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Global Forcing Number for Maximal Matchings under Graph Operations

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Abstract. Let $S = \{e_1, e_2, \ldots, e_m\}$ be an ordered subset of edges of a connected graph G. The edge S-representation of an edge set $M \subseteq E(G)$ with respect to S is the vector $r_e(M|S) = (d_1, d_2, \ldots, d_m)$, where $d_i = 1$ if $e_i \in M$ and $d_i = 0$ otherwise, for each $i \in \{1, \ldots, k\}$. We say S is a global forcing set for maximal matchings of G if $r_e(M_1|S) \neq r_e(M_2|S)$ for any two maximal matchings M_1 and M_2 of G. A global forcing set for maximal matchings of G with minimum cardinality is called a minimum global forcing set for maximal matchings, and its cardinality, denoted by φ_{gm}, is the global forcing number (GFN for short) for maximal matchings. Similarly, for an ordered subset $F = \{v_1, v_2, \ldots, v_k\}$ of $V(G)$, the F-representation of a vertex set $I \subseteq V(G)$ with respect to F is the vector $r(I|F) = (d_1, d_2, \ldots, d_k)$, where $d_i = 1$ if $v_i \in I$ and $d_i = 0$ otherwise, for each $i \in \{1, \ldots, k\}$. We say F is a global forcing set for independent dominatings of G if $r(D_1|F) \neq r(D_2|F)$ for any two maximal independent dominating sets D_1 and D_2 of G. A global forcing set for independent dominatings of G with minimum cardinality is called a minimum global forcing set for independent dominatings, and its cardinality, denoted by φ_{gi}, is the GFN for independent dominatings. In this paper we study the GFN for maximal matchings under several types of graph products. Also, we present some upper bounds for this invariant. Moreover, we present some bounds for φ_{gm} of some well-known graphs.

Keywords. Global forcing set, Global forcing number, Maximal matching, Maximal independent dominating, Product graph.

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1 Introduction

All graphs considered in this paper are connected and simple. For a graph G we denote by V_G the set of its vertices and by E_G the set of its edges. The number of vertices of G is the **order** of G and the number of edges of G is the **size** of G.

Let $G = (V_G, E_G)$ be a graph. A subset M_G of E_G is called a **matching** of G if no two edges of M are adjacent. The vertices incident to the edges of a matching M_G are said to be **saturated**(or M_G-saturated) by M_G; the others are said to be **unsaturated** (or M_G-unsaturated). If there does not exist a matching M'_G in G such that $|M_G| < |M'_G|$, then M_G is called a **maximum matching** of G, and its cardinality is denoted by $\upsilon(G)$. A matching M_G is **maximal** if it cannot be extended to a larger matching in G, see [17]

The concept of forcing set is one of the most applicable graph-theoretical concepts which was first introduced by Klein and Randić in [10]. One can see [19, 20] for application of forcing set in large-scale computations. Also, [6, 10] are recommended to get information about relation between the innate degree of freedom in mathematical chemistry and the forcing set in graph theory. On the other hand, several purely graph-theoretical literatures on forcing set, such as [1, 2, 14, 16, 22], show importance of this parameter in graph theory. Recently, Vukičević et al. [13] have extended the concepts of global forcing set and global forcing number to maximal matchings as follows.

Let $S = \{e_1, e_2, \ldots, e_m\}$ be an ordered subset of edges of a connected graph G. The **edge S-representation** of an independent edge set $M \subseteq E(G)$ with respect to S is the vector $r_e(M|S) = (d_1, d_2, \ldots, d_m)$, where $d_i = 1$ if $e_i \in M$ and $d_i = 0$ otherwise, for $i \in \{1, \ldots, k\}$. We say S is a **global forcing set for maximal matchings** of G if $r_e(M_1|S) \neq r_e(M_2|S)$ for any two maximal matchings M_1 and M_2. A global forcing set for maximal matchings of G with minimum cardinality is called a **minimum global forcing set for maximal matchings**, and its cardinality, denoted by φ_{gm}, is the **global forcing number (GFN) for maximal matchings**.

A set of non-adjacent vertices of a graph G is called **independent set**. The size of a largest independent set is called the **independence number** of G and denoted by $\alpha(G)$.

For a graph $G = (V_G, E_G)$, we say $D_G \subseteq V_G$ is an **independent dominating set** in G if D_G is a set of non-adjacent vertices and each vertex of $V_G \setminus D_G$ is adjacent to at least one vertex in D_G. The **independent domination number** of G, denoted by $i(G)$, is the minimum cardinality of an independent dominating set of G, see [18]. If we drop the requirement of independence, we obtain **dominating sets**, and the smallest cardinality of a dominating set in G is the **domination number** of G, denoted by $\gamma(G)$, see [8].

For stating our results, we need to expand the concept of forcing independent spectrum of graphs which was introduced in [15] as follows.

Let $F = \{v_1, v_2, \ldots, v_k\}$ be an ordered subset of vertices of a connected graph G. The **F-representation** of an independent set $I \subseteq V(G)$ with respect to F is the vector $r(I|F) = (d_1, d_2, \ldots, d_k)$, where $d_i = 1$ if $v_i \in I$ and $d_i = 0$ otherwise, for $i \in \{1, \ldots, k\}$. We say F is a **global forcing set for independent dominatings** of G if $r(D_1|F) \neq r(D_2|F)$ for any two maximal independent dominating sets D_1 and D_2. A global forcing set for independent dominatings of G with minimum cardinality is called a **minimum global forcing set for independent dominatings**, and its cardinality, denoted by φ_{gi}, is the **GFN for independent dominatings**.

According to this fact that computing GFN even for quite restricted classes of graphs is algorithmically difficult, we are interested in studying this invariant via graph products. As applications of our results, we compute the GFN for maximal matching number of some fullerene graphs. We also present some upper bounds for this invariant by line and the GFN of maximal independent dominatings.

We remind that all notations and terminologies are standard here and taken mainly from the standard books of graph theory. For instance, as usual we denote the maximum degree and the minimum degree of a graph G by Δ (or Δ_G) and δ (or δ_G), respectively. Also, the hypercube Q_n is a graph in which vertices are n-tuples $(t_1, t_2, \ldots, t_n)$ where $t_i \in \{0, 1\}$ and two vertices are adjacent when their n-tuples differ in exactly one coordinate. Moreover, we denote the path and cycle graphs of order n by P_n and C_n, respectively.

2 Main Results

For stating our results, we need the below results:

Proposition 1. [13] Let $S \subseteq E_G$ be a set of edges such that the graph induced by $E_G \setminus S$ has only one maximal matching. Then S is a global forcing set for maximal matchings.

Corollary 1. [13] Let M be any matching in G. Then $E_G \setminus M$ is a global forcing set for maximal matchings in G.

Theorem 1. [13] Let G be a simple graph on n vertices and m edges. Then $\varphi_{gm}(G) \leq m - \nu(G)$.

2.1 Generalized hierarchical product

Let G and H be two graphs and U be a nonempty subset of V_G. The **generalized hierarchical product** of G and H, $G(U) \sqcap H$, is a graph whose vertex and edge sets are defined as follow:

$$V_{G(U)\sqcap H} = \{(g, h) \mid g \in V_G \text{ and } h \in V_H\},$$
$$E_{G(U)\sqcap H} = \{(g, h)(g', h') \mid (gg' \in E_G \text{ and } h = h') \text{ or } (g = g' \in U \text{ and } hh' \in E_H)\}.$$

This product has several applications in other branches of science such as computer science. We refer interested readers to study [3, 9, 12].

Theorem 2. If G and H are two graphs with n and m vertices, respectively, and U is a nonempty subset of V_G, then

$$\varphi_{gm}(G(U) \sqcap H) \leq m|E_G| + |U||E_H| - \max_{X \subseteq U}\{m\nu(G - X) + |U \setminus V_{G(M_{G-X})}|\nu(H)\}.$$

Proof. Let $X \subseteq U$, M_{G-X} be a maximum matching of $G-X$ (which has minimum meet with U among all maximum matchings of $G-X$), and M_H be a maximum matching of H. Set

$$M = \{(g,h)(g',h') \mid \ (gg' \in M_{G-X} \ \text{ and } h \in V_H) \text{ or } (g = g' \in X \text{ and } hh' \in M_H\}.$$

It is clear that M is a matching in $G(U) \sqcap H$. We claim M is maximal. To prove our claim, let $e = (g,h)(g',h')$ be an edge of $G(U) \sqcap H$ which is not in M. Thus, e can be in the following two possible forms:

case 1. $g = g' \in U$ and $hh' \in E_H$. In order to e is not in M then $hh' \in E_H \setminus (M_H)$ and so hh' cannot be added to M_H for constructing a larger matching, and consequently e cannot be added to M to obtain a larger matching.

case 2. $h = h'$ and $gg' \in E_G$. Since e is not in M, then $gg' \in E_G \setminus M_{G-X}$. Thus M_G cannot be extended to $M_{G-X} \cup \{gg'\}$ as a matching for G, and so M cannot be extended to $M \cup \{e\}$ as a matching in $G(U) \sqcap H$.

Therefore, M is a maximal matching in $G(U) \sqcap H$. So, according to Corollary 1, $E_{G(U)\sqcap H} \setminus M$ is a global forcing maximal matching in $G(U) \sqcap H$. On the other hand, the cardinality of M is equal to $m\nu(G-X) + |U \setminus V_{G(M_{G-X})}|\nu(H)$. Thus, by Theorem 1,

$$\varphi_{gm}(G(U) \sqcap H) \leq m|E_G| + |U||E_H| - \max_{X \subseteq U}\{m\nu(G-X) + |U \setminus V_{G(M_{G-X})}|\nu(H)\},$$

which completes our proof. □

Corollary 2. Let G and H be two graphs with n and m vertices, respectively, and U be a nonempty subset of V_G. If G has a maximum matching which has no meet with U, then

$$\varphi_{gm}(G(U) \sqcap H) \leq m(|E_G| - \nu(G)) + |U|(|E_H| - \nu(H)).$$

A **Zig-Zag Polyhex Lattice**, $H_{r,2t+1}$, is a planar graph with $2t+1$ rows of hexagonals such that there are r and $r-1$ hexagonals in each row, alternatively. Look at Figure 1 for more illustration.

Let $P_{2r+1} := v_1, v_2, \ldots, v_{2r+1}$ be a path. The graph $H_{r,1}$ is isomorphic to $P_{2r+1}(U) \sqcap P_2$ where $U = \{v_i \in V_{P_{2r+1}} \mid i \text{ is an odd number}\}$. Thus, the function $f = 2\nu(P_{2r+1} - X) + |U \setminus V_{P_{2r+1}(M_{P_{2r+1}-X})}|$, from the power set of U to $\mathbb{N}$, attains its maximum at $X = \{v_1\}$; because v_1 is the just vertex of P_{2r+1} which is not saturated; on the other hand, $\nu(H_{r,1} - X)$ is decreasing when $|X|$ is increasing. Therefore, by replacing $\nu(P_{2r+1}) = r$ and $X = \{v_1\}$ in Theorem 2, we have $\varphi_{gm}(H_{r,1}) \leq 3r$.

Parts (a) and (b) of Figure 1 show $H_{7,1}$ and $H_{7,3}$, respectively. Black vertices in these figures are elements of U and bold edges in this figure are elements of the global forcing set for maximal matchings of $H_{7,1}$ and $H_{7,3}$, respectively, which is defined in the proof of Theorem 2. By replacing $r = 7$ in $\varphi_{gm}(H_{r,1}) \leq 3r$, we have $\varphi_{gm}(H_{r,1}) \leq 21$; on the other hand, one can check that the exact value of $\varphi_{gm}(H_{r,1})$ is equal to 21. This shows the presented upper bound for $\varphi_{gm}(H_{r,1})$ is sharp.

Corollary 3. For every positive integer number r and $t \in \{2^i - 1\}_{i=1}^{\infty}$,

$$\varphi(H_{r,2t+1}) \leq 6rt + 5r + t + 1 - 2^{log_2^{t+1}}(2r+1).$$

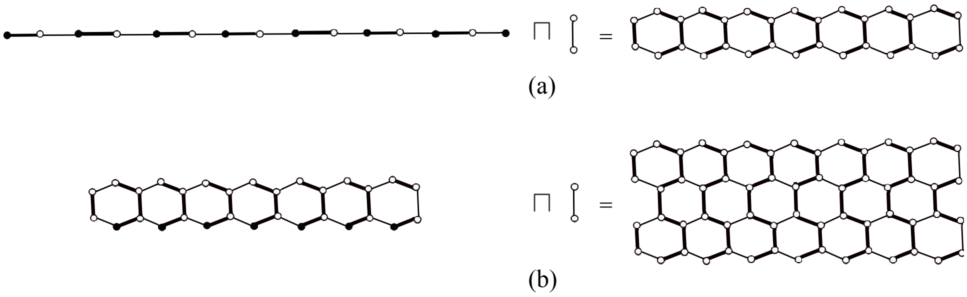

Figure 1: Graphs $H_{7,1}$ and $H_{7,3}$ with their global forcing set for maximal matchings.

Proof. It is not difficult to check that $H_{r,2t+1} = H_{r,t}(U) \sqcap P_2$. Then, by Theorem 2,

$$\varphi_{gm}(H_{r,2t+1}) = \varphi_{gm}(H_{r,t}(U) \sqcap P_2) \leq 2|E_{H_{r,t}}| + |U| \\ - \max_{X \subseteq U}\{2\nu(H_{r,t} - X) + |U \setminus V_{H_{r,t}(M_{H_{r,t}-X})}|\}.$$

So, it is enough to obtain $|E_{H_{r,t}}|$, $\nu(H_{r,t})$ and X. Consider the function $f = 2\nu(H_{r,t} - X) + |U \setminus V_{G(M_{H_{r,t}-X})}|$, from the power set of U to $\mathbb{N}$. Function f is decreasing; because if the size of X increases, then $\nu(H_{r,t} - X)$ decreases (as much as $|U \setminus V_{G(M_{H_{r,t}-X})}|$ increases). Therefore, f attains its maximum at $X = \emptyset$. On the other hand, $|E_{H_{r,t}}| = 3rt + 2r + \frac{t+1}{2}$ and $\nu(H_{r,t}) \geq 2^{log_2^{t+1}-1}(2r+1)$ which completes our proof. □

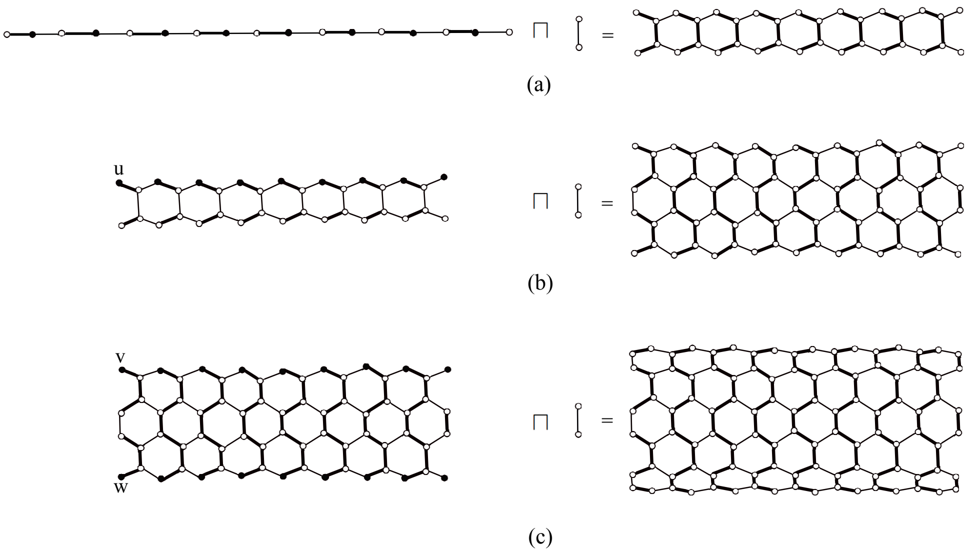

Figure 2: Graphs $G_{7,1}$, $G_{7,2}$ and $A_{7,8}$ with their global forcing maximal matching.

A **Zig-Zag Polyhex Lattice-like**, $G_{r,k}$ is a planar graph with $2^k - 1$ rows of hexagonals such that there are r and $r+1$ hexagonals in each row, alternatively, and there is a pendent vertex at both ends of its first and last level. See parts (a) and (b) of Figure 2. In part (a), $G_{7,1}$ has one row of hexagonals and two levels (note that each row is formed by two levels).

Armchair graph $A_{r,k}$ is a tube whose surface is covered with hexagonals such that there are k rows of hexagonals on it such that there are r and $r+1$ hexagonals in the rows, alternately. Part (c) of Figure 2 shows armchair graph $A_{7,8}$.

Corollary 4. If r is a positive integer number and $k \in \{2^{i+1}\}_{i=1}^{\infty}$, then $\varphi(A_{r,k}) \leq 2^{2+log_2^{\frac{k}{2}}}(r+1)$.

Proof. By the definition of generalized hierarchical product, we can say $A_{r,k}$ isomorphic to $G_{r,log_2^{\frac{k}{2}}}(U) \sqcap P_2$ where U is independent vertices of the first and last level of $G_{r,log_2^{\frac{k}{2}}}$. So, by applying Theorem 2, we have

$$\varphi_{gm}(A_{r,k}) = \varphi_{gm}(G_{r,log_2^{\frac{k}{2}}}(U) \sqcap P_2) \leq 2|E_{G_{r,log_2^{\frac{k}{2}}}}| + |U|$$
$$- \max_{X \subseteq U}\{2\nu(G_{r,log_2^{\frac{k}{2}}} - X) + |U \setminus V_{G(M_{G_{r,log_2^{\frac{k}{2}}}} - X)}|\}.$$

So, it is sufficient to obtain X and compute the value of $|E_{G_{r,log_2^{\frac{k}{2}}}}|$ and $\nu(G_{r,log_2^{\frac{k}{2}}})$. On the oher hand, $|E_{G_{r,log_2^{\frac{k}{2}}}}| = 2^{log_2^{\frac{k}{2}}}(3r + \frac{7}{2}) - (r+2)$ and $\nu(G_{r,log_2^{\frac{k}{2}}}) \leq 2^{log_2^{\frac{k}{2}}}(r + \frac{3}{2}) - 1$, and so we obtain X. If $f = \nu(G_{r,log_2^{\frac{k}{2}}} - X) + |U \setminus V_{G(M_{G_{r,log_2^{\frac{k}{2}}}} - X)}|$ is a function from the power set of U to $\mathbb{N}$, then f attains its maximum at $X = \{v, w\}$ where v and w are two pendent vertices of $G_{r,log_2^{\frac{k}{2}}}$. For more illustration, see part (c) of Figure 2. In this figure, there is the generalized hierarchical product of $G_{7,2}$ and P_2 where U is the set of all back vertices in $G_{7,2}$. Also, bold edges in $A_{7,8}$ are elements of the global forcing maximal matching in $A_{7,8}$ which is defined in the proof of Theorem 2. Moreover, part (a) of Figure 2 shows constructing $G_{7,1}$ from P_{17} and P_2 where U is the set of all back vertices in P_{17}; similarly, part (b) of Figure 2 shows constructing $G_{7,2}$ from $G_{7,1}$ and P_2 where U is the set of all back vertices in $G_{7,1}$. □

Generalized hierarchical product of $G(U) \sqcap H$ is known as hierarchical product where $|U| = 1$. The **hierarchical product** of G and H is usually denoted by $G \sqcap H$. By the previous theorem, we can say the next result about GFN for maximal matchings of hierarchical product of graphs.

Theorem 3. Let G and H be two graphs with n and m vertices, respectively, and $U = \{r\}$ be a nonempty subset of V_G. Then

$$\varphi_{gm}(G \sqcap H) \leq \begin{cases} m(|E_G| - \nu(G)) + |E_H|, & \text{if } \nu(G) - \nu(G-r) \geq 1, \\ m(|E_G| - \nu(G)) + |E_H| - \nu(H), & \text{if } \nu(G) - \nu(G-r) = 0. \end{cases}$$

Octanitrocubane is the most powerful chemical explosive. Let G be the graph of this molecule, see Figure 2. As shown in Figure 2, G is formed by hierarchical product of P_2 and Q_3 where U is a vertex of P_2. Since $\nu(G) - \nu(G-r) = 1$, then by Theorem 3, $\varphi_{gm}(G) = \varphi_{gm}(P_2 \sqcap O_3) \leq 12$. On the other hand, the exact value of $\varphi_{gm}(G)$ is equal to 12 which shows the upper bound in Theorem 3 is sharp.

Generalized hierarchical product of $G(U) \sqcap H$ is known as **Cartesian product** where $U = V_G$. The Cartesian product of G and H is usually denoted by $G \times H$.

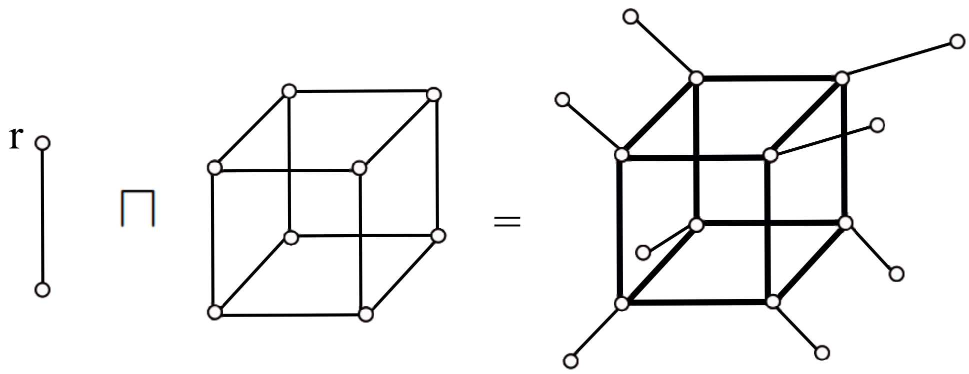

Figure 3: Octanitrocuban with its global forcing maximal matching.

Theorem 4. Let G and H be two graphs with n and m vertices, then

$$\varphi_{gm}(G \times H) \leq m|E_G| + n|E_H| - \max_{X \subseteq V_G} \{m\nu(G - X) + |V_G \setminus V_{G(M_{G-X})}|\nu(H)\}.$$

3 Global Forcing Maximal Matching Number as Global Forcing Maximal Independent Domination Number

For a graph G, the **line graph** of G, denoted $L(G)$, is a graph whose vertices are edges of G and two vertices are adjacent if and only if their corresponding edges are adjacent in G.
It is clear that matchings in G correspond to independent sets in $L(G)$. Also, it is not difficult to show that maximal matchings in a graph G are in a one-to-one correspondence with independent dominating sets in $L(G)$. Thus, if V' is a minimum global forcing set for independent dominatings of $L(G)$, then the edges corresponding to the vertices of V' form a global forcing set E' for maximal matchings of G. Moreover, E' is minimum, because if there was a minimum global forcing set E'' for maximal matchings of G such that $|E''| < |E'|$, then the corresponding vertices in $L(G)$ would be a global forcing set V" for independent dominatings in $L(G)$ of cardinality smaller than $|V'|$, a contradiction. Hence we can say the following result.

Theorem 5. For each graph G,

$$\varphi_{gm}(G) = \varphi_{gi}(L(G)).$$

Theorem 6. Let G be a simple graph on n vertices. Then $\varphi_{gi}(G) \leq n - \alpha(G)$.

Proof. At first, we prove that if V' is a subset of V_G such that $G - V'$ is an empty graph, then V' is a global forcing set for independent dominatings in G. To do this, assume to the contrary that D_1 and D_2 are two different maximal independent dominating sets in G such that $r(D_1|G - V') = r(D_2|G - V')$. Since $D_1 \neq D_2$, then there exists a vertex $v_i \in (V_G \setminus V') \cap D_1$ which is not in D_2. Thus, there must be a vertex v_j in $V' \cap D_1$ that $v_iv_j \in E_G$, since D_1 is a dominating set; and so d_j is equal to zero in $r(D_1|G - V')$. On the other hand, d_j is equal to one in $r(D_2|G - V')$, since D_2 is a dominating set and $v_i \notin D_2$, a contradiction.

By above argument $G - I$ is global forcing independent dominating set where I is a largest independent set of G. Therefore, $\varphi_{gi}(G) \leq n - \alpha(G)$. □

Theorem 7. [11] If G is a graph of order n containing no clique of size q, then $\alpha(G) \geq \dfrac{2n}{\Delta_G + q}$.

Applying Theorem 6 and Theorem 7 we have:

Theorem 8. If G is a graph of order n containing no clique of size q, then $\varphi_{gi}(G) \leq n - \dfrac{2n}{\Delta_G + q}$.

Using Theorem 5 and Theorem 8 leads to the next theorem.

Theorem 9. If G is a graph of order n whose line containing no clique of size q, then $\varphi_{gm}(G) \leq m - \dfrac{2m}{\Delta_{L(G)} + q}$.

Theorem 10. [5, 21] If G is a graph, then

$$\alpha(G) \geq \sum_{u \in V_G} \frac{1}{deg_G(u) + 1}.$$

By Theorem 6 and Theorem 10 we can write:

Theorem 11. If G is a graph of order n, then

$$\varphi_{gi}(G) \leq n - \sum_{u \in V_G} \frac{1}{deg_G(u) + 1}.$$

Proof. By Theorem 6, we have

$$\varphi_{gi}(G) \leq n - \alpha(G). \tag{1}$$

Also, by Theorem 10, we have

$$\alpha(G) \geq \sum_{u \in V_G} \frac{1}{deg_G(u) + 1}. \tag{2}$$

By replacing relation (2) in relation (1), $\varphi_{gi}(G) \leq n - \sum_{u \in V_G} \dfrac{1}{deg_G(u) + 1}$. □

Combining Theorem 5 and Theorem 11 leads to the next theorem.

Theorem 12. If G is a graph of size m, then

$$\varphi_{gm}(G) \leq m - \sum_{u \in V_{L(G)}} \frac{1}{deg_{L(G)}(u) + 1}.$$

In following, let $\omega(G)$ show the clique number of G.

Theorem 13. [11] If G is a graph of order n, then

$$\alpha(G) \geq \frac{2n}{\Delta_G + \omega(G) + 1}.$$

According to Theorem 6 and Theorem 13 we can say:

Theorem 14. If G is a graph of order n, then

$$\varphi_{gi}(G) \leq n - \frac{2n}{\Delta_G + \omega(G) + 1}.$$

Based on Theorem 5 and Theorem 14 we can conclude that:

Theorem 15. If G is a graph of size m, then

$$\varphi_{gm}(G) \leq m - \frac{2m}{\Delta_{L(G)} + \omega(L(G)) + 1}.$$

Theorem 16. [7] Let G be a graph of order n. If p is an integer such that for every clique C of G, there is a vertex u in C with $deg_G(u) + |C| + 1 \leq p$, then $\alpha(G) \geq \frac{2n}{p}$.

By Theorem 6 and Theorem 16 we can say:

Theorem 17. Let G be a graph of order n. If p is an integer such that for every clique C of G, there is a vertex u in C with $deg_G(u) + |C| + 1 \leq p$, then $\varphi_{gi}(G) \leq n - \frac{2n}{p}$.

Using Theorem 5 and Theorem 17 we conclude that:

Theorem 18. Let G be a graph of size n. If p is an integer such that for every clique C of $L(G)$, there is a vertex u in C with $deg_{L(G)}(u) + |C| + 1 \leq p$, then $\varphi_{gm}(G) \leq m - \frac{2m}{p}$.

4 Concluding Remarks

Global forcing number for maximal matchings of graphs is algorithmically difficult to compute and very applicable. In this paper we have studied this invariant under three graph products. We have also obtained some sharp bounds. It would be interesting to study this invariant under other graph operations such as lexicographic, splice and link.

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عدد فورسینگ عمومی برای تطابق‌های ماکسیمال تحت اعمال گراف

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چکیده

فرض کنید $S = \{e_1, e_2, \ldots, e_m\}$ یک زیرمجموعه مرتب از یال‌های یک گراف همبند G باشد. S-نمایش یالی از یک مجموعه یال $M \subseteq E(G)$ نسبت به S بردار $r_e(M|S) = (d_1, d_2, \ldots, d_m)$ است که $d_i = 1$ اگر $e_i \in M$ و درغیر اینصورت $d_i = 0$. مجموعه S یک مجموعه فورسینگ عمومی برای تطابق‌های ماکسیمال از G است هرگاه برای هر دو تطابق ماکسیمال M_1 و M_2 از G داشته باشیم $r_e(M_1|S) = r_e(M_2|S)$. یک مجموعه فورسینگ عمومی برای تطابق‌های ماکسیمال از G با مینیم اندازه را مجموعه فورسینگ عمومی مینیمم برای تطابق‌های ماکسیمال نامیده می‌شود و تعداد عضوهای آن را با نماد φ_{gm} نمایش داده و عدد فورسینگ عمومی (به اختصار GFN) برای تطابق‌های ماکسیمال نامیده می‌شود. به‌طور مشابه، برای یک زیرمجموعه مرتب $F = \{v_1, v_2, \ldots, v_k\}$ از $V(G)$، F-نمایش از یک مجموعه رئوس $I \subseteq V(G)$ نسبت به F بردار $r(I|F) = (d_1, d_2, \ldots, d_k)$ است که $d_i = 1$ اگر $v_i \in I$ و درغیر اینصورت $d_i = 0$. مجموعه F یک مجموعه فورسینگ عمومی برای غالب‌های مستقل از G است هرگاه برای هر دو مجموعه غالب مستقل D_1 و D_2 از G داشته باشیم $r(D_1|F) \neq r(D_2|F)$. یک مجموعه فورسینگ عمومی برای غالب‌های مستقل از G با مینیم تعداد عضو را مجموعه فورسینگ عمومی مینیمم برای غالب‌های مستقل نامیده شده و تعداد عضوهای که با نماد φ_{gi} نشان داده می‌شود GFN برای غالب‌های مستقل می‌باشد. در این مقاله GFN را برای تطابق‌های ماکسیمال تحت چندین نوع از ضرب‌های گراف مطالعه می‌کنیم. همچنین، کران‌های بالایی برای این متغیر ارائه می‌کنیم. علاوه‌براین، کران‌هایی برای φ_{gm} تعدادی از گراف‌های معروف ارائه می‌دهیم.

کلمات کلیدی

مجموعه فورسینگ سراسری، عدد فورسینگ سراسری، تطابق ماکسیمال، غالب مستقل ماکسیمال، ضرب گراف.