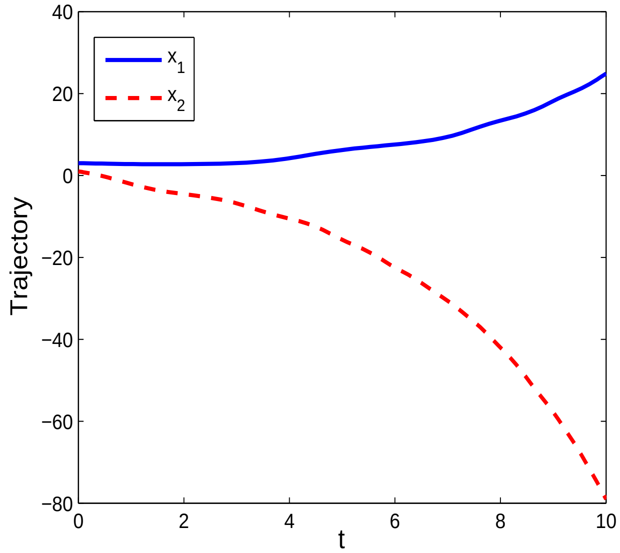

Figure 1: The treatment of main system (13) with u=0.

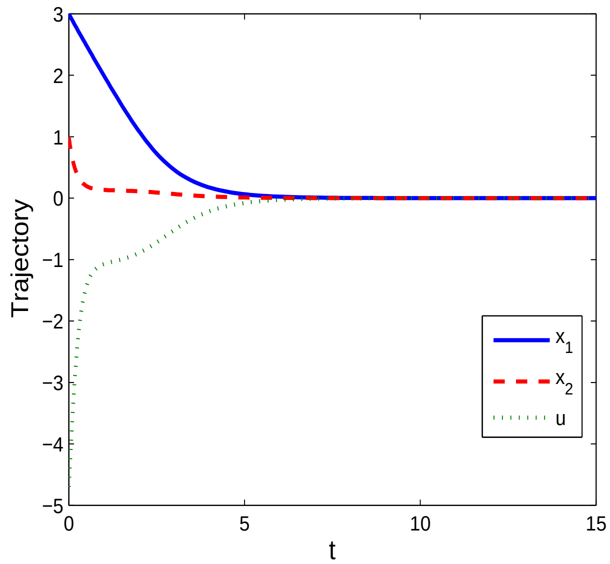

Figure 2: The treatment of hyperbolic system (14).

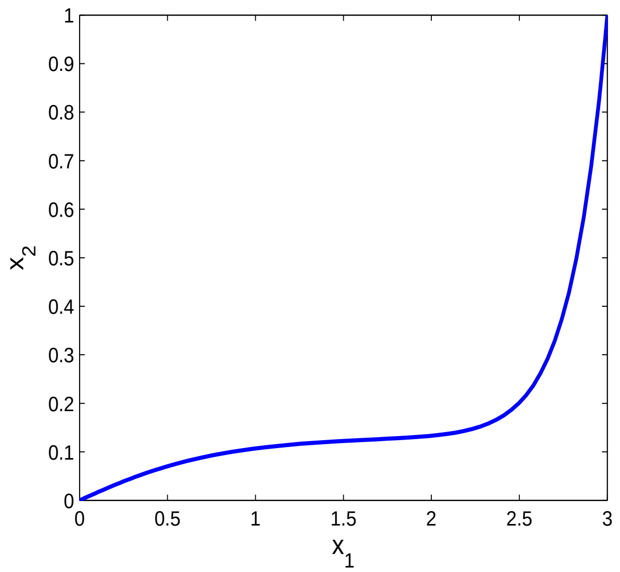

Figure 3: The phase plane of hyperbolic system (14).

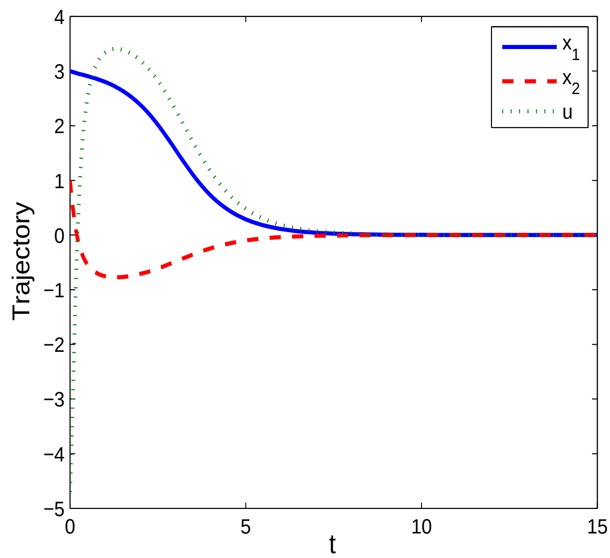

Figure 4: The treatment of main system (13).

5 Conclusions and Suggestions

The space of hyperbolic tangent functions is a universal approximator. This space can be utilized to design stabilizer control for the nonlinear dynamical systems. For future works, this

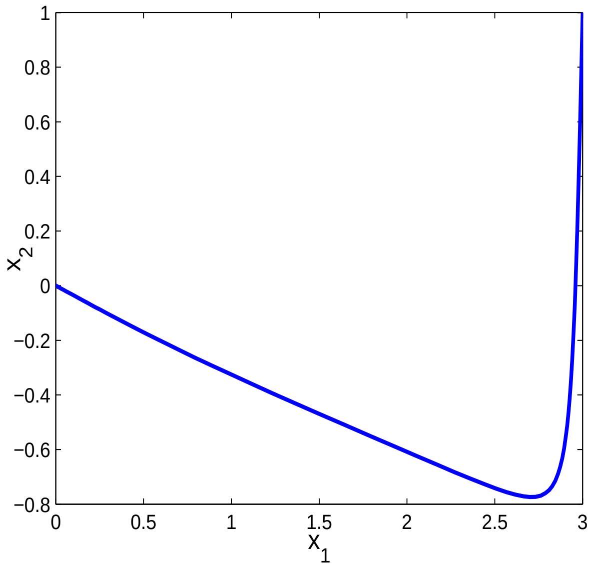

Figure 5: The phase plane of main system (13).

space will be suggested to design adaptive control. Also, it can be applied to stabilize the nonlinear delay control systems.

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خاصیت تقریبگر کلی فضای توابع تانژانت هایپربولیک

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چکیده

در این مقاله، ابتدا فضای توابع تانژانت هایپربولیک معرفی شده و سپس خاصیت تقریبگر کلی این فضا اثبات می‌شود. درواقع با استفاده از این فضا هر تابع پیوسته غیرخطی می‌تواند به طور یکنواخت با هر دقتی تقریب زده شود. همچنین به عنوان یک کاربرد، این فضای توابع به منظور طراحی کنترل بازخورد برای یک سیستم دینامیکی غیر خطی استفاده می‌شود.

کلمات کلیدی

توابع تانژانت هایپربولیک، تقریبگر کلی، کنترل پایدار ساز.