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MQ-Radial Basis Functions Center Nodes Selection with PROMETHEE Technique

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Abstract. In this paper, we decide to select the best center nodes of radial basis functions by applying the Multiple Criteria Decision Making (MCDM) techniques. Two methods based on radial basis functions to approximate the solution of partial differential equation by using collocation method are applied. The first is based on the Kansa's approach, and the second is based on the Hermite interpolation. In addition, by choosing five sets of center nodes: Uniform grid, Cartesian, Chebyshev, Legendre and Legendre-Gauss-Lobato (LGL) as alternatives and achieving the error, condition number of interpolation matrix and memory time as criteria, rating of cases with the help of PROMETHEE technique is obtained. In the end, the best center nodes and method is selected according to the rankings. This ranking shows that Hermite interpolation by using non-uniform nodes as center nodes is more suitable than Kansa's approach with each center nodes.

Keywords. Multiple Criteria Decision Making, Radial basis functions, PROMETHEE, Hermite interpolation, Optimal selecting.

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1 Introduction

Radial basis functions (RBFs) interpolation is a technique for representing a function starting with data on scattered points. This technique first appears in the literature as a method for scattered data interpolation, and interest in this method exploded after the review of Franke [1], who found it to be the most impressive of the many methods he tested. Later, Kansa [2, 3] proposed a scheme for the estimation of partial derivatives using RBFs. The main advantage of radial basis functions methods is the meshless characteristic of them. The use of radial basis functions as a meshless method for the numerical solution of partial differential equations (PDEs) is based on the Collocation method. These methods have recently received a great deal of attention from researchers [4, 5, 6, 7, 8, 9].

Recently, RBFs methods were extended to solve various ordinary and partial differential equations including the high order ordinary differential equations [10], second-order parabolic equation with nonlocal boundary conditions [11, 12], the nonlinear Fokker-Planck equation [13], optimal control problems [14], the viscous flow over nonlinearly stretching sheet with chemical reaction, heat transfer and magnetic field [15], the unsteady flow of gas in a semi-infinite porous medium [16] nonlinear differential and integral equations [17, 18, 19], Second-order hyperbolic telegraph equation [20], the solution of 2D biharmonic equations [21], the case of heat transfer equations [22] and so on [23, 24, 25].

An RBF $\psi(\|\mathbf{x}-\mathbf{x}_i\|):\mathbb{R}^+\longrightarrow\mathbb{R}$ depends on the separation between a field point $\mathbf{x}\in\mathbb{R}^d$ and the data centers $\mathbf{x}_i$, for $i=1,2,...,N$, and N data points. The interpolants are classed as radial due to their spherical symmetry around centers $\mathbf{x}_i$, where $\|.\|$ is the Euclidean norm. One of the most powerful interpolation method with analytic two-dimensional test function is the RBFs method based on multiquadric (MQ) basis function

$$\psi(r)=\sqrt{r^2+c^2}\ , \tag{1}$$

suggested by R.L. Hardy [26], where $r=\|\mathbf{x}-\mathbf{x}_i\|$ and c is a free positive parameter, often referred to as the shape parameter, to be specified by the user. Madych and Nelson [27] showed that interpolation with MQ is exponentially convergent based on reproducing kernel Hilbert space. Convergence property of the MQ has been also showed by Buhman [28, 29]. Too large or too small shape parameter c in (1) make the MQ too flat or too peaked. Despite many research works presented to finding algorithms for selecting the optimum values of c [30, 31, 32, 33, 34], the optimal choice of shape parameter is an open problem which is still under intensive investigation.

The interested reader is referred to the recent books and paper by Buhmann [28, **?**] and Wendland [35] for more basic details about RBFs, compactly and globally supported and convergence rate of the radial basis functions.

Center nodes $\{\mathbf{x}_i\}_{i=1}^N$ are not necessarily structured, that is, they can have an arbitrary distribution. The arbitrary grid structure is one of the major differences between the RBFs methods and other global methods. Such a mesh-free grid structure yields high flexibility especially when the domain is irregular. Finding the Center nodes in RBF methods is too important an open problem. In this work, we aim to select the best center nodes based on convergence, condition number of interpolation matrix, time and memory with a famous MCDM method named PROMETHEE.

Today, complex decisions in various conditions are under influence of frequent and different factors and criteria which have a significant and deniable role in consequence and effects of decisions and we cannot simply and base of the common methods find response for them but we should use (hang on to) modern scientific methods. MCDM problem is a well known branch of decision theory. It has been found in real life decision situations [36, 37, 38, 39]. In general, decision-making is the study of identifying and choosing alternatives based on the values and preferences of the decision-maker. Making a decision implies that some alternatives are to be considered, and that one chooses the alternative(s) that possibly best fits with the goals, objectives, desires and values of the problem. MCDM is a powerful tool used widely for evaluation and ranking problems containing multiple, usually conflicting, criteria [40], as how it is in finding the best center nodes in RBF methods. A lot of researchers have devoted themselves to solve MCDM [41, 42, 43, 44, 45, 46, 47, 48, 49, 50].

Several approaches have been proposed for multicriteria decision and the relevant methods were developed and applied with more or less success depending on the specific problem [51]. Among numerous methods of MCDM, The Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) is significantly suitable for ranking applications [40]. PROMETHEE brings together flexibility and simplicity for the user [52] and is quite simple in conception and application compared to other methods for multicriteria analysis [53]. The PROMETHEE method and their applications has attracted much attention from academics and practitioners [54]. It is well adapted to problems where a finite number of alternative actions are to be ranked considering several, sometimes conflicting, criteria [51]. This method is a relatively simple ranking method, which is perfectly intelligible for the decision maker and is accepted as one of the most intuitive MCDM methods [55]. It is one of the best known and most widely applied outranking method because it follows a transparent computational procedure and can be easily understood by actors and DMs [56]. The PROMETHEE method has found a vast scope of application such as logistics and transportation [57, 58], environment management [59, 60], finance [61, 62], chemistry [63], production planning [64, 65, 66], energy management [67], service [68, 69], sport [70] and supply chain management [71, 72].

The PROMETHEE model has many advantages, in comparison to other MCDM models, such as structuring the issue, the amount of data that could be processed, the possibility to quantify the qualitative values, software support and presentation of the results [73]. Hence we used PROMETHEE Technique to rank possible alternatives due to its coordination with the structure of the issue, popularity, vast usage, remarkable outcomes, being easy to use and professional software.

This paper is arranged as follows: in Section 2, we describe the properties of radial basis functions. Two approaches based on radial basis functions for approximate the solution of linear operation by using collocation method are applied. In section 3, the PROMETHEE methodology is described. we give computational results of numerical experiments with methods based on preceding sections, to support our theoretical discussion in section 4. The conclusions are discussed in the final Section.

2 Radial basis functions

2.1 Definition of radial basis functions

Let $\mathbb{R}^+ = \{x \in \mathbb{R}, x \geq 0\}$ be the non-negative half-line and let $\psi : \mathbb{R}^+ \to \mathbb{R}$ be a continuous function with $\psi(0) \geq 0$. A radial basis function on $\mathbb{R}^d$ is a function of the form

$$\psi(\|\mathbf{x} - \mathbf{x}_i\|) \ ,$$

where $\mathbf{x}, \ \mathbf{x}_i \in \mathbb{R}^d$ and $\|.\|$ denotes the Euclidean distance between $\mathbf{x}$ and $\mathbf{x}_i$s. If one chooses N points $\{\mathbf{x}_i\}_{i=1}^N$ in $\mathbb{R}^d$ then by custom

$$s(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i \psi(\|\mathbf{x} - \mathbf{x}_i\|); \quad \lambda_i \in \mathbb{R}$$

is called a radial basis function as well [74].

2.2 RBFs interpolation based on Kansa approach

We now discuss Kansa's collocation method. Assume we are given a domain $\Omega \subset \mathbb{R}^d$, and a linear operator of the form

$$L[u](\mathbf{x}, t) = H(\mathbf{x}, t) \ , \qquad \mathbf{x} \in \Omega \ , t \in [0, T), \tag{2}$$

with initial and boundary conditions

$$I[u](\mathbf{x}) = f(\mathbf{x}) \ , \quad \mathbf{x} \in \Omega \ , t = 0, \tag{3}$$

$$B[u](\mathbf{x}) = g(\mathbf{x}, t) \ , \qquad \mathbf{x} \in \partial\Omega \ , t \in [0, T). \tag{4}$$

Then we approximate u by radial basis functions as

$$u(\hat{\mathbf{x}}) = \sum_{i=1}^{N} \lambda_i \psi(\|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i\|) \ , \tag{5}$$

where $\hat{\mathbf{x}} = (\mathbf{x}, t)$. The simplest possible setting is shown in expansion (5). The Collocation matrix is constructed by matching the differential equation (2) and the initial and boundary conditions (3) and (4) at the collocation nodes $\{\hat{\mathbf{x}}_j\}_{j=1}^N$ of the form

$$A = \begin{bmatrix} B[\Psi] \\ I[\Psi] \\ L[\Psi] \end{bmatrix}, \tag{6}$$

where the blocks of matrix is generated in Appendix 1.

Kansa's method is an unsymmetric RBF Collocation method based upon the MQ interpolation functions. Although the above approach has been applied successfully in several cases [6, 7, 10, 11, 22, 75], no existence of solution and convergence analysis is available in the literature and, for some cases, it has been reported that the resulting matrix was extremely ill–conditioned. The condition number of the above interpolation matrix for smooth RBFs like Gaussian or multiquadrics are extremely large.

Several techniques have been proposed to improve the conditioning of the coefficient matrix and the solution accuracy. Fasshauer [76] suggested an alternative approach to the unsymmetric scheme based on the Hermite interpolation property of the radial basis functions. The advantage of the Hermite-based approach is that the matrix resulting from the scheme is symmetric, as opposed to the completely unstructured matrix of the same size resulting from unsymmetric schemes.

2.3 RBFs interpolation based on Hermite approach

It is possible to represent the solution u of the above boundary value problem in terms of the following Hermite RBF (HRBF) interpolation:

$$\begin{aligned} u(\hat{\mathbf{x}}) = \ & \sum_{i=1}^{N_0} \lambda_i B^*[\psi](\|\hat{\mathbf{x}}-\hat{\mathbf{x}}_i\|) + \sum_{i=N_0+1}^{N_1} \lambda_i I^*[\psi](\|\hat{\mathbf{x}}-\hat{\mathbf{x}}_i\|) \\ & + \sum_{i=N_1+1}^{N} \lambda_i L^*[\psi](\|\hat{\mathbf{x}}-\hat{\mathbf{x}}_i\|) \ , \end{aligned}$$

where N_0 and $N_1 - N_0$ denote the number of nodes on $\partial\Omega \times [0,T)$ and $\Omega \times \{0\}$ and $N - N_1 - N_0$ the number of internal nodes. In the above expression L^*, I^* and B^* are the operators used in (2), (3) and (4), but acting on ψ viewed as a function of the second argument $\hat{\mathbf{x}}_i$ [76]. This expansion for $u(\hat{\mathbf{x}})$ leads to a collocation matrix A which is of the form

$$A = \begin{bmatrix} B\big[B^*[\Psi]\big] & B\big[I^*[\Psi]\big] & B\big[L^*[\Psi]\big] \\ I\big[B^*[\Psi]\big] & I\big[I^*[\Psi]\big] & I\big[L^*[\Psi]\big] \\ L\big[B^*[\Psi]\big] & L\big[I^*[\Psi]\big] & L\big[L^*[\Psi]\big] \end{bmatrix} , \qquad (7)$$

where the blocks generated in Appendix 2.

The matrix (7) is of the same type as the scattered HRBF interpolation matrices and thus non-singular as long as is ψ chosen appropriately. A major point in favour of the HRBF approach is that the matrix resulting from the scheme is symmetric, as opposed to the completely unstructured matrix (6) of the same size. The convergence proof for HRBF interpolation was given by Wu [77] who also recently proved the convergence of this approach when solving PDEs [78]; see also [79]. A comparison analysis between unsymmetric and symmetric radial basis function collocation methods for the numerical solution of partial differential equations is described in paper by Power [80].

3 PROMETHEE Methodology

PROMETHEE is a MCDM method developed by Brans et al. [81]. It is a ranking method quite simple in conception and application compared to other methods for multi-criteria analysis [82].

Let A be a set of alternatives and $g_j(a)$ represent the value of criterion $g_j(a),\ j = 1, 2, \cdots, J$ of alternative $a \in A$. As the first step in PROMETHEE a preference function $F_j(a, b)$ is defined for each pair of actions for criterion g_j .Assuming that more is preferred to less. Where q_i and p_i are indifference and preference thresholds for ith criterion respectively.

$$F_j(a,b) = 0 \qquad \texttt{if} \;\; g_j(a) - g_j(b) \leq q_j$$

$$F_j(a,b) = 1 \qquad \texttt{if} \;\; g_j(a) - g_j(b) \geq p_j$$

$$0 < F_j(a,b) < 1 \qquad \texttt{if} \;\; q_j < g_j(a) - g_j(b) < p_j$$

Different shapes (six types) for F_j have been suggested. If a is better than b according to jth criterion, $F_j(a, b) > 0$, otherwise $F_j(a, b) = 0$. Using the weights w_j assigned to each criterion (where $\sum w_j = 1$), one can determine the aggregated preference indicator as follows:

$$\Pi(a,b) = \sum w_j f_j(a,b).$$

If the number of alternatives is more than two, overall ranking is done by aggregating the measures of pair wise comparisons. For each alternative $a \in A$, the following two outranking dominance flows can be obtained with respect to all the other alternatives $x \in A$:

$$\varphi^+(a) = \frac{1}{n-1} \sum_{x \in A} \Pi(a,x) \quad \texttt{leaving flow}.$$

The leaving flow is the sum of the values of the arcs leaving node a and therefore provide a measure of the outranking character of a. The higher $\varphi^+(a)$, is the better alternative a,

$$\varphi^-(a) = \frac{1}{n-1} \sum_{x \in A} \Pi(x,a) \quad \texttt{entering flow}.$$

The entering flow measures the outranked character. The smaller $\varphi^-(a)$, is the better alternative a [83]. For each alternative a, it is obvious that we can also determine the net flow for each criterion separately. Let us define the net flow for criterion g_j as follows:

$$\varphi_j(a) = \frac{1}{n-1} \sum_{x \in A} (F_j(a,x) - F_j(x,a)).$$

$\varphi_j(a)$ quantifies the position of alternative a according to criterion j with respect to all the other alternatives in the set A. The larger the single criterion net flow the better alternative a on criterion g_j.
According to PROMETHEE I, action a is superior to action b if the leaving flow of a is greater than the leaving flow of b and entering flow of a is smaller than the entering flow of b.

$$a \quad \texttt{outranks} \quad b \quad \texttt{if}: \; \varphi^+(a) \geq \varphi^+(b) \quad \texttt{and} \quad \varphi^-(a) \leq \varphi^-(b).$$

Equality in φ^+ and φ^- indicates indifference among the two compared alternatives. Two alternatives are considered incomparable if alternative a is better than alternative b in terms of leaving flow, while the entering flows indicate the reverse [82]:

$$[\varphi^+(a) > \varphi^+(b) \texttt{ and } \varphi^-(a) > \varphi^-(b)] \texttt{ or } [\varphi^+(a) < \varphi^+(b) \texttt{ and } \varphi^-(a) < \varphi^-(b)].$$

PROMETHEE II provides a complete ranking of the alternatives from the best to the worst one by

$$\Phi(a) = \varphi^+(a) - \varphi^-(a).$$

The implementation of PROMETHEE requires two additional types of information, namely: (1) information on the relative importance that is the weights of the criteria considered, (2) information on the decision-maker s preference function, which he/she uses when comparing the contribution of the alternatives in terms of each separate criterion [84]. This function is used to compute the degree of preference associated to the best action in case of pairwise comparisons [85]. When we compare two alternatives a and B, we must be able to express the result of these comparisons in terms of preference. Then we consider a preference function Φ [84]. There are six basics types of preference functions proposed by Brans and Vincke [86]. with the aim of enabling the selection of specific preference function, which can be listed as usual function, U-shape function, V-shape function, level function, linear function and Gaussian function.

4 Algorithm explain with examples

The proposed approach is applied in two partial differential equations. we aim to choose best centers nodes of RBFs by applying Kansa and HRBF collocation method. Finding the best nodes between the set of nodes for example: uniform, cartesian, Chebyshev for these methods is an open problem. Thus ranking or choosing the appropriate methods by using suitable center nodes is so important in RBFs approximation.
In order to learn more about using of mentioned techniques in real environment, we impediment the proposed algorithms steps with a concrete examples.
In the process of using the model, we perform the three following steps:
1st step: Determination of fundamental criteria and Alternatives.
2nd step: Rating of cases with the help of PROMETHEE technique.
3rd step: Analyzing of consequences.

4.1 Determination of fundamental criteria and Alternatives

Here, two following classical heat equation is solved by using Kansa and HRBF method with MQ function.

$$
\begin{aligned}
&u_t(\mathbf{x},t) = \nabla u(\mathbf{x},t) + f(\mathbf{x},t), && \text{in } \Omega \times J,\\
&u(\mathbf{x},0) = g(\mathbf{x}), && \mathbf{x} \in \Omega,\\
&Bu(\mathbf{x},t) = h(\mathbf{x},t), && \text{on } \partial\Omega \times J,
\end{aligned}
$$

`Example 1: the Homogeneous one-dimensional case:`

$g(x_1) = \sin(x_1), \qquad 0 < x_1 < \pi\ , \qquad t > 0,$

$u(0,t) = 0, \qquad u(\pi,t) = 0.$

`Exact solution:` $u(x_1,t) = \sin(x_1)\ e^{-2t}$.

`Example 2: the Inhomogeneous two-dimensional case:`

$f(x_1,x_2,t) = \sin(x_1)\sin(x_2)e^{-t} - 4,$

$g(x_1,x_2) = \sin(x_1)\sin(x_2) + x_1^2 + x_2^2, \qquad 0 < x_1, x_2 < \pi\ , \qquad t > 0,$

$u(0,x_2,t) = x_2^2, \qquad u(x_1,0,t) = x_1^2,$

$u(\pi,x_2,t) = x_2^2 + \pi^2, \qquad u(x_1,\pi,t) = x_1^2 + \pi^2.$

`Exact solution:` $u(x_1,x_2,t) = \sin(x_1)\sin(x_2)e^{-t} + x_1^2 + x_2^2$.

The error is root mean square (RMS) and obtained as:

$$
RMS = \sqrt{\frac{\sum_{k=1}^{M}\left(u(\mathbf{x}_k,t_k) - u_N(\mathbf{x}_k,t_k)\right)^2}{M}}\ .
$$

where $u(\mathbf{x}_k,t_k)$ and $u_N(\mathbf{x}_k,t_k)$ are achieved by exact and approximate solution on $(\mathbf{x}_k,t_k)$, and M is number of test points. Also we consider shape parameter equals one for the both examples and all cases.

Tables (1) and (2) show determination of fundamental criteria and Alternatives for each two examples.

Tables (3) and (4) show grading of cases in example 1 for $N = 36,\ 100$. Table (5) shows grading of cases in example 2 for $N = 512$.

Table 1: Fundamental criteria

Label	C_1	C_2	C_3
Criteria	Error	Condition Number	Time.Memory

Table 2: Alternatives in nodes and methods

Label	A_1	A_2	A_3	A_4	A_5
Kansa nodes	Uniform Grid	Legendre	Chebyshev	LGL	Cartesian
Label	A_6	A_7	A_8	A_9	A_{10}
HRBF nodes	Uniform Grid	Legendre	Chebyshev	LGL	Cartesian

Table 3: Grading of cases in example 1 for $N = 36$.

N	C_1	C_2	C_3
Min/Max	Min	Min	Min
Preference Function	Usual	Usual	Usual
Unit	Numerical$\times 10^{+7}$	Numerical$\times 10^{-5}$	Kbs
A_1	580	370	337.04
A_2	480	300	365.75
A_3	410	170	323.13
A_4	330	105	323.10
A_5	500	100	328.99
A_6	10	2.9	373.09
A_7	5	3.6	310.40
A_8	4	2.6	328.18
A_9	5	1.5	346.19
A_{10}	7	1.2	324.72

Table 4: Grading of cases in example 1 for $N = 100$.

N	C_1	C_2	C_3
Min/Max	Min	Min	Min
Preference Function	Usual	Usual	Usual
Unit	Numerical$\times 10^{+7}$	Numerical$\times 10^{-9}$	Kbs
A_1	4.100	190	1109.12
A_2	2.700	340	1409.06
A_3	2.600	390	1249.81
A_4	0.520	120	1285.24
A_5	20.00	17000	1124.74
A_6	0.031	15.0	1457.08
A_7	0.010	1.7	2061.45
A_8	0.003	1.1	1985.16
A_9	0.004	12.3	1984.85
A_{10}	0.090	13.0	2084.07

4.2 Rating of the cases with the help of PROMETHEE technique

In our study, one of the most frequently used preference function type in the literature and the most suitable preference function type to the characteristic of our problem, the usual function (it was introduced at Section 3) is selected for the evaluation. In next step we should evaluate

Table 5: Grading of cases in example 2 for $N = 512$.

N	C_1	C_2	C_3
Min/Max	Min	Min	Min
Preference Function	Usual	Usual	Usual
Unit	Numerical$\times 10^{+7}$	Numerical$\times 10^{-9}$	Kbs
A_1	500.00	810.0	5400
A_2	37.00	230.0	5914
A_3	71.00	130.0	6101
A_4	10.00	110.0	6010
A_5	83.00	510.0	5913
A_6	3.20	9.7	6310
A_7	0.31	3.4	6897
A_8	0.48	1.2	6911
A_9	0.17	1.1	7110
A_{10}	0.87	4.7	6981

them by analyzing the cases in each criterion, and finally by correct rating of cases, choose the best case. For this purpose, he can perform steps of PROMETHEE technique to the end or for ease of calculation; he can use the relevant software like DECISION LAB.
After completing the grading table, we can easily derive the rating consequences of the cases by using of PROMETHEE technique, Also we can evaluate and analyze the consequences by using of graphical capabilities of the software DECISION LAB, like Gaia planes.
Figure 1 displays ranking of cases with the help of PROMETHEE II technique with $N = 36$ for example 1. This ranking shows that HRBF method by using Legendre points are the most suitable choices as RBF methods and center nodes. The output figure listing the outsourcers with $N = 100$ for example 1 is given in Figure 2. As seen in the figure, the best choice in the center nodes may be changed in big number of nodes, but HRBF is the more appropriate than Kansa's method yet. Figure 3 shows PROMETHEE II output for all two scenarios $N = 36$ and $N = 100$. This ranking shows that HRBF method by using Chebyshev points as center nodes is the best choice. In Figure 4, the outsourcers are listed with $N = 512$ for example 2. This ranking shows that HRBF method by using Legendre or Legendre-Gauss-Lobatto (LGL) points as center nodes are the most suitable choices. Moreover, The geometrical analysis for interactive aid (GAIA) plane which displays the relative position of the alternatives graphically, in terms of contributions to the various criteria is given in Figures 5, 6 and 7.

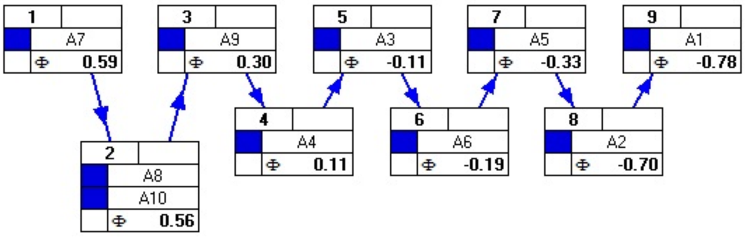

Figure 1: Example 1: Rating of cases with the help of PROMETHEE II technique with $N = 36$.

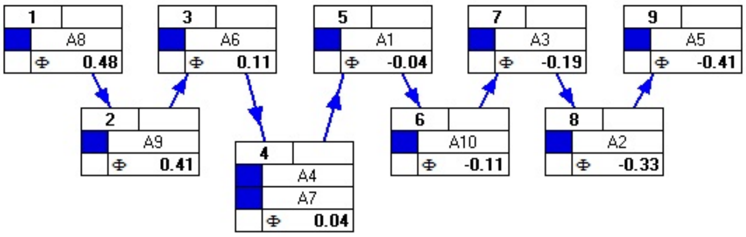

Figure 2: Example 1: Rating of cases with the help of PROMETHEE II technique with $N = 100$.

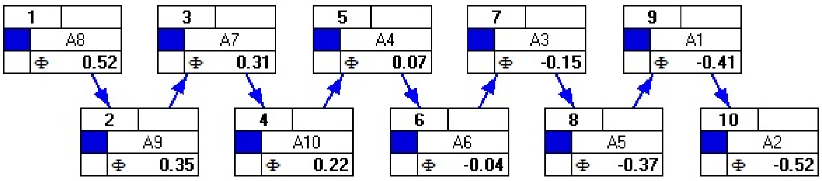

Figure 3: Example 1: PROMETHEE II output: final scores of Alternatives.

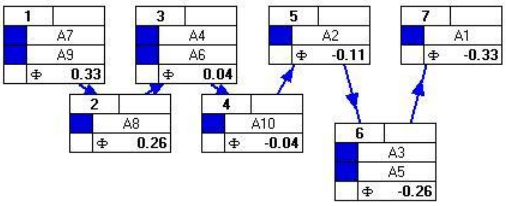

Figure 4: Example 2: Rating of cases with the help of PROMETHEE II technique with $N = 512$.

The GAIA plane was used in order to determine discriminating power of each criterion, aspects of correspondence and conflicts as well as the quality of each alternative by each criterion. Alternatives are presented by triangles and criteria by axes with square ends. Eccentric position of square of the criterion represents the volume of influence of that criterion, while correspondence between some criteria is defined by approximately the same direction of axe of those criteria. Criteria vectors expressing similar preferences on the data are oriented in the same direction, while conflicting criteria are pointing in opposite directions. The length of each

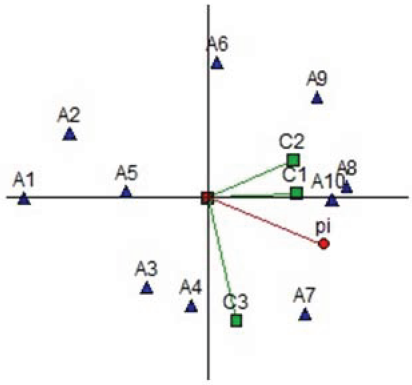

Figure 5: Example 1: Gaia planes with $N = 36$.

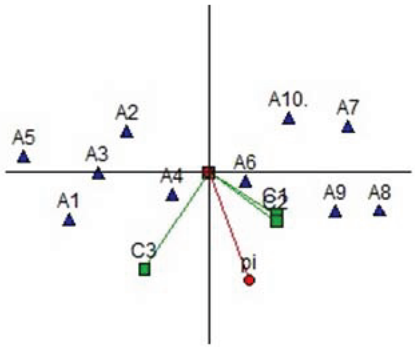

Figure 6: Example 1: Gaia planes with $N = 100$.

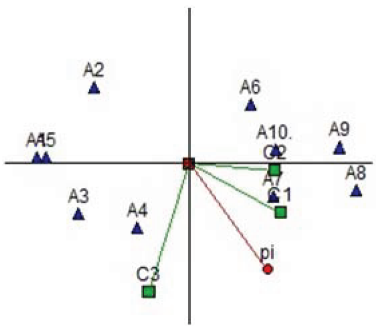

Figure 7: Example 2: Gaia planes with $N = 512$.

vector is a measure of its power in options' differentiation. Vector φ (decision axis) represents the direction of the compromise derived from the weights assignment.

4.3 consequences analysis with the help of DECISION LAB soft ware

Despite we can use potential adverse of the software in analyzing the sensitivity and determination of effectiveness of criteria validity. This capability help decision maker to observe the results of ranking when wights of criteria changed. For example, because of importance of the error in function approximations, the following figures show the consequences of rating of cases in 2 different forms with validities changed in first criteria.
Figure 8 displays of the cases according to the first weights of the criteria. Figure 9 shows of the cases according to the increase weight first criteria (0.33 to 0.50).

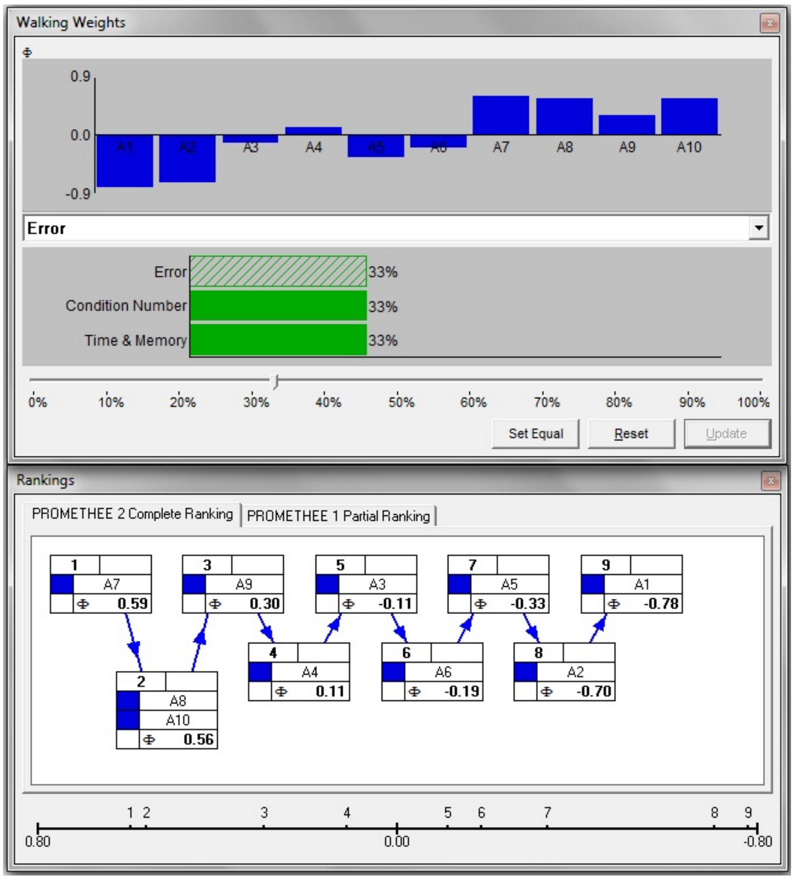

Figure 8: Example 1: Position of the cases according to the first weights of the criteria.

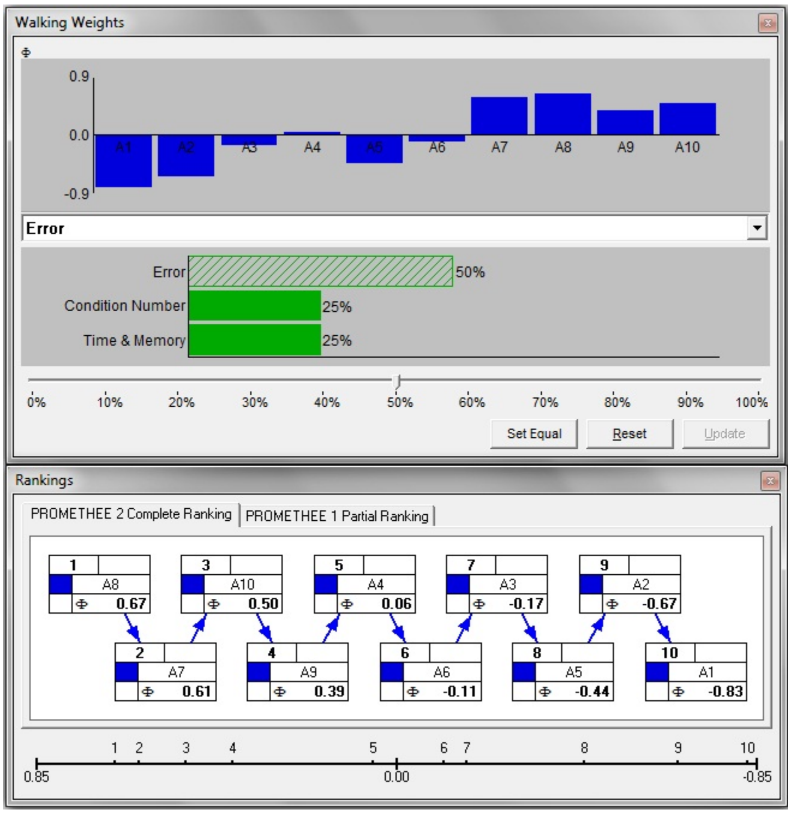

Figure 9: Example 1: Position of the cases according to the increase weight first criteria (0.33 to 0.50).

As observed, by changing the validity of the criteria, rating of the cases totally will be changed, so the applicants can evaluate the consequences of the factors validities changed in the final rating of the cases by using this method and also evaluate the effectiveness of each criterion.

5 Conclusions

Humans always are deciding in different conditions of their life and follow to find an appropriate solution for their problems; but decision making process is sometimes very complicated and necessity to assistance and counseling is unavoidable. So in the recent years, mathematical methods and knowledge of computer, as a helping decision making system has helped decision maker and create new branches and methods like MCDM techniques and decision support systems. Thus, we has used these technique in this research to optimize decision making of selecting the best radial basis functions methods and centers nodes.
Here, Two methods based on radial basis functions for approximate the solution of partial differential equation by using collocation method are applied. By choosing five sets of center

nodes: Uniform grid, Cartesian, Chebyshev, Legendre and LGL as Alternatives and achieving the error, Condition number of interpolation matrix and memory time as criteria, rating of cases with the help of PROMETHEE II technique is obtained. This ranking shows that Hermite interpolation by using non-uniform nodes as center nodes is appropriate when we applied RBF methods for solving partial differential equations.

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Appendix 1.

$$
\begin{aligned}
B[\Psi]_{ji} &= B[\psi](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \partial\Omega \times [0,T) , \hat{\mathbf{x}}_i \in \Omega \times [0,T) ,\\
I[\Psi]_{ji} &= I[\psi](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \Omega \times \{0\} , \hat{\mathbf{x}}_i \in \Omega \times [0,T) ,\\
L[\Psi]_{ji} &= L[\psi](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \Omega^\circ \times [0,T) , \hat{\mathbf{x}}_i \in \Omega \times [0,T) .
\end{aligned}
$$

Here we identify the collocation points same as center points. Ω° is interior of Ω. The problem is well-poses if the linear system $A\Lambda = C$ has unique solution [76]. C is defined of the form

$$
C = \begin{bmatrix} g(\hat{\mathbf{x}}_j) \\ f(\hat{\mathbf{x}}_j) \\ H(\hat{\mathbf{x}}_j) \end{bmatrix} . \tag{8}
$$

We note that a change in boundary conditions (4) is as simple as changing rows in matrix A in (6) as well as on the right hand side C in (8).

Appendix 2.

$$
\begin{aligned}
B\big[B^*[\Psi]\big]_{ji} &= B\big[B^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j, \hat{\mathbf{x}}_i \in \partial\Omega \times [0,T) ,\\
B\big[I^*[\Psi]\big]_{ji} &= B\big[I^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \partial\Omega \times [0,T) , \hat{\mathbf{x}}_i \in \Omega \times \{0\},\\
B\big[L^*[\Psi]\big]_{ji} &= B\big[L^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \partial\Omega \times [0,T) ,\ \hat{\mathbf{x}}_i \in \Omega^\circ \times [0,T) ,\\
I\big[B^*[\Psi]\big]_{ji} &= I\big[B^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \Omega \times \{0\} , \hat{\mathbf{x}}_i \in \partial\Omega \times [0,T),\\
I\big[I^*[\Psi]\big]_{ji} &= I\big[I^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j, \hat{\mathbf{x}}_i \in \Omega \times \{0\},\\
I\big[L^*[\Psi]\big]_{ji} &= I\big[L^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \Omega \times \{0\} ,\ \hat{\mathbf{x}}_i \in \Omega^\circ \times [0,T) ,\\
L\big[B^*[\Psi]\big]_{ji} &= L\big[B^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \Omega^\circ \times [0,T) , \hat{\mathbf{x}}_i \in \partial\Omega \times [0,T),\\
L\big[I^*[\Psi]\big]_{ji} &= L\big[I^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j \in \Omega^\circ \times [0,T), \hat{\mathbf{x}}_i \in \Omega \times \{0\},\\
L\big[L^*[\Psi]\big]_{ji} &= L\big[L^*[\psi]\big](\|\hat{\mathbf{x}}_j - \hat{\mathbf{x}}_i\|) , && \hat{\mathbf{x}}_j,\ \hat{\mathbf{x}}_i \in \Omega^\circ \times [0,T).
\end{aligned}
$$

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انتخاب نقاط مرکزی توابع پایه ای شعاعی با کمک تکنیک پرامیتی

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چکیده

در این مقاله تلاش می شود که بهترین نقاط مرکزی توابع پایه شعاعی را با استفاده از تکنیک‌های تصمیم‌گیری چند معیاره (MCDM) انتخاب کنیم. دو روش مبتنی بر توابع پایه‌ای شعاعی برای حل معادلات دیفرانسیل با مشتقات جزئی مورد استفاده قرار می‌گیرد. روش اول مبتنی بر روش کانسا و روش دوم مبتنی بر درون‌یابی هرمیتی می‌باشند. علاوه بر این، با انتخاب پنج مجموعه از نقاط مرکزی: کارتزین، هم‌فاصله، چبیشف، لژاندر و لژاندر گاوس لوباتو به عنوان گزینه‌های تحقیق و متغیرهای: خطا، عدد حالت ماتریس درون‌یاب و زمان اجرا به عنوان معیارهای تاثیرگذار، گزینه‌ها با کمک تکنیک پرامیتی رتبه‌بندی گردیدند. در نهایت بهترین نقاط مرکزی بر اساس رتبه بدست آمده انتخاب گردید. این رتبه‌بندی نشان می‌دهد که روش درون‌یابی هرمیتی با استفاده از نقاط غیر یکنواخت به عنوان نقاط مرکزی مناسب‌تر از روش کانسا با هر نقطه مرکزی است.

کلمات کلیدی

تصمیم گیری چندمعیاره، توابع مرکزي شعاعي، پرامیتي، درون یابي هرمیت، انتخاب بهینه .