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Development of RMPC Algorithm for Compensation of Uncertain Time-Delay and Disturbance in NCS

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Abstract. In this paper, a synthesis method based on robust model predictive control is developed for compensation of uncertain time-delays in networked control systems with bounded disturbance. The proposed method uses linear matrix inequalities and uncertainty polytope to model uncertain time-delays and system disturbances. The continuous system with time-delay is discretized using uncertainty polytope. Then, the discretized model together with model disturbance is compensated. Uncertain parameters and additive disturbances are included in the controller design explicitly and robust stability is guaranteed in this method. The proposed method is applied on a level process. It is simulated by applying conventional RMPC as well. The simulation results show the effectiveness of the proposed method compared with conventional algorithm of the RMPC method.

Keywords. Networked control system, robust model predictive control, disturbance, network-induced delay, uncertainty polytope.

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1 Introduction

Nowadays, Networked control systems (NCS) have been one of the main research focuses in academia as well as in industrial applications [1]. When a traditional feedback control system is closed via a communication channel, which may be shared with other nodes outside the control system, then the control system is called an NCS [1]. Despite the great benefits of networks in control loops, they can introduce unreliable and time-dependent levels of service in terms of, for example, delays, jitter, or losses [2]. The time to read a sensor measurement and to send a control signal to an actuator through the network depends on network characteristics such as topology and routing schemes. Therefore, the overall performance of an NCS can be affected significantly by network delays. The severity of the delay problem is aggravated when data loss occurs during a transmission. Moreover, the delays do not only degrade the performance of a network-based control system, but they also can destabilize the system. [3], [4].

In the real world, NCSs with time delays exist extensively in industrial control applications, for example fieldbus and PROFIBUS based control systems [5]. Some papers focuses on the delay analysis in NCSs for example In [6] The delay in FF-H^1 control loops is analysed. Another example is [5] where in it, The delay for PROFIBUS PA control loops is estimated and evaluated. In [7], the delay in FF H^1 networks is evaluated analytically. Steam generator control loops with FF H^1 networks are analysed in [8]. Implementation and evaluation of network induced delays and packet loss of the Modbus protocol for wireless networked control systems are presented in [9].

Because of the time-delay characteristics, the design of a closed-loop controller is difficult for a dynamical system with time-delay. Various control approaches have been developed for compensation of delays in networked control systems. Predictive control is a good candidate for this goal [8], [10, 11]. In [12], NCSs with random time-delays are compensated using a modified MPC method. In [13], a developed model predictive controller is used for nonlinear NCSs with delays and packet dropouts .A networked MPC is utilised to compensate constant delays in [14]. It is shown in [4], [15] that the robust control approaches are very effective in delay compensation of the NCSs. Uncertainty polytope is used to describe NCSs with delay uncertainty [16]. In [6], the robust model predictive control is used for delay compensation in FF H^1 Fieldbus control loops. A robust approach based on H∞ control is used in [17] and [18] to compensate delays. Finite – time H∞ control is developed in [19] for NCSs with random time variable delays. In [20], a completely model-free adaptive controller is designed in the presence of parametric and non-parametric uncertainties and time varying delay to reduce delay effects. Although, there are significant methods for compensation network induced delays, a few of them concentrate on the other aspects of control such as model uncertainty and disturbance rejection. The problem of performance control with disturbance attenuation for time-delay systems has gathered much attention in recent years [21]. In [22], the problem of robust tracking control of networked control systems with external disturbance and network-induced delay using networked predictive control is presented. An RMPC algorithm is proposed in [23] for NCSs with packet dropouts modelled as disturbances. In [24] a synthesis approach based on RMPC algorithm is proposed for systems with model uncertainty and external disturbance but without any time-delay.

The aim of this paper is to investigate and development of RMPC algorithm to compensate network – induced delays together with attenuation of disturbances. To support this investigation, a synthesis algorithm based on uncertainty polytope and robust disturbance rejection is developed. There are some papers which consider the disturbance rejection in NCSs but the delay is not considered simultaneously or an especial case of constant time delays is included [25] and [26].

In this work, the control algorithm is developed in a way that encompasses the disturbance attenuation in addition to the uncertainty in the time delay, compared to [6] which consider only uncertain time delay and [24] which consider only disturbance attenuation.

This paper is organized as follows. Introduction is the 1st section. Problem statement and model of the networked control system is presented in section 2. Development of RMPC algorithm to include disturbance attenuation is proposed in section 3. In the 4th Section, the simulation results of the developed RMPC in a process with external disturbance is presented. and finally, the conclusion is given in section 5.

2 Problem Statement and Model of the Networked Control System

The block diagram of the NCS with uncertain time-delay in the measurement and control channel, is shown in Fig. 1 the τ_k^m is the time-delay in the measurement channel and τ_k^a is the time-delay in the control channel. In this case we assume that $\tau_k^m = 0$ and τ_k^a is uncertain but bounded.

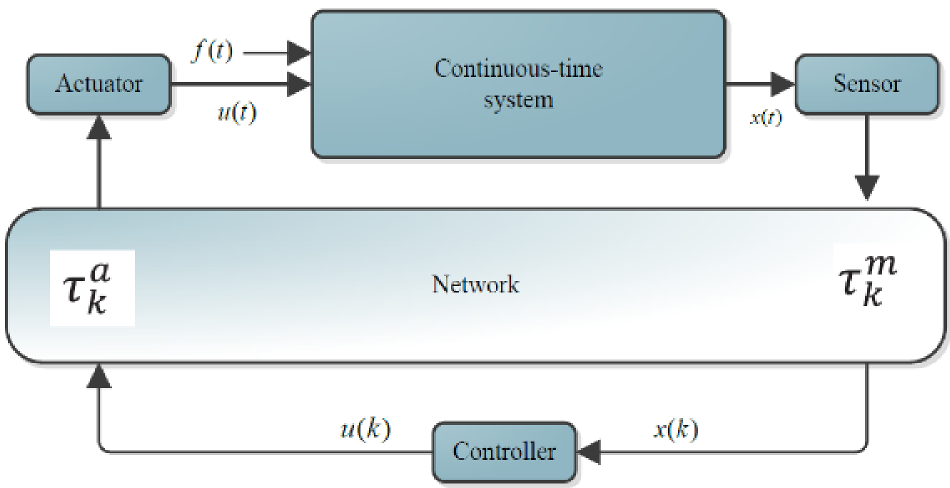


Figure 1: Block diagram of the NCS with external disturbance.

The open loop model of the continuous-time system with networks and signal interfaces is represented in the state-space form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t-\tau) + Df(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $\in R^n$, $u \in R^m$, $f \in R^d$, $y \in R^r$, are state, input, disturbance and output vector, respectively. The parameter $\tau \in [\underline{\tau}\ ,\ \overline{\tau}]$ represents uncertain delay with known lower and upper bounds. With selection of an appropriate sampling time t_s, the (1) is discretized as follows based on uncertainty polytope [27]:

$$\begin{cases} x_{k+1} = \left(I + \tilde{A}\right) x_k - \tilde{A}x_{k-1} + \tilde{B}_{\underline{d}} \triangle u_{k-\underline{d}} + \cdots + \tilde{B}_{\overline{d}} \triangle u_{k-\overline{d}} + \tilde{D}F_k \\ y_k = Cx_k \end{cases} \tag{2}$$

Where x_k is $x(t_k)$ and the symbols in (2) are defined in table 1

Table 1: Definitions of symbols in (2).

Symbols	Details
I	Identity Matrix
$\tilde{A}$	$\tilde{A} = e^{At_s}$
$\tilde{B}_{\underline{d}}, \ \ldots, \ \tilde{B}_{\overline{d}}$	$\tilde{B}_i = \int_0^{t_S} e^{As} Bds$
$\tilde{D}$	$\tilde{D} = \int_0^{t_s} e^{AS} Dds$
$\tilde{B}_i$	$\tilde{B}_i + \tilde{B}_{i+1} = \int_0^{t_s} e^{AS} Bds$
τ	$\tau \in [\underline{\tau}\ ,\ \overline{\tau}] \subset (\ \underline{d}t_s\ \ ,\ \ \overline{d}t_s)$
t_s	$t_s = t_k - t_{k-1}$, the time in seconds between two consecutive samples
k	Time index
$\triangle u_j$	$\triangle u_j = u_j - u_{j-1}\ \ ,\ \ j = k - \overline{d}\ ,\ \ldots,\ k - \underline{d}$
$\underline{d}$	$\underline{d} = fl(\underline{\tau}/t_{PSA})$
$\overline{d}$	$\overline{d} = cl(\overline{\tau}/t_{PSA})$
$fl(\cdot)$	A function in which the nearest integer less than $(\cdot)$ is selected
$cl(\cdot)$	A function where the nearest integer larger than $(\cdot)$ is selected

Then, the discrete time model can be represented as follows [6]:

$$x^a_{k+1} = A^\alpha x^a_k + B^\alpha \triangle u^a_k + D^\alpha F_k; \quad D^\alpha = \tilde{D}, \tag{3}$$

Periodic Signal Acquisition time or sampling time. It is equal to $t_s (t_{PSA} = t_s)$

where, x_k^a is the state vector define as:

$$x_k^a = [x_k,\ x_{k-1},\ \triangle u_{k-\overline{d}},\ \ldots,\ \triangle u_{k-\underline{d}},\ \ldots,\ \triangle u_{k-1}], \tag{4}$$

where the delay uncertainty are included in matrices (A^α, B^α) and $x_k^a \in R^s$; $s = 2n + \overline{d}$

With synthesis of models descripted in [16], [28], when τ varies in the interval of $[\underline{\tau}\ ,\ \overline{\tau}]$, any (A^α, B^α) belong to a convex polytope:

$$\Omega = \left\{ (A^\alpha, B^\alpha) \middle| (A^\alpha, B^\alpha) = \sum_{i=\underline{d}}^{\overline{d}} \lambda_i \left(A_i^a, B_i^a\right);\ \sum_{i=\underline{d}}^{\overline{d}} \lambda_i = 1\ ;\ \ \lambda_i \geq 0 \right\};$$
$$\forall i\ \in \left[\underline{d}\ ,\ \ldots,\ \overline{d}-1\right], \tag{5}$$

where

$$A_i^a = \begin{bmatrix} I+\tilde{A}\ ,\ -\tilde{A}\ ,\ 0_{n\times(\overline{d}-i)}\ ,\ \tilde{B}\ ,\ 0_{n\times(i-1)} \\ I_{n\times n}\ \ ,\ 0_{n\times(\overline{d}+n)} \\ 0_{(\overline{d}-1)\times(2n+1)}\ ,\ I_{(\overline{d}-1)\times(\overline{d}-1)} \\ 0_{1\times(2n+\overline{d})} \end{bmatrix}, \tag{6}$$

$$B_i^a = \begin{bmatrix} 0_{(2n+\overline{d}-1)\times 1} \\ 1 \end{bmatrix}, \quad i=\underline{d},\ldots,\overline{d}, \tag{7}$$

A_i^a, B_i^a are the vertices of Ω. The disturbance F_k is persistent, bounded and contained in a convex polytope:

$$\Omega_F = \left\{ F \middle| F = \sum_{j=1}^{n_d} \beta_j F_j;\ \sum_{j=1}^{n_d} \beta_j = 1\ ;\ \ \beta_j \geq 0 \right\} \tag{8}$$

Where F_j are the vertices of Ω_F and n_d is the number of F_j .
Therefore, the continuous system (1) with lower and upper limits lie in the uncertainty range $\tau \in [\underline{\tau}\ ,\ \overline{\tau}]$ and disturbance f can be discretized and presented by (3)-(8).

3 Development of RMPC Design

In this section, the RMPC method which is proposed in [6] and [28] is developed to compensate the delay uncertainty and disturbances. The proposed method is called as DRMPC in this paper. The aim of this section is to design a state feedback control law that is able to guarantee robust stability for the closed-loop system in the presence of bounded delay and disturbance.

Consider the linear time-invariant system (3) at each sampling time, a state feedback control law as:

$\triangle u^\alpha\,(k+h/k) = Kx^\alpha(k+h/k)$ that (i) minimizes the upper bound γ on $J_\infty(k)$ and (ii) guarantees robust stability for delay uncertainty and disturbance within a positively invariant set $Z = \left\{x^\alpha \in R^s / \|x^\alpha\|_P^2 \leq \gamma\right\}$, can be calculated by solving following optimization problem [24]

$$\min_{\gamma,\ P,\ K}\ \max_{[A^{\alpha}(k+h),B^{\alpha}(k+h)\in\Omega]} \quad J_{\infty}(k)=\sum_{h=0}^{\infty}\left[\|x_s^{\alpha}(k+h/k)\|_{\psi}^{2}+\|Kx_s^{\alpha}(k+h/k)\|_{\delta}^{2}\right], \tag{9}$$

$$s.t.$$

$$x_s^{\alpha}(k+h+1/k)=\left[A^{\alpha}(k+i)+B^{\alpha}(k+h)K\right]x_s^{\alpha}(k+h/k), \tag{10}$$

$$\begin{aligned} x^{\alpha}(k+h+1/k) &= \left[\left[A^{\alpha}(k+h)+B^{\alpha}(k+h)K\right]\right]x^{\alpha}(k+h/k)\\ &\quad+\tilde{D}(k+h)F(k+h), \end{aligned} \tag{11}$$

$$\begin{aligned} &\|x_s^{\alpha}(k+h+1/k)\|_{P}^{2}-\|x_s^{\alpha}(k+h/k)\|_{P}^{2}\\ &\quad\le-\left[\|x_s^{\alpha}(k+h/k)\|_{\psi}^{2}+\|Kx_s^{\alpha}(k+h/k)\|_{\delta}^{2}\right], \end{aligned} \tag{12}$$

$$\|x^{\alpha}(k)\|_{P}^{2}\le\gamma, \tag{13}$$

$$\|x^{\alpha}(k+h+1/k)\|_{P}^{2}\le\gamma, \tag{14}$$

where $x_s^{\alpha}(k+h/k)$ is the predicted state without disturbance corruption. ψ and δ are weighing matrices. The positively invariant set is computed by (13). All possible predicted states are restricted to lie in the positively invariant set by (14).

3.1 DRMPC Formulation

Proposition 1. at each sampling time k, the (12) and (13) are satisfied if there exists matrices Q_k and Y_k and a scalar γ such that the following LMIs are satisfied

$$\begin{bmatrix} Q_k & * & * & * \\ A_i^a Q_k + B_i^a Y_k & Q_k & * & * \\ \psi^{\frac{1}{2}} Q_k & 0 & \gamma I & * \\ R^{\frac{1}{2}} Y_k & 0 & 0 & \gamma I \end{bmatrix} \ge 0\ ,\quad i{=}\underline{d},\ldots,\overline{d} \tag{15}$$

$$\begin{bmatrix} 1 & * \\ x_k^a & Q \end{bmatrix} \ge 0. \tag{16}$$

Notation: "$*$" is used to complete symmetric matrices. The superscript "T" denotes the transpose for vectors or matrices.

Then it follows that γ is the upper bound of $J_{\infty}(k)$.

Proof. by following [28], (12) and (13) are ensured by (15) and (16), respectively. By summing (12) from $i{=}\ 0$ to $i{=}\infty$ and applying (13), it follows that $J_{\infty}(k)<\gamma$. □

Proposition 2. (14) is satisfied if there exists matrices Q_k and Y_k such that the following LMIs satisfied

$$\begin{bmatrix} \theta Q_k & * \\ A_i^a Q_k + B_i^a Y_k & Q_k \end{bmatrix} \ge 0,\quad i{=}\underline{d},\ldots,\overline{d} \tag{17}$$

$$\begin{bmatrix} \xi & * \\ D^{\alpha} F_{jk} & Q_k \end{bmatrix} \ge 0,\quad j{=}1\ ,\ldots,n_d \tag{18}$$

Where $F_{jk} = F_j(k)$ and $0 < \theta < 1$ is a pre-specified scalar. Then all possible predicted states are restricted to lie in a positively invariant set by (14) (A positively invariant set containing the measured state at each sampling time is computed by (12)).

Proof. see appendix A. □

By considering Propositions 1 and 2, a state feedback control law that guarantees robust stability in the present of both uncertain delay and disturbance, can be calculated. Consider the linear time-invariant system (3) (that is discretized form of (1)) at each sampling instant k, a state feedback control law $\triangle u^{\alpha}(k+h/k) = Kx^{\alpha}(k+h/k)$, $K = Y_k Q_k^{-1}$ that guarantees robust stability for both uncertain delay and disturbance within a positively invariant set $Z = \left\{ x^{\alpha} \in R^s / \|x^{\alpha}\|_P^2 \leq \gamma \right\}$, is obtained by solving following optimization problem

$\min_{\gamma,\ Y,\ Q} \gamma$ subject to (15)-(18) .

By applying the proposed MPC algorithm, all future states evolving from the initial state are guaranteed to stay within a positively invariant set.

4 Simulation Results

In this section, the simulation results are demonstrated for a level process with uncertain, but bounded delay together with external disturbance. The upper and lower bounds of delays evaluated experimentally. These upper and lower bounds are used to model the system. The simulations are performed for two scenarios. In scenario I the small amplitude disturbance (constant and time varying) is considered. In scenario II the disturbance with large amplitude (constant and time varying) is considered. In both scenarios, the delays are uncertain but bounded in the known upper and lower range. The main parameters of two scenarios are shown in Table 2.

Table 2: Main parameters used in the simulation.

Scenarios I	**Parameter** ($\underline{\mathbf{d}}$, $\overline{\mathbf{d}}$)	**Sample time** ($\mathbf{t_s}$)	ψ	σ	θ
$\tau \in [0.15 \quad 0.35]$	$(2, 4)$	100ms	$I_{s\times s}$ (Ident Mat with dimension s)	0.5	0.97

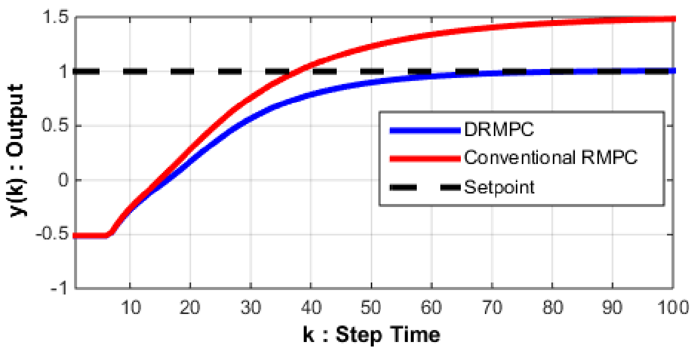
4.1 Small Disturbance

The second-order level process is expressed as follow:

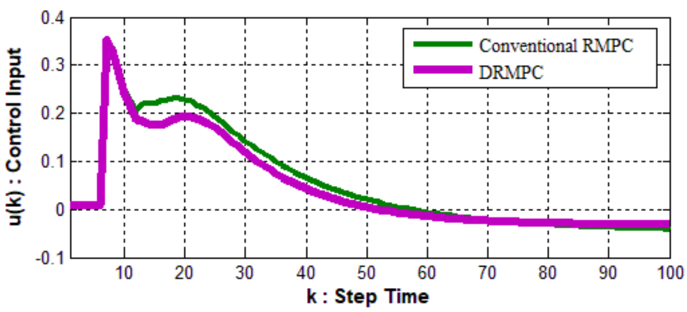
$$G_0(s) = \frac{k}{s(s+\alpha)} e^{-\tau s}, \tag{19}$$

such a model can be obtained from identification procedure, where step input is applied to the system and second order approximation is obtained from system identification toolbox in MATLAB. In (19) $k = 1$ and $\alpha = 0.9$ for a simple level process. The discrete time model of (19) with disturbance input can be represented as (3) in the general form. The focus of this paper is compensation of uncertain delay together with disturbance input and this simple process is considered only for showing the effectiveness of the proposed method (DRMPC).

To show the effectiveness of the proposed RMPC algorithm (DRMPC), the conventional RMPC algorithm is compared with the proposed method. As mentioned earlier the disturbance has small amplitude in scenario I. Fig. (2a,2b) illustrates the output signals and control inputs of closed loop system (conventional RMPC vs DRMPC) with constant disturbance $F(k) = 0.05$. Fig. (3a,3b) illustrates the output signals and control inputs respectively for two methods (conventional RMPC vs DRMPC) when the time varying disturbance is applied ($F(k) = 0.05\sin(0.5k)$). One can see that the performance of proposed DRMPC is better than the conventional RMPC for constant disturbances. When the disturbance is the time-varying one, it can be seen that, both cases have identical responses approximately. However, the performance gets worse for conventional RMPC when both the delay and disturbance are considered, but it remains stable for disturbances. The maximum of step time in all cases is $k = 100$.

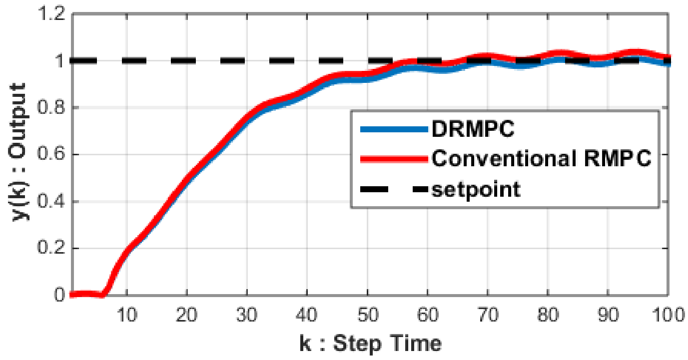


(a)

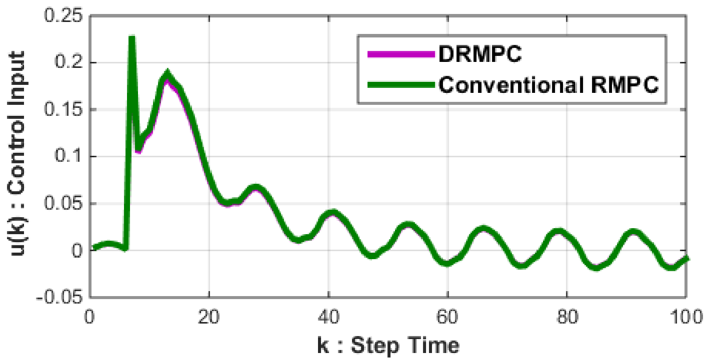


(b)

Figure 2: Comparison of step responses for two control algorithms when constant disturbance is applied. (a) process output (b) control input.



(a)



(b)

Figure 3: Comparison of step responses for two control algorithms when time-varying disturbance is applied. (a) process output (b) control input.

4.2 Large Disturbance

In order to investigate the effects of larger disturbances together with uncertain time-delay, the simulation is performed for scenario II. The main parameters of scenario II are the same as scenario I in Table 2. In this case the constant disturbance is $(k) = 2.5$, the time-varying disturbance is $F(k) = 0.5sin(0.5k)$.

Fig. (4a,4b) illustrates the output signals and control inputs respectively for two methods (conventional RMPC vs DRMPC) when constant disturbance is applied $(F(k) = 2.5)$. Fig. (5a,5b) illustrates the output signals and control inputs respectively for two methods (conventional RMPC vs DRMPC) when time varying disturbance is applied $(F(k) = 0.5\sin(0.5k))$. One can see that the performance of proposed DRMPC is much better than the conventional RMPC. However, the performance gets worse for conventional RMPC when both the delay and disturbance are exist, but it remains stable for large disturbances. The maximum of step time in all cases is $k = 100$.

Simulation Results for Conventional RMPC and DRMPC are illustrated in Fig. 2 to Fig. 5 It can be seen that; the performance degrades in Conventional RMPC comparing to DRMPC

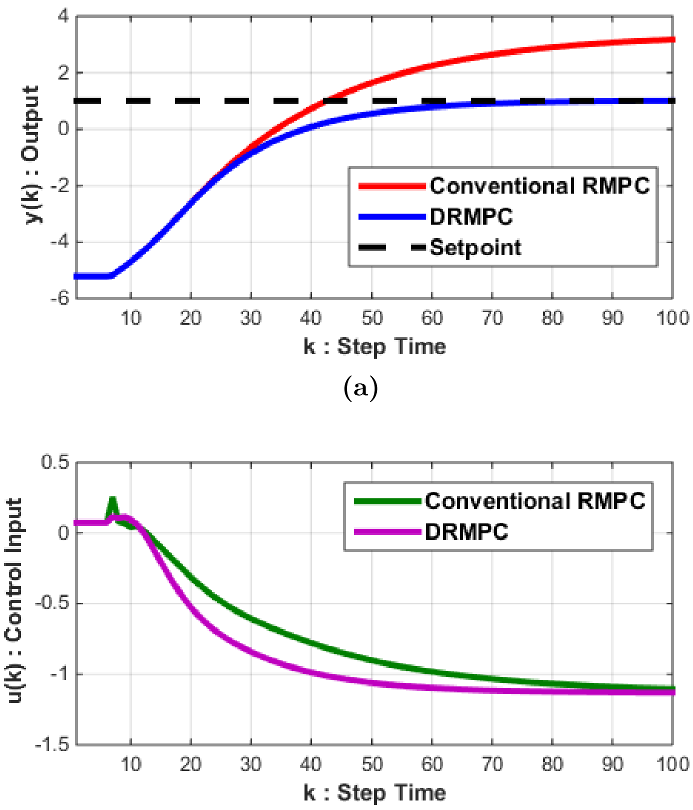


Figure 4: Comparison of step responses for two control algorithms when constant disturbance is applied. (a) process output (b) control input.

for small disturbances (scenario I). For larger value of disturbances (scenario II) together with uncertain time delay, the performance of controlled system using Conventional RMPC becomes more degraded comparing to DRMPC but in both cases, the closed loop systems are stable. Thus, the most important differences of DRMPC over the Conventional RMPC is that the proposed method guarantees the stability of the closed-loop system for different kinds of disturbances in the presence of uncertain time-delay. For comparison of the two control methods, the following indexes are selected [6]:

$$E_{\text{perf1}}=\left(\frac{1}{n}\right)\sqrt{\sum_{k=1}^{n}\left(\left|xref_k-y_k\right|^2+{u_k}^2\right)}, \tag{20}$$

$$E_{\text{perf2}}=\left(\frac{1}{n}\right)\sum_{k=1}^{n}\left(xref_k-y_k\right), \tag{21}$$

where n is the last step of simulation (100 in this case), the (20) and (21) states the overall performance on the simulation interval. In both cases, the reference signal is unit step. The table. 3 illustrates the performance indexes of two control methods when time-varying disturbance is applied.

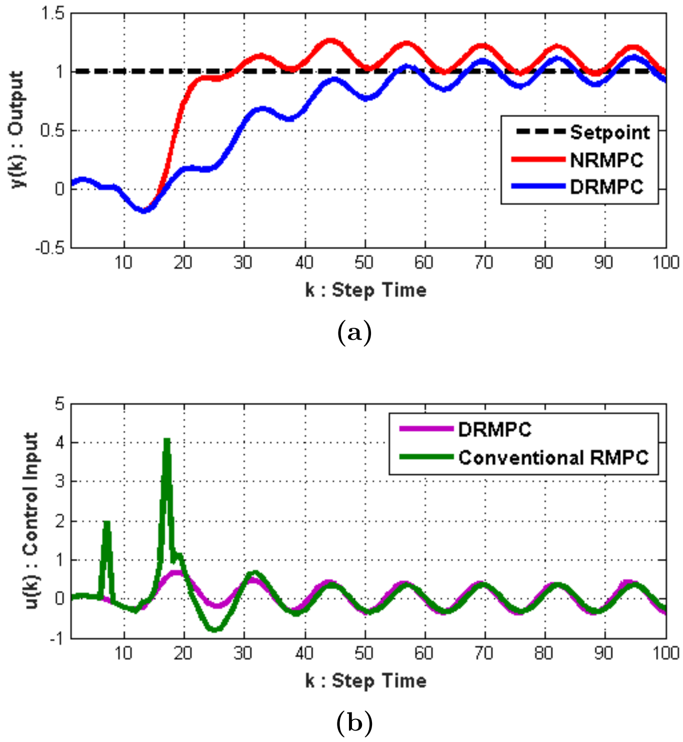


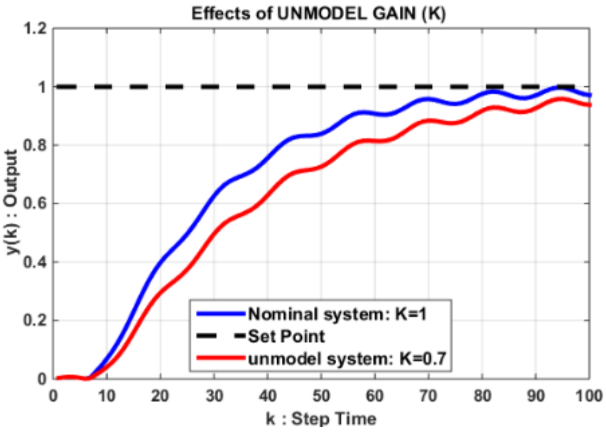
Figure 5: Comparison of step responses for two control algorithms when time-varying disturbance is applied. (a) process output (b) control input.

Table 3: The performance index of the two control methods.

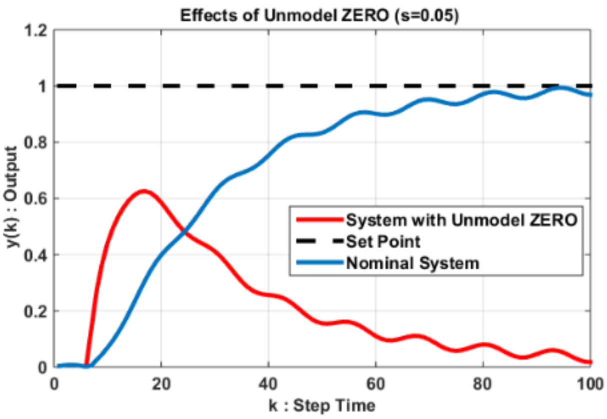
	NRMPC with Time-Varying Disturbance in Scenario I	DRMPC with Time-Varying Disturbance in Scenario I	NRMPC with Time-Varying Disturb-acne in Scenario II	DRMPC with Time-Varying Disturbance in Scenario II
E_{perf1}	0.1925	0.1898	0.3342	0.2671
E_{perf2}	0.29	0.26	0.47	0.32
$t_{settling\ time}(s)$	53	53.5	-	71
$Overshoot\ \%$	4	2	26	12

4.3 Unmodeled Dynamics

Although the purpose of developing and designing the compensator in section 3 is to compensate the effects of time-delay and input disturbance, the controller is applied to a system with unmodeled dynamics to analyze the controller's behavior against unmodeled dynamics.



(a)



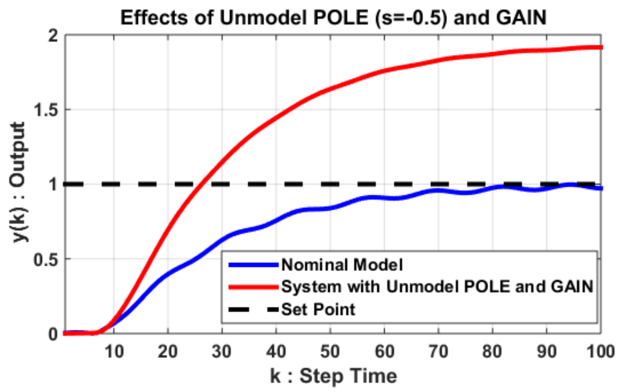
(b)

Figure 6: Comparison of step responses when DRMPC is applied to the system with nominal and unmodeled dynamics. (a) uncertain gain (b) unmodeled zero.

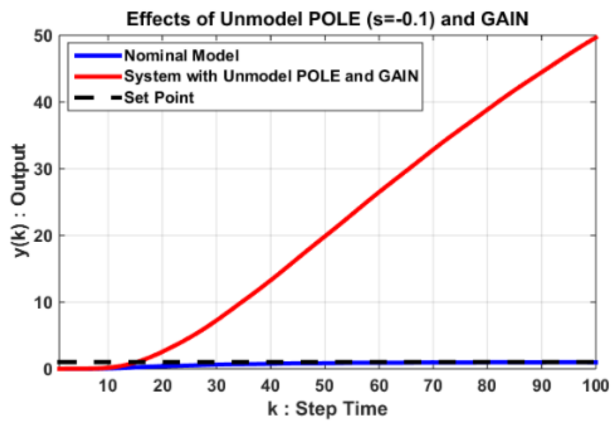
In this section, the DRMPC is implemented to system (19) for various unmodeled dynamics (gain, zero, and pole). To this end, simulations are done in four different cases. Fig. 6 to Fig. 7 illustrate the simulation results. In Fig. 6a, the DRMPC is applied to the system (19) with uncertain gain. In this case, it is assumed that the gain k is changed 30 percent from the nominal one ($k = 1$ for nominal model and $k = 0.7$ for system with uncertain gain). It is worth to mention that the DRMPC state feedback gain is designed for nominal system, but applied to two systems (with nominal and unmodeled dynamic). As shown in Fig. 6a, the performance of the closed loop system is degraded when the gain k is changed.

The red curve in Fig. 6b represented the step response of (19) with unmodeled zero in $s = 0.05$. As can be seen in Fig. 6b, in this case, the performance is severely impaired.

Fig. 7 represented the step response comparison of nominal and system with unmodeled dynamics in pole and gain together. The red curve in Fig. 7a represented the step response of (19) with unmodeled pole in $s = -0.5$. As it is shown in this figure, the system is still stable but the performance of the closed loop system is degraded. The blue curve in Fig. 7b, represented the step response of (19) with unmodeled pole in $s = -0.1$. As it is shown in this figure, when the unmodeled pole move close to the origin, the margin of stability is decreased. For near origin poles, it gets unstable.



(a)



(b)

Figure 7: Comparison of step responses when DRMPC is applied to the system with nominal and unmodeled dynamics in pole and gain. (a) unmodeled pole in $s = -0.5$ (b) unmodeled pole in $s = -0.1$.

5 Conclusion

Industrial networks cause time delays in control loops. When the network is a part of a control loop, the network-induced delay has effects on the system's characteristics such as stability and performance. In real time networked control systems, due to the asynchrony between devices and network characteristics, time delays are uncertain, but bounded. In this paper, the conventional RMPC algorithm is developed for systems with disturbance and uncertain delay. It was demonstrated that the proposed RMPC algorithm is synthesis of two methods. Each of these methods used for compensation of only uncertain delays and only Disturbances. In this paper, a control loop is investigated in which the architecture is prevalence in new automation systems. Simulation results demonstrated that the proposed developed RMPC is more effective than conventional RMPC when both the disturbance and uncertain delay exist. It was seen that when the amplitude of disturbance is small, the results of two methods are similar. The difference of two algorithms is shown when the amplitude of disturbance is big together with time-delay is uncertain.

Appendices

Appendix A: proof of proposition 2

Lemma 1. Suppose $M > 0$ is a symmetric matrix while a and b are vectors with appropriate dimensions. Then, $\|a+b\|_M^2 \leq (1+\delta)\|a\|_M^2 + (1+\frac{1}{\delta})\|b\|_M^2$ for any scalar $\delta > 0$.

By substituting (11), $P = \gamma Q^{-1}$ in to (14) and applying Lemma 1, for any $\delta_1 > 0$, we can see that (14) is satisfied if

$$\begin{aligned}(1+\delta_1)\left\|\left[A^\alpha(k+h)+B^\alpha(k+h)K\right]x^\alpha(k+h/k)\right\|_{Q^{-1}}^2 \\ +(1+\frac{1}{\delta_1})\left\|\tilde{D}(k+h)F(k+h)\right\|_{Q^{-1}}^2 \leq 1.\end{aligned} \tag{22}$$

Consider the term $\|[A^\alpha(k+h)+B^\alpha(k+h)K]x^\alpha(k+h/k)\|_{Q^{-1}}^2$ in (18), let $\theta\|x^\alpha(k+h/k)\|_{Q^{-1}}^2$ be the maximum value of this term where $0 < \theta < 1$ is a pre-specified scalar

$$\|[A^\alpha(k+h)+B^\alpha(k+h)K]x^\alpha(k+h/k)\|_{Q^{-1}}^2 \leq \theta\|x^\alpha(k+h/k)\|_{Q^{-1}}^2. \tag{23}$$

Substituting $K = YQ^{-1}$, pre multiplying by Q^T, post multiplying by Q and applying schur complement [29] leads to

$$\begin{bmatrix} \theta Q & * \\ A^\alpha(k+h)Q+B^\alpha(k+h)Y & Q \end{bmatrix} \geq 0. \tag{24}$$

From the convexity of the polytopic description, (24) is equivalent to (14).

Consider the term $\left\|\tilde{D}(k+h)F(k+h)\right\|_{Q^{-1}}^2$ in (22), let ξ be the maximum value of this term

$$\left\|\tilde{D}(k+h)F(k+h)\right\|_{Q^{-1}}^2 \leq \xi. \tag{25}$$

Applying the schur complement leads to

$$\begin{bmatrix} \xi & * \\ \tilde{D}(k+h)F(k+h) & Q \end{bmatrix} \geq 0. \tag{26}$$

From the convexity of the polytopic description, (26) is equivalent to (18). From (23) , (25) and $\|x^\alpha(k+h/k)\|_{Q^{-1}}^2$, (22) is equivalent to

$$(1+\delta_1)\theta + (1+\frac{1}{\delta_1})\xi \leq 1. \tag{27}$$

The maximum allowable value of ξ can be calculated by solving

$$\xi = \max_{\delta_1} \frac{1-(1+\delta_1)\theta}{(1+\frac{1}{\delta_1})}. \tag{28}$$

From (28), ξ can be obtained as $\xi = (1-\theta^{\frac{1}{2}})^2$.

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توسعه الگوریتم کنترل پیش بین مدل مقاوم به منظور جبران‌سازی تاخیر زمانی نامعین و اغتشاش در سیستم کنترل تحت شبکه

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چکیده

در این مقاله، روشی تلفیقی بر پایه کنترل پیش بین مدل مقاوم و به منظور جبران سازی تاخیرهای زمانی نامعین در سیستم های کنترل تحت شبکه‌ای که ورودی اغتشاش نیز دارند، توسعه می یابد. روش ارائه شده از نامساوی‌های ماتریسی خطی و چند ضلعی‌های عدم قطعیت به منظور مدل‌سازی تاخیر زمانی، اغتشاش و محاسبه بهره فیدبک حالت استفاده می کند. سیستم زمان پیوسته که شامل تاخیر زمانی است، با استفاده از چند ضلعی‌های عدم قطعیت گسسته‌سازی می شود. سپس مدل گسسته‌سازی شده همراه با اغتشاش جبران‌سازی می شود. پارامترهای نامعینی و ورودی اغتشاش به صورت صریح در طراحی کنترل کننده وارد می شود و پایداری سیستم کنترل حلقه بسته تضمین می گردد. روش ارائه شده بر روی یک فرآیند سطح اعمال می شود. همچنین روش مرسوم در کنترل پیش بین مقاوم نیز مورد شبیه‌سازی قرار می گیرد. نتایج شبیه‌سازی نشان دهنده موثر بودن روش ارائه شده در مقایسه با روش مرسوم کنترل پیش بین مقاوم است.

کلمات کلیدی

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