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Single Facility Goal Location Problems with Symmetric and Asymmetric Penalty Functions

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Abstract. Location theory is an interstice field of optimization and operations research. In the classic location models, the goal is finding the location of one or more facilities such that some criteria such as transportation cost, the sum of distances passed by clients, total service time and cost of servicing are minimized. The goal Weber location problem is a special case of location models that has been considered recently by some researchers. In this problem the ideal is locating the facility in the distance r_i, from the i-th client. However, in most instances, the solution of this problem doesn't exist. Therefore, the minimizing sum of errors is considered. In the previous versions of the goal location problem the penalty functions have been considered by some symmetric functions such as square and absolute errors of distances between clients and ideal point. In this paper, we consider the asymmetric linex function as the error function. We consider the case that the distances are measured by L_p norm. Some iterative methods are used to solve the problem and the results are compared with some previously examined methods.

Keywords. Continuous location, Goal Weber problem, Weiszfeld-like method, Single facility, Linex function, BFGS method.

MSC. 90B90.

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1 Introduction

Nowadays location theory has many applications in the real life models. The Fermat-Weber single facility location problem is one of the fundamental models in location theory. In this problem, n existing demand points are located at known distinct points $A_1, \cdots, A_n$, and a new facility should be located at a point X such that the sum of weighted distances from it to the demand points is minimized. Let $d(X, A_i)$ represents the distance between points X and A_i and w_i be the weight of demands on point A_i. Then the Fermat-Weber single facility location problem is minimizing the following function:

$$F(X) = \sum_{i=1}^{n} w_i d(X, A_i). \tag{1}$$

Weiszfeld [18] in (1937) presented an iterative procedure for solving this problem with Euclidean distance. This algorithm is widely used because of its simplicity and effectiveness. The algorithm can be generalized to other location problems where the cost is a function of the Euclidean distance rather than just being proportional to the distance (see e.g. [5, 6, 8, 9, 16]).

Fathali et al. [11], introduced a special case of single facility location problem in which a specified radius for every demand point is considered and we want the distance between new facility and demand point A_i be equal to the corresponding radius, r_i. However, since in most of instances this point does not exist, they tried to minimize the sum of square errors. Jamalian and Fathali [12] presented a linear model for the problem with absolute error. Then in 2017, Fathali and Jamalian [10] called this problem Goal Weber Facility Location Problem (GWFLP) and presented a particle swarm optimization method for the model with square error. Recently, Soleimani et al. [15] considered the square error model with L_p norm and proposed a Gauss-Newton and imperialist competitive algorithm for solving the problem. Table 1 shows a brief literature review of GWFLP.

As mentioned in [11] one of application of GWFLP is finding the location of a company in the vicinity of some cities with respect to the establishing and transportation costs. Suppose that the cost of establishment a facility in the regions that is farther than a given distance r_i from the city i is very low. On the other hand, moving away from a city causes increasing transportation costs. Therefore a trade-off between establishing and transportation costs seems to be reasonable. Some other applications of GWFLP are in desirable and undesirable facility location models, where the facility shouldn't be close than a specified distance to the facility center, because of its undesirability. On the other hand, if the facility is so far from the city center, the cost of providing security, human forces, transportation and other costs will be increased.

Table 1: The literature review of GWFLP models.

	Norm	Penalty function		
Authors		Symmetric		Asymmetric
		Absolute error	Square error	Linex
Fathali et al.[11]	L_2	-	*	-
Jamalian et al. [12]	L_1	*	-	-
Fathali et al. [10]	L_2	-	*	-
Soleimani et al. [15]	L_p	*	*	-
Present paper	L_p	*	*	*

In all previously presented papers on goal facility location problem the objective function is symmetric, i.e. the cost of positive and negative errors are the same. However, in real applications these costs may not equal. Therefore, in this paper we consider the Linex loss function as the objective function. By changing the parameters of this function, it can cover many symmetric and asymmetric cases.

There are many iterative methods for solving nonlinear programming models (see e.g. [13]). Among them we used BFGS method and some of its modifications which are the most efficient quasi-Newton methods for solving unconstrained nonlinear models (see [4] and references therein). The BFGS and six modified versions of it are examined for solving GWFLP with Linex function and the results are compared with some other previously presented methods.

In what follows, the preliminary and definition of loss functions are given in Section 2. In Section 3, the main properties of the presented model are proposed. Section 4 contains iterative methods and the imperialist Competitive Algorithm to solve the considered problem. In Section 5, the computational results obtained by the proposed methods are given.

2 Penalty Functions

Definition 1. Let θ be a given parameter and D be the set of all estimators of θ. Then the error of estimation θ by $\delta \in D$ is shown by the penalty or loss function $E(\theta, \delta)$ that is satisfied in the following conditions:

1. $E(\theta, \delta) \geq 0, \quad \forall \delta \in D,$

2. $E(\theta, \delta) = 0, \iff \theta = \delta.$

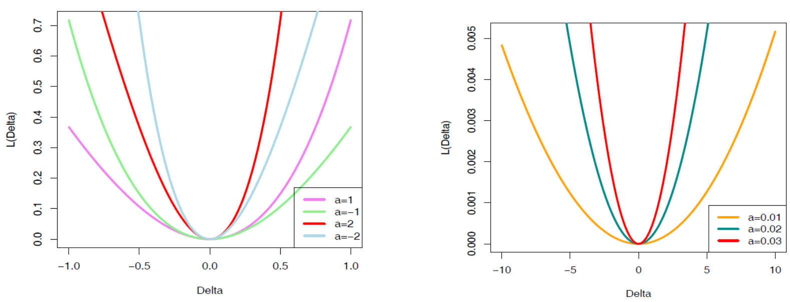
Two applicable symmetric loss functions are: 1- square error, $E_1(\theta, \delta) = (\theta - \delta)^2$, 2- absolute error, $E_2(\theta, \delta) = |\theta - \delta|$.

As an asymmetric loss function we consider the following well known Linex function, which is introduced by Varian [17].

$$E_3(\Delta) = b(e^{a\Delta} - a\Delta - 1),$$

where $\Delta = \theta - \delta$, $a \neq 0$ and $b > 0$.

The Linex function is convex. Figure 1 shows the loss function $E_3(\Delta)$ for $b = 1$ and varying values of a. For the small values of $|a|$, this function is nearly symmetric (see Figure 1 part b), however, for the case $|a| = 1$ it is not a symmetric function (see Figure 1 part a).



(a) **(b)**

Figure 1: The loss function $E_3(\Delta)$ for $b = 1$ and varying values of a.

3 Goal Weber Facility Location Problems

Let n points $A_1, ..., A_n$ be given in the plane. The coordinate and weight of point A_i, for $i = 1, ..., n$, are (a_i, b_i) and w_i, respectively. Let r_i, for $i = 1, ..., n$, be the given ideal distance between the point A_i and the server. In the Goal Weber Facility Location Problem (GWFLP) we want to find the location of a new facility $X = (x, y)$ in the plane such that the following function is minimized:

$$F(X) = \sum_{i=1}^{n} w_i E(d(X, A_i) - r_i), \tag{2}$$

where $d(X, A_i)$ for $i = 1, ..., n$, is the distance between points A_i and X.

In the case $E = E_2$, Fathali et al. [11] showed that the objective function of model (2) with L_2 norm is non convex and the optimal solution is in the extended rectangular hull of the demand points. Extended rectangular hull of demand points is the smallest rectangle which contains all the demand points with their circles. The same property for L_p norm, in the cases $E = E_1$ and $E = E_2$ have been shown in [15]. In this paper we consider the L_p norm with asymmetric linex loss function, i.e. the case $E = E_3$.

The model of GWFLP with linex loss function under L_p norm is as follows.

$$\min F_L(X) = \sum_{i=1}^{n} w_i(e^{a(L_p(X-A_i)-r_i)} - a(L_p(X - A_i) - r_i) - 1). \tag{3}$$

Theorem 1. The optimal solution of problem (3) is in the extended rectangular hull of the demand points.

Proof. Suppose ERH be the extended rectangular hull of the existing points. Then $RH_1 = (a_{min}, b_{min})$, $RH_2 = (a_{min}, b_{max})$, $RH_3 = (a_{max}, b_{max})$ and $RH_4 = (a_{max}, b_{min})$ are the extreme points of ERH, where

$$a_{min} = \min \{a_i - r_i | i = 1, \ldots, n\},$$
$$a_{max} = \max \{a_i + r_i | i = 1, \ldots, n\},$$
$$b_{min} = \min \{b_i - r_i | i = 1, \ldots, n\},$$
$$b_{max} = \max \{b_i + r_i | i = 1, \ldots, n\}.$$

Suppose $\bar{X} = (\bar{x}, \bar{y}) \notin ERH$, then the following cases may happen: 1- $\bar{x} > a_{max}$, 2- $\bar{x} < a_{min}$, 3- $\bar{y} > b_{max}$ and 4- $\bar{y} < b_{min}$. It is not so difficult to show that in each of these cases $\bar{X}$ is not optimal. We present the proof for the first case. The proofs of the other cases are the same.

Consider the case $\bar{x} > a_{max}$, then let $X' = (a_{max}, \bar{y})$. For $i = 1, \ldots, n$, we have

$$L_p(\bar{X} - A_i) = (|\bar{x} - a_i|^p + |\bar{y} - b_i|^p)^{\frac{1}{p}} > (|a_{max} - a_i|^p + |\bar{y} - b_i|^p)^{\frac{1}{p}} = L_p(X' - X_i) > r_i.$$

Therefor,

$$(|\bar{x} - a_i|^p + |\bar{y} - b_i|^p)^{\frac{1}{p}} - r_i > (|a_{max} - a_i|^p + |\bar{y} - b_i|^p)^{\frac{1}{p}} - r_i > 0.$$

Since $w_i \geq 0$, for $i = 1, \ldots, n$, thus

$$w_i(e^{a(L_p(\bar{X}-A_i)-r_i)} - a(L_p(\bar{X} - A_i) - r_i) - 1) > w_i(e^{a(L_p(X'_i)-r_i)} - a(L_p(X'_i) - r_i) - 1).$$

So $F_L(\bar{X}) > F_L(X')$ and $\bar{X}$ is not an optimal solution of (3). □

Now we want to find the optimal solution of problem (3). Since Linex function is convex and $w_i \geq 0$, for $i = 1, \cdots, n$, then the following lemma holds.

Lemma 1. The objective function of (3) is convex.

Note that as Fathali et al. [11] showed, the GWFLP with Euclidean norm is not convex and local optimum may not be global. However, in the case of Linex function if we could find a local optimum then it is also global.

3.1 The Weiszfeld Like Method

To find the optimal solution of our considered problem, we can use the necessary condition of optimality, i.e. $\frac{\partial F_L(X)}{\partial X} = 0$ to obtain a candidate optimal solution. But $F_L(X)$ is not differentiable in the existing points. Therefore, we use the following hyperbolic approximation of L_p norm.

$$L_p^h(X - A_i) = (((x - a_i)^2 + \epsilon)^{\frac{p}{2}} + ((y - b_i)^2 + \epsilon)^{\frac{p}{2}})^{\frac{1}{p}},$$

where ϵ is a small positive number.

Consequently, the objective function of GWFLP with this norm is defined as follows.

$$F_L^h(X) = \sum_{i=1}^{n} w_i(e^{a(L_p^h(X-A_i)-r_i)} - a(L_p^h(X - A_i) - r_i) - 1) \tag{4}$$

Lemma 2.

$$\lim_{\epsilon \to 0} |F_L^h(X) - F_L(X)| = 0.$$

Proof. By Mankowski inequality

$$\begin{aligned}
&|w_i(e^{a(L_p^h(X-A_i)-r_i)} - a(L_p^h(X - A_i) - r_i) - 1) - w_i(e^{a(L_p(X-A_i)-r_i)} - a(L_p(X - A_i) - r_i) - 1)| \\
&\quad = |w_i(e^{a(L_p^h(X-A_i)-r_i)} - e^{a(L_p(X-A_i)-r_i)}) - aw_i(L_p^h(X - A_i) - L_p(X - A_i))| \\
&\quad \leq w_i(|e^{a(L_p^h(X-A_i)-r_i)} - e^{a(L_p(X-A_i)-r_i)}| + |a||L_p^h(X - A_i) - L_p(X - A_i)|) \\
&\quad \leq w_i(|e^{a(L_p(X-A_i)-r_i)}(e^{(2\epsilon)^{\frac{1}{p}}} - 1)| + |a|(2\epsilon)^{\frac{1}{p}}).
\end{aligned}$$

Thus

$$|F_L^h(X) - F_L(X)| \leq \sum_{i=1}^{n} w_i|e^{a(L_p(X-A_i)-r_i)}(e^{(2\epsilon)^{\frac{1}{p}}} - 1)| + |a|(2\epsilon)^{\frac{1}{p}} \sum_{i=1}^{n} w_i.$$

Therefore

$$\lim_{\epsilon \to 0} |F_L^h(X) - F_L(X)| = 0.$$

□

Now we use the necessary optimality condition for $F_l^h(X)$.

$$\frac{\partial F_L^h}{\partial x} = \sum_{i=1}^{n} aw_i \left(\frac{Ra_i(e^{a(L_p^h(X-A_i)-r_i)} - 1)}{DX_i}(x - a_i) \right) = 0, \tag{5}$$

and

$$\frac{\partial F_L^h}{\partial y} = \sum_{i=1}^{n} aw_i \left(\frac{Rb_i(e^{a(L_p^h(X-A_i)-r_i)} - 1)}{DX_i}(y - b_i) \right) = 0, \tag{6}$$

where

$$DX_i = (((x-a_i)^2+\epsilon)^{\frac{p}{2}} + ((y-b_i)^2+\epsilon)^{\frac{p}{2}})^{\frac{p-1}{p}},$$
$$Ra_i = ((x-a_i)^2+\epsilon)^{\frac{p-2}{2}},$$

and

$$Rb_i = ((y-b_i)^2+\epsilon)^{\frac{p-2}{2}}.$$

Therefore, by starting with an initial point $X^0 = (x^0, y^0)$, we can use the following iterative method.

$$x^{t+1} = \frac{\sum_{i=1}^{n}\left(\frac{w_i Ra_i^t (e^{a(L_p^h(X^t-A_i)-r_i)}-1)}{DX_i^t} a_i\right)}{\sum_{i=1}^{n}\left(\frac{w_i Ra_i^t (e^{a(L_p^h(X^t-A_i)-r_i)}-1)}{DX_i^t}\right)}, \tag{7}$$

and

$$y^{t+1} = \frac{\sum_{i=1}^{n}\left(\frac{w_i Rb_i^t (e^{a(L_p^h(X^t-A_i)-r_i)}-1)}{DX_i^t} b_i\right)}{\sum_{i=1}^{n}\left(\frac{w_i Rb_i^t (e^{a(L_p^h(X^t-A_i)-r_i)}-1)}{DX_i^t}\right)}. \tag{8}$$

The main idea of this method is the same as Weiszfeld algorithm, therefore we call it the Weiszfeld Like Algorithm (WLA).

Since $L_p^h(X^t-A_i) = (DX_i^t)^{\frac{1}{p-1}}$, therefore,

$$\begin{aligned} x^{t+1} &= \frac{\sum_{i=1}^{n}\left(\frac{w_i Ra_i^t (e^{a((DX_i^t)^{\frac{1}{p-1}}-r_i)}-1)}{DX_i^t} a_i\right)}{\sum_{i=1}^{n}\left(\frac{w_i Ra_i^t (e^{a((DX_i^t)^{\frac{1}{p-1}}-r_i)}-1)}{DX_i^t}\right)} \\ &= x(t) - \frac{\frac{\partial F_L^h(X^t)}{\partial x}}{a\sum_{i=1}^{n}\left(\frac{w_i Ra_i^t (e^{a((DX_i^t)^{\frac{1}{p-1}}-r_i)}-1)}{DX_i^t}\right)}. \end{aligned} \tag{9}$$

and

$$\begin{aligned} y^{t+1} &= \frac{\sum_{i=1}^{n}\left(\frac{w_i Rb_i^t (e^{a((DX_i^t)^{\frac{1}{p-1}}-r_i)}-1)}{DX_i^t} b_i\right)}{\sum_{i=1}^{n}\left(\frac{w_i Rb_i^t (e^{a((DX_i^t)^{\frac{1}{p-1}}-r_i)}-1)}{DX_i^t}\right)} \\ &= y(t) - \frac{\frac{\partial F_L^h(X^t)}{\partial y}}{a\sum_{i=1}^{n}\left(\frac{w_i Rb_i^t (e^{a((DX_i^t)^{\frac{1}{p-1}}-r_i)}-1)}{DX_i^t}\right)}. \end{aligned} \tag{10}$$

The presented method in this section indeed is a fixed point iteration method which is a linear convergence method. In the next section, we use a quasi-Newton method which is faster than Wieiszfeld like algorithm.

4 The BFGS Method and Its Modified Versions

The BFGS method is the most popular quasi-Newton algorithm, named for its discoverers Broyden, Flecher, Goldfarb, and Shanno. Quasi-Newton methods only used the gradient of the objective function in each iteration and the second derivatives are not required. Therefore, these methods are sometimes more efficient than Newton's methods (see [13]).

The BFGS method use the following iterative method

$$x_{k+1} = x_k - \alpha_k H_k g_k,$$

where $g_k = \nabla f(x_k)$, H_k is an approximation of the Hessian of f in x_k, and α_k is the step length that can be obtained by line search methods. Usually, α_k is chosen to satisfy the following conditions, called Wolfe conditions,

$$f(x_k + \alpha_k d_k) \leq f(x_k) + \rho \alpha_k \nabla f_k^T d_k, \tag{11}$$

$$f(x_k + \alpha_k d_k)^T d_k \geq \sigma \nabla f_k^T d_k, \tag{12}$$

where $0 < \rho < \sigma < 1$ and d_k is the search direction in the iterative method.

In each iteration, H_k is updated as the following

$$H_{k+1} = (I - \rho_k s_k y_k^T) H_k (I - \rho_k y_k s_k^T) + \rho_k s_k s_k^T, \tag{13}$$

where $y_k = g_{k+1} - g_k$, $s_k = x_{k+1} - x_k$ and $\rho_k = \frac{1}{y_k^T s_k}$.

The BFGS method can be stated as the following algorithm.

There are many modifications of BFGS method for solving unconstrained optimization problems in the literature. In the next section, we use six modified versions of BFGS for solving GWFLP. These methods are listed in Table 2.

5 Computational Results

In this section we compare the results of BFGS methods and WLA, with Imperialist Competitive Algorithm (ICA) and Gauss-Newton method (GN) [15].

Algorithm 1 (BFGS)[13].

Input: The starting point x_0, convergence tolerance $\epsilon > 0$, inverse Hessian approximation H_0.

Set $k := 0$ (iteration counter).

Iteration step:

While $||\nabla f_k|| > \epsilon$ **do** the following:

1. **Compute** search direction $d_k = -H_k \nabla f_k$.
2. **Set** $x_{k+1} = x_k + \alpha_k d_k$, where α_k is computed from a line search procedure to satisfy the Wolfe conditions (11) and (12).
3. **Define** $s_k = x_{k+1} - x_k$ and $y_k = \nabla f_{k+1} - \nabla f_k$.
4. **Compute** H_{k+1} using (13).
5. **Set** $k := k + 1$.

end while

Table 2: The modifications of BFGS and their references.

Method	Author/Authors
SBFGS	Andrei [1]
TSBFGS	Andrei [2]
MBFGS	Yuan and Zengxin [19]
MMLSBFGS	Babaie-Kafaki [3]
TMLSBFGS	Babaie-Kafaki [4]
SMMLSBFGS	Oren and Luengerger [14]

Consider the problem with $n = 30$ points that their coordinates, weights and ideal distances are given in Table 3. In this table the columns with the heading ID show the three cases of ideal distances.

In the case that the distances are measured by L_2 norm, i.e. $p = 2$, all methods converged to the points $(8.34, 8.08)$, $(8.30, 7.74)$ and $(8.34, 8.09)$ with objective functions 45121, 12816 and 5928, respectively for the three cases of ideal distances. Table 4 contains the average CPU times of each of the examined methods.

As the results show, BFGS methods are faster than the other methods with the same objective functions. Table 5 shows the results obtained by BFGS for varying norms and the second case of ideal distances (ID2).

Table 3: The coordinates, weights and ideal distances of existing points.

(x, y, w)	ID	(x, y, w)	ID	(x, y, w)	ID
(1,3,3)	(1,2,3)	(14,15,1)	(1,2,3)	(10,8,1)	(1,1,3)
(1,4,2)	(1,2,3)	(14,3,1)	(1,1,3)	(10,10,3)	(1,2,3)
(2,15,1)	(1,3,3)	(14,1,2)	(1,2,3)	(11,4,2)	(1,2,3)
(2,4,3)	(1,2,3)	(15,8,3)	(1,2,3)	(11,13,3)	(1,1,3)
(3,6,2)	(1,2,3)	(15,10,3)	(1,3,3)	(13,3,1)	(1,2,3)
(7,15,2)	(1,2,3)	(3,2,1)	(1,3,3)	(3,10,2)	(1,2,3)
(8,3,1)	(1,2,3)	(4,6,1)	(1,2,3)	(9,11,2)	(1,2,3)
(8,6,3)	(1,2,3)	(4,3,2)	(1,3,3)	(15,15,2)	(1,2,3)
(8,5,1)	(1,2,3)	(6,8,3)	(1,3,3)	(7,14,2)	(1,2,3)
(8,2,1)	(1,3,3)	(6,11,1)	(1,1,3)	(13,7,3)	(1,3,3)

Table 4: The CPU times of examined methods.

Method	CPU Time in second
ICA	14.25
GN	4.30
WLA	160.21
BFGS	2.31
SBFGS	2.29
TSBFGS	2.51
MBFGS	2.05
MMLSBFGS	1.95
TMLSBFGS	1.96
SMMLSBFGS	2.01

Table 5: The results of BFGS for varying values of p and the ideal distance ID2.

p	X	$F(X)$
1	(8.58,7.72)	386540
1.5	(8.42, 7.68)	32950
2	(8.30, 7.74)	12816
5	(8.18, 8.12)	4162
10	(8.13, 8.26)	3484

We also examined the examples with 100, 200 and 500 existing points. The coordinates, weights and ideal distances are generated randomly. Table 6 shows the optimal points obtained by all examined methods. Since all methods find the same solution, we compare the CPU time and number of iterations in Table 7. Since for the generated instances the CPU times of ICA and WLA methods are extraordinarily large and BFGS methods are very faster than GN, we just compare the results of BFGS methods.

Table 6: The results of large examples for the varying values of p.

n	p	$X = (x, y)$
100	1.5	(31.38,33.89)
100	2	(31.53,33.65)
100	3	(31.96,32.78)
200	1.5	(30.94,31.94)
200	2	(30.94,32.02)
200	3	(31.02,31.96)
500	1.5	(32.23,31.22)
500	2	(32.22,31.16)
500	3	(32.11,31.06)

Table 7: The CPU times and the number of iterations for the large examples with the varying values of p.

Method	n	$p = 1.5$		$p = 2$		$p = 3$	
		CPU time	iteration	CPU time	iteration	CPU time	iteration
BFGS	100	11.18	8	6.20	7	8.31	6
	200	18.73	5	11.04	6	17.16	5
	500	41.02	6	27.28	5	44.73	6
SBFGS	100	11.11	8	6.18	7	8.41	6
	200	18.10	5	12.56	7	17.15	5
	500	40.91	6	27.13	5	44.63	6
TSBFGS	100	11.59	9	7.74	9	9.32	6
	200	21.95	6	13.74	7	21.84	8
	500	45.19	8	30.26	6	41.25	5
MBFGS	100	9.87	7	5.33	4	9.39	5
	200	17.92	6	13.54	7	17.28	5
	500	34.73	4	27.20	6	42.39	5
MMLSBFGS	100	7.94	6	6.03	7	8.21	6
	200	16.74	5	12.03	5	13.54	4
	500	35.50	5	27.23	5	36.52	4
TMLSBFGS	100	9.39	7	6.43	7	9.10	6
	200	16.61	5	12.06	5	16.46	5
	500	35.65	5	27.43	5	39.06	5
SMMLSBFGS	100	9.38	7	6.45	7	9.10	6
	200	16.95	5	12.32	5	16.51	5
	500	35.82	5	27.27	5	38.80	5

Efficiency comparisons were drawn using the Dolan-Mor´e performance profile [7] on the running time and the total number of function. Performance profile gives, for every $t \geq 1$, the proportion $p(t)$ of the test problems that each considered algorithmic variant has a performance within a factor of t of the best. Figure 2 shows the performance profiles of the total number of function and the CPU time for BFGS methods. The results show that the MMLSBFGS and TMLSBFGS are computationally preferable to the other methods.

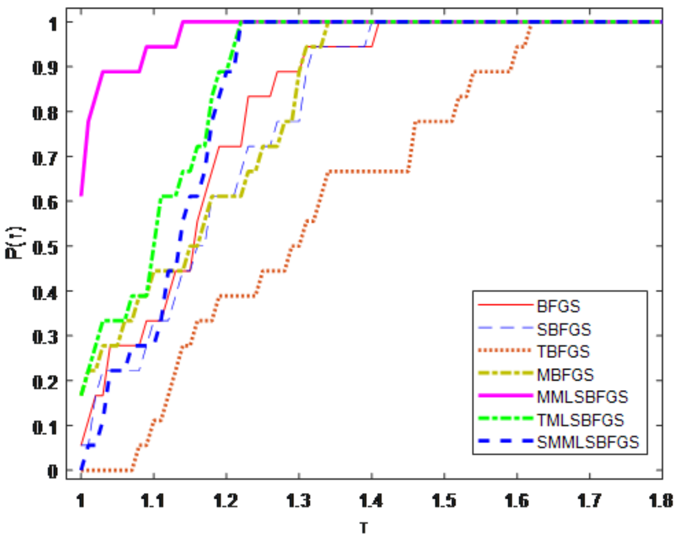


Figure 2: Performance profiles of the total number of function and the CPU time for BFGS methods.

The properties of system were used during the tests are: core i5, 2.6 GHz and 6 GB RAM. In all instances, the parameters in the BFGS method are chosen as follows.

$$\rho = 10^{-4}, \ \sigma = 0.1, X_0 = \binom{2}{2}, \ \epsilon = 10^{-5}, \ H_0 = I.$$

6 Summary and Conclusion

In this paper we study a version of the single facility location problem in which we want to find the location of a new facility such that the sum of weighted errors under linex loss function is minimized. This problem is convex and we showed that optimal solution of the problem is in the extended convex hull of demand points. We used the iterative Weiszfeld-like and BFGS algorithms for solving the considered problem. The results are compared with those obtained by ICA and GN algorithms. The results show that the BFGS iterative procedures are the best in the time of solving problem.

Other kinds of objective functions such as Hamming norm and minimax criteria and also multiple facility location problem can be considered in the future works.

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مسائل مکانیابی تک وسیله‌ای وبر آرمانی تحت توابع جریمه متقارن و نامتقارن

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چکیده

نظریه مکانیابی یکی از مباحث جذاب در بهینه‌سازی و تحقیق در عملیات است. در مدل‌های کلاسیک مکانیابی، هدف پیدا کردن مکان یک یا چند سرویس دهنده است به قسمی که معیارهایی از قبیل هزینه حمل و نقل، مجموع فاصله پیموده شده توسط مشتریان، زمان نهایی سرویس و هزینه سرویس‌دهی کمینه شود. مساله مکانیابی وبر آرمانی یک حالت خاص از مسائل مکانیابی است که اخیرا مورد توجه پژوهشگران قرار گرفته است. در این مساله ایده‌آل این است که سرویس دهنده دقیقا در فاصله r_i از مشتری iام قرار گیرد. اما در اغلب موارد این مساله دارای جواب نیست. لذا در مساله مکانیابی آرمانی به دنبال کمینه کردن مجموع وزنی خطا هستیم. در مقالات قبلی، تابع جریمه به صورت توابع متقارن، از قبیل مجذور و قدر مطلق مجموع خطای فاصله بین مشتریان و نقطه ایده‌آل در نظر گرفته شده است. در این مقاله تابع خطا را به صورت تابع لینکس در نظر می‌گیریم که می‌تواند نامتقارن باشد. حالتی که فاصله‌ها با نرم L_p اندازه گرفته می‌شود را در نظر می‌گیریم. چند روش تکراری را برای حل مساله بررسی کرده و روش‌های ارائه شده را با استفاده از چند مثال با هم مقایسه می‌کنیم.

کلمات کلیدی

مکانیابی آرمانی، تابع جریمه لینکس، روش وایزفلد، روش BFGS، مکانیابی پیوسته.