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Analysis of Students' Mistakes in Solving Integrals to Minimize their Mistakes

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Abstract. Experiences of teaching Integral have indicated that the vast majorities of Iranian university students commit numerous errors while solving integral problems and have weak skills in this field; we might even say that they hide away from integral and consider it the nightmare of mathematics. On the other hand, Integral is the base of pure and applied mathematics for all students of science, especially engineering, which some of their lessons are dependent on it directly or indirectly, so it is important to pay attention to it. Through descriptive method-exposed factor, an exam has been conducted in the form of three questions, the first of which is consisted of 4 sections on fifty students from different fields, and then interviews were conducted with a few of those students about their answers in order to study the students' behaviors when solving integral problems and to determine the type of their errors. By analyzing the performance of students in this test, we can see that students often struggle with integral and mostly have a feeble performance in solving trigonometric integrals. They want to learn computational integral instead of how to conceptualize integral in their minds correctly. The error most committed by university students was procedural errors, which arise from using derivative instead of integral. Besides most of the mistakes happen in solving definite integrals, and calculating finite areas between two curves. This is due to a lack of understanding of integrals and a lack of information in other areas of mathematics.

Keywords. Calculus, Integral, Student understanding, Understanding mathematical education, Initial function concept, Riemann concept, Teaching, Challenge.

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1 Introduction

The mathematical concepts, according to Dane, Faruk Cetin, Ba & Özturan Saçgırulilı (2016), have complex, abstract, and hierarchical levels. In formal education, we face an increase in these aforementioned levels of mathematical concepts and also the class level(Cetin, Dane & Bekdemir (2012)). Also, The fragmentation of the students' thinking structure had been shown by Adi Wibawa, Nusantara, Subanji & Parta (2017) in solving the problems of the application of the definite integral in area. For all students and especially university students, the integral and its related concepts are one of the important, fundamental, and necessary topics in learning basic mathematics. On the one hand, the integral is an Irrefutable tool in solving applied problems for university students, especially in the majors of basic sciences and engineering (Dancis, 2001). The mathematics education system in Iran is planned in such a way that in the last years of high school and the first year of university and after learning the concept of derivative and differential, students get acquainted with the concept of integral with an overemphasis on a symbolic form of the primary function (anti-derivative), in contrast, they should be first introduced to integral with the conceptualization of perimeter and area symbolic form, the Riemann symbolic form or by the adding up pieces symbolic form. Therefore, considering the approach taken by teachers and professors, the ability of students to solve routing integral problems is far greater than their ability to solve practical issues, and even these routine integral problems are solved with numerous procedural errors. Now, we look to analyze these problems and identify their roots and offer solutions where it is possible. Experience has shown that Iranian students learn integral superficially and like a parrot; their goal is to learn how to calculate an integral. However, the teaching method of professors is also useful in this regard. According to the above issues, several studies have been conducted worldwide regarding the issues students face in integral problems; one of the best studies has been done by Seah Eng Kia (2005), also Avital & Libeskind (1978), Ronaldson (1963), Chou (2002), and Everton (1983) have done some research and presented their results for their own countries and with different samples; but unfortunately, nothing fundamental has been done in this regard in Iran. With a thorough study of these researches, it can be said that students have a fundamental problem in conceptualizing integrals using Riemann sum or the sum of infinite rectangles. For example, Thomas & Yi (1996) sought to examine the misconceptions of learners when confronted with the Riemann integral, and they found that learners do not have a precise understanding of the Riemann integral and use an algorithm-based learning process to solve problems. We are now looking to localize the work of previous people, especially S.E.Kiat, in Iran and implement it following the Iranian educational system and reference books. Therefore, we have modified the test he has designed with a little reduction and change in order to analyze the students' problems and students' mistakes for solving integrals. In order to achieve this goal, we implement the conceptual framework of S.E.Kiat, which itself was developed from the interrogative framework of Ronaldson (1963), Libeskind (1978), Everton (1983). This developed interrogative framework is given in Appendix (I).

2 Research Methodology and Participants

Students participating in this research have selected from 280 engineering students who study at Islamic Azad University of Mashhad, such that at first, these students were trained for two semesters in pre-university mathematics and general Math I and II, (their training in these lessons based on the proposed exam include all details and preparations). All have participated in two midterm and final exams for each lesson, and after reviewing the results of these exams, and examining their high school education records, fifty students were selected who the selected students are among the average students in the field of mathematics. The present study was conducted after the passing of these students in General Mathematics 2 and was carried out in the second semester of the 2018-19 academic years in the Islamic Azad University of Mashhad such that fifty students majoring in engineering and sciences (physics and chemistry) and of which ten students are electrical students, ten students are civil engineering students, ten students are mechanical students, and the rest are physics and chemistry students. There were ten students in each field and a total of 30 male and 20 female students. All these fifty people have been trained with the same teaching method in the calculus 1 and 2 and have an average score of 13 in the calculus 1 and 13.5 in the calculus 2 (Scores in Iran's universities are calculated from 20). They have also taken introductory physics courses. Since the research follows the responses from the number of students in terms of the number and type of errors that are made in integral problems. In order to study the above topics carefully, we based our research on the observation and through a descriptive method-exposed factor of this fifty students who are currently studying the major. On the other hand, we have proposed an exam with eight problems in the first phase. In the second phase, eight problems are studied in terms of content validity by six professors of pure mathematics and mathematics education in the Islamic Azad University of Mashhad. With regard to views and content validity indexes, three questions are accepted finally (see Appendix II). Note that the purpose of each question is different, and some questions are designed for different purposes. Nevertheless, it can be said that the main primary purpose of the questions is to examine the level of students' capability in solving integrals, which are based on the primary function and Riemann or infinite sum of rectangles. Other minor goals include the ability of students to solve in solving integrals that have different powers of x. (Objective I), which is also pursued in the second and third questions, and another goal is to study the ability of students in solving integrals of functions as $(a+bx)^n$, as well as trigonometric and exponential functions (Objective II) and the study of students' ability in using certain integrals to calculate the area of the next objective (Objective III). We note that while pursuing the above goals, the basic necessary skills of students are also examined. Also, all problems have taken from Apostol (1967). The reliability of all six problems is proved on the same samples of students then its reliability is determined that was more than 0.75. Participants were asked to present their responses along with the complete explanation of their responses' details in (written manners).

3 Data Collection

The above test consists of two stages. One is a written test in which all students have participated, and the other is an interview with some of them based on their written answers. The students interviewed often fall into one of three categories:

- Students who have given complete answers to the questions.
- Students who have performed poorly in solving questions.
- Students who have an excellent educational background but had a disappointing performance in answering questions.

Each session held in presence of four persons that one of them is a student participating in the interview and three interviewers who are experienced and successful faculty members in the department of mathematics, Islamic Azad University of Mashhad. It is important to mention that the students' oral responses to the problems were much longer than what would come, but we tried to convey their meaning as much as possible. The interview has done through questions that proposed by one of the authors about solving integral's. The interview was purposeful, i.e. the interviewer presented questions according to their possible responses to get nearer to the purpose of the interview. The following items were considered in the test:

- The test was taken simultaneously from all 50 people.
- Before the exam, students are informed to have enough time to study and prepare for the exam.
- The duration of the written test is 1 hour.
- The interviews were filmed for accuracy in and later analysis.
- Before the interview, each student is allowed to think about their answers.
- An attempt has been made to conduct a fully structured interview, i.e., the questions have been designed according to the written answers of students and their assumed answer to the interviewer's questions.

4 Results and Related Discussion

A summary of the written test results is given in Table 1.
Note that the score of each question is calculated according to the types of error that students may make for solving it, which is listed in Table 1. By studying the above table, we see that 20 students have solved question 1a entirely and accurately, and 10 have solved more than half of it. In other words, 60% of students answered question 1a, so it can be said that students can solve integrals of the functions $(a+bx)^n$ even though they have difficulty writing the details. Also, regarding the integration of trigonometric functions that are included in the first question of part 1bi and 1bii, only 32 people were able to solve it, among which only 20 people solved

Table 1: Test results summary

Percentage pass	Number of students who scored (in%)			S.D.	Main	Total mark	Q
	100	50-99	0-49				
60%	20	10	20	1.92	2.66	5	1a
44%	15	7	28	1.48	2.52	5	1bi
20%	5	5	40	1.28	1.8	5	1bii
50%	15	10	25	1.59	2.95	5	1biii
46%	5	18	27	2.6	3.72	7	2
20%	3	7	40	1.8	1.64	7	3

these two questions accurately, and 12 people were able to solve more than half of it. In other words, in these two questions, they performed much worse than in question 1a. In fact, by examining their answers, we find that in integrating trigonometric functions, many students have used derivatives instead of integrals (the most common procedural mistake), which is due to the lack of accurate and complete conceptualization of integrals and their incomplete understanding of functions; and that is why some students have written $2\sin(2x-1)$ in response to question 1bi. In question 1bii, they have not been able to use the trigonometric formulas to convert the function under integral and then integrate. They have made numerous mistakes in solving it, and therefore have the lowest efficiency among the questions. Only five people have been able to solve this question completely. On the other hand, we note that in this question, how to perform calculations is also essential, and students have answered this part of the question almost acceptably. In question 1biii, in which the integration of exponential functions is included and aims to examine the performance of students, we see that only 15 people were able to thoroughly answer it and 10 people managed to solve more than half of it. They had a more satisfying performance compared to questions 1bi and 1bii. We should note that some people have made some procedural severe mistakes (using the derivative instead of the integral). In general, and by analyzing the first question, we can say that the performance of students in integrating functions of $(a+bx)^n$ is much better than integrating exponential functions, and their performance in integrating exponential functions is much better than that of trigonometric functions and they make fewer mistakes. However, there have been interrogative and technical errors in doing all four sections of this question. In the second and third questions, students' skills in solving definite integrals and mainly calculating the area are considered. According to the given function, integration of different powers x is also considered (Objective I). By studying the table, we see that due to the closeness of these two questions, the way students have answered them is so different; such that students answering the third question are less than half of the students who were able to answer the second question, i.e., $42\%, 20\%$. Only 10% of students were able to answer the second question thoroughly, and only 6% of them managed to answer the third question accurately. In fact, in the second question, the analysis of the answers shows that they did not understand the need to draw a graph of the function and even marking it, and went straight to integrating the function (x^2-4x) at a distance of $x=0$ to $x=5$ and have not recognized that part of the function graph is above and a part of it is below the x-axis. Regarding the third question, they had to calculate the ratio of the two hatched areas, and again, there is a part above and a part below the x-axis. Even the diagram of this question is presented, and yet again, students had many problems. They had the weakest efficiency in this question, such that some were not able to calculate the area under the x-axis, some had difficulty in determining the integral boundaries, and some did not know from which function they should integrate, which all three refer to fundamental weaknesses in the conceptual understanding of integrals. In general, it can be said that students perform much better in problems that directly require integral computation than problems in which the application of integrals is considered. In fact, in performing direct definite and indefinite integrals, they have a relatively better ability than using integrals to calculate the area. Nevertheless, the exciting thing is that they easily integrate different x functions.

5 Error Analysis

The types of mistakes made by students in solving the test questions is shown in Table 2. Examining Table 2, we see that the most technical errors occurred in the first question, However,

Table 2: Type and quantity of errors made by students

ZERO	No Errors	Technical Errors	Procedural Errors	Conceptual Errors	Q
0	20	7	20	3	1a
0	15	6	24	5	1bi
1	5	30	11	4	1bii
3	15	6	22	4	1biii
0	5	4	8	32	2
1	3	13	5	28	3
5	63	66	90	76	Total

the interrogative(Conceptual) errors occurred in the second and third questions, and there are procedural errors in all the questions. Column zero in Table 2 also means that students did not answer the question. The table above shows that students committed 76 interrogative errors, 90 procedural errors, and 66 technical errors. For example, the first interrogative error occurred in question 1a, where students had to calculate the integral of function $2(3+4x)^4$, and three persons made this error by using 3 in the answer instead of 5, for example. Hirad replied that:

$$\int 2\,(3+4x)^4 dx = \frac{2}{4}(3+4x)^3 + C,$$

Hirad combined the derivative and the integral and provided this answer, but since only three people made such a mistake, it can be said that most of them understood this question. Table 2 also shows that most of the interrogative errors occurred in the second and third questions. Even though the graph of the function and the regions are presented, most of the students have had problems calculating the area under the x-axis and have used either the wrong bounds or the wrong functions to solve these questions. For example, Negin wrote this in response to the third question:

$$
\begin{aligned}
S_A &= \int_0^2 \left(x^2 - 6x + 8\right) dx + \int_2^4 (8 - 2x)\, dx \\
&= \left(\frac{x^3}{3} - 3x^2 + 8x\right)_0^2 - \left(8x - x^2\right) \\
&= \left(\frac{64}{3} - 12 + 32\right) + (16 - 12) = \cdots, \\
S_B &= \int_2^4 \left(x^2 - 6x + 8\right) dx = \left(\frac{x^3}{3} - 3x^2 + 8x\right)_2^4 \\
&= \ (\frac{64}{3} - 12 + 32) - \left(\frac{8}{3} \ - 12 + 16\right) = \cdots .
\end{aligned}
$$

When we ask Negin why she used these two integrals to calculate zone A, she points to the area between the diagonal line and the curve and says that we have to calculate this part, and therefore, we have to take integrals from these two functions. Hence, this mistake is due to the lack of precise and complete conceptualization of definite integral in her mind. Even though we continue to guide her on how to calculate the area of zone A, she insists on her answer. Hooman also wrote this in response to this question:

$$
\begin{aligned}
S_A &= \int_0^4 (8 + 2x)\, dx = \left(8x + x^2\right)_0^4 \quad = 32 + 16 = 48 \\
S_B &= \int_2^4 \left(x^2 - 6x + 8\right) dx = \left(\frac{x^3}{3} - 3x^2 + 8x\right)_2^4 = \cdots .
\end{aligned}
$$

Hooman's answer in calculating area B is the same as Negin's answer. Both made the same mistake in not understanding that the function negative and under the x-axis, but in calculating area A, Hooman acted very differently and ignored the curve function; when discussing with him how to calculate the area of A, the extreme lack of conceptualization of the integral is evident in his mind, whereas we note that if only the linear function $y = 8 + 2x$ was involved in calculating the area of A, he could have easily calculated it. Hooman: To calculate the area A, it is sufficient to integrate the function $y + 2x = 8$ at a distance of 0 to 4. (points to the hatched part between the line and the x-axis.) Interviewer: But how do we calculate the part between the function $y + 2x = 8$ and the curve $y^2 - 6x + 8$: Hooman? After about 3 minutes of thought and silence, Hooman only states that it is no different from before and that we must integrate directly from the linear function. Mahshid's answer to the second question is as follows:

$$
S = \int_0^5 \left(x^2 - 4x\right) dx = \ \left(\frac{x^3}{3} - 2x^2\right)_0^5 = \frac{125}{3} - 50 = \cdots .
$$

Mahshid did not understand that part of the $x^2 - 4x$ function is below the x-axis and that she has to integrate it twice, and when we ask her the reason for this action, she says: "There is no need to mark the function or even draw it, and I did not think at all that such a thing could happen, but now I remember a similar problem that I solved before and I should have done this." We find that if Mahshid had marked the integral, she could have solved the question entirely. Table 2 tells us that the highest number of errors was the procedural error, which is 90 times. For example, Negar wrote this in response to question 1bi:

$$\int \text{Cos}\,(2x-1)\,dx = \frac{1}{2}\text{Sin}(2x-1).$$

Besides Negar, 20 other people have made the same mistake and have not written the constant value of C, and in response, they only say that they forgot; in fact, it can be said that they do not have an exact concept of this constant C in their minds that will help them not forget. In the same question, Hamin has written this:

$$\int \text{Cos}\,(2x-1)\,dx = -2\text{Sin}\,(2x-1)\ +C.$$

Although she wrote C, she has used the derivative instead of the integral (this is the most common type of mistake) Negin has given the following answer to question 1biii:

$$\int e^{(2x+3)}dx = 2e^{2x+3}.$$

Here, in addition to using the derivative instead of the integral, Negin did not write down the constant C and made two mistakes at the same time. and In response to why she used 2 instead of 1/2, she immediately admitted her mistake and said that she had mistaken it for the derivative. Examining all the answers, we find that students often have problems in the process of calculating integrations in a direct way, which depends on the conceptualization of integrals based on the primary forms of the functions such as trigonometric or exponential ones. Hence, and they use the derivative instead of the integral. Regarding the third type of error, i.e., technical errors that have occurred 66 times in the all test, we can say that the main reason is the lack of sufficient information in other mathematical fields such as algebra, geometry, trigonometry, or even carelessness. These errors have nothing to do with the correct and accurate conceptualization of integrals by students. Whether the conceptualization is complete or not, these errors may occur, while it can be said that students with fewer conceptual and procedural errors certainly make fewer technical errors. However, because technical errors make it impossible to answer the questions related to the integral fully, they are worth reviewing. Hesam has written this in response to question 1a:

$$\int 2(3+4x)^4dx = 2\int \left(3^4+(4x)^4\right)dx = 2\left(3^4x+\ \frac{4}{5}x^5\right)+C.$$

insufficient knowledge of algebra has caused this error. In response to question 1biii and due to weakness in trigonometry and lack of sufficient information in this field, Kimia did not know that $\text{Sec}^2\ x = 1+\text{Tan}^2 x$ and used $1+\text{Sec}^2\ x = \text{Tan}^2 x$ and has written the following:

$$\int_0^{\frac{\pi}{2}} \text{Tan}^2\ 2x\ dx = \ \int_0^{\frac{\pi}{2}} \left(1+\text{Sec}^2\,2x\right)dx = (x+\tan 2x)_0^{\frac{\pi}{2}} = \frac{\pi}{2}.$$

(Note that there is also a procedural error in this question) In response to the second question, Zohreh did not pay enough attention as well and has written the following:

$$S = \int_0^5 \left(x^2-4x\right)dx = \frac{x^3}{3}+4x^2 = \frac{125}{3}+100.$$

She also forgot to divide 4 by 2. When we ask students about these issues, they immediately realize their mistakes and correct them. Finally, Table 3 provides a list of all the mistakes

Table 3: Types of mistakes

Errors	Description	Numbers
Conceptual) (Interrogative)	Combining derivatives and integrals	**3**
	Not recognizing the need to mark the function	**29**
	Failure to select the correct function	**25**
	Failure to select the correct bounds	**19**
Procedural (Executive)	Failure to write the constant C	**24**
	Mistake between derivative and integral	**66**
Technical	Coordinate geometry	**18**
	Computational Algebra	**16**
	Trigonometry	**15**
	Inaccuracy	**17**

students made during the test and the interview that followed. Examining Table 3, we find that most errors are procedural ones. One of the most important reasons for this error is the use of derivatives instead of integrals, which can be related to lack of experience in this field, which itself is because of not having enough practice. The type of error lease made by the students was a technical error, which means that if students were taught to conceptualize integrals more wholly and accurately, they would be more able to solve problems. Finally, we point out that although students are among the average students at the university under study and even though they were given enough time to study before the exam, they are still weak in the field of integral and have much work to do.

6 Discussion

Considering the conditions and questions of the exam and the interviews conducted and also considering that only 50 people have participated in this exam and they have also been selected from average students, we cannot say that this exam is a complete, accurate and perfect test and therefore performing similar tests in this subject is always recommended. We also need to look at how integrals should be taught and how students should learn them. However, it can be said that in general, students have a fundamental problem in understanding integrals. They should be re-taught the basic concepts related to it, and its prerequisites, especially the derivative, before the subject of integral is brought up. Although the fact that teaching hours in Iran are limited in these subjects, and it is not possible to emphasize the prerequisite topics should be considered in this regard, one solution can be to hold extracurricular or problem-solving classes in this field because basic concepts like algebra, geometry, and trigonometry are important concepts for understanding integrals correctly and the large number of technical errors made by students shows the importance of these concepts. On the other hand, teaching mathematics in Iran, especially in the first year of university, is done without the necessary and sufficient emphasis on drawing diagrams and various graphs of functions and this subject is discussed in a maximum of 2 hours and is very vital and essential for learning the concept of integral and determining and calculating different areas of different functions, and is taught to strengthen students' visual perspectives so they would not make many mistakes in solving relevant integrals. In other words, useful emphasis on visual explanations by teachers and strengthening students' visual perspectives facilitates the interrogative comprehension of integrals. However, if all these subjects are completed, the test will show us that students have difficulty in conceptualizing and understanding integrals, the many procedural errors made prove this, and in other words, their interrogative understanding of integrals is incomplete, especially since in Iranian universities, and especially for engineering students, integrals are often conceptualized through the primary function, and the emphasis is on calculating the integral rather than understanding its concept, so terrible mistakes have occurred in calculating different areas and even in choosing the appropriate bounds for the integral or in the selection of the appropriate function to calculate the area using integral. Therefore, teaching methods and how to teach students is effective in reducing students' mistakes. For example, although the function diagram for the desired area is given in the third question, students still have many interrogative and procedural problems in solving the problem. Therefore, we emphasize that it is better to conceptualize the integral with the help of Riemann's concept or the summation of infinite areas. With the help of the primary function, it was found in the interviews that many students, unfortunately, read mathematics to write, so we emphasize the importance of strengthening the communication skills of writing compared to the communication skills of reading in students. One suggested solution is to write an example of calculating a finite area for students and force them to provide a written answer, and then ask them to discuss their answers so that they can understand the integral. Writing mathematics greatly reduces a variety of errors, primarily procedural and technical ones. Note, however, that the most massive largest error made by students participating in the test is procedural errors that are caused by mistakes in using integrals and derivatives, which requires much emphasis when teaching this subject. The main suggestion is to use derivatives and integrals parallelly so they will become engraved in the minds of students. Finally, although integral is one of the most essential concepts in mathematics for students, especially in the field of engineering, on one hand, many of them are dealing with the fundamental difficulties with this concept to understand. On the other hand, they do not have any outstanding effort to learn it (why? Of course, there are many reasons for this issue that are beyond the scope of this research article).

Appendix I: Classification of students' errors

Appendix II: Test on Integration

First question:

a) Solve this integral: $\int 2(3+4x)^4 dx$.

b) Evaluate the following integrals:

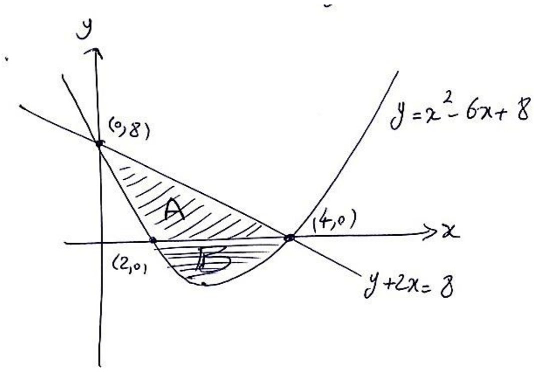
Error type:	Description of the error with an example
Conceptual error: Errors due to inability of learners to understand the relations in the question or due to their inability to understand its meaning.	Calculate the area between the curve y $= x^2 - 4x$ and the x-axis from $x = 0$ to $x = 5$ $\int_0^5 \left(x^2 - 4x\right) dx = \left(\frac{x^3}{3} - 2x^2\right)_0^5 = \frac{-25}{3}$ They did not realize that part of the curve is below the x-axis.
Procedural error: Error due to inability to perform calculations or algorithms despite learners' understanding of the concepts in the question.	$\int \tan^2 2x dx = \int (\text{Sec}^2 2x - 1) dx$ $\tan^2 2x - x + C$ Not writing $\frac{1}{2}$ behind the $\tan^2 2x$ function
Technical error: Error due to carelessness or lack of sufficient information on other issues.	$\int 2(3+4x)^4 d = \int (6+8x)^4 dx$ $= \frac{(6+8x)^5}{5\times 8} + C$ Wrong multiplication of $(3+4x)$

i) $\int \cos(2x-1) dx$,

ii) $\int_0^{\pi/2} \tan^2 2x dx$,

iii) $\int e^{(2x+3)} dx$.

Second question: Find the area bounded by the curve $y = x^2 - 4x$ and the x-axis from $x = 0$ to $x = 5$. **Third question:** The following diagram shows a part of the line $y + 2x = 8$ and the curve of the function $y = x^2 - 6x + 8$. Find the ratio of the area of zone A to the area of zone B.



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تجزیه و تحلیل اشتباهات دانش‌آموزان در حل انتگرال برای به حداقل رساندن اشتباهات آن‌ها

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چکیده

اکثر دانشجویان ایرانی در حل مسائل مربوط به انتگرال دارای مشکلات عدیده‌ای می‌باشند و در این مبحث ضعیف‌اند و حتی می‌توان گفت که از انتگرال فراری هستند و یا به عبارتی، انتگرال را کابوس ریاضیات خود می‌دانند. لذا می‌خواهیم با برگزاری آزمون و سپس مصاحبه، مشکلات پیش‌روی دانشجویان مهندسی و علوم پایه درحل مسایل انتگرالی را تجزیه و تحلیل کرده، و سپس در حد امکان راه حل‌هایی برای رفع آنها ارائه دهیم . این تحقیق شامل سه سوال می‌باشد که سوال اول خود از ۴ بخش تشکیل شده است، که از دانشجویان منتخب گرفته شده و سپس مصاحبه‌ای کوتاه با چند نفر از آنها در مورد پاسخ هایشان انجام شده است. با بررسی عملکرد دانشجویان در این آزمون، مشاهده می‌شود که دانشجویان غالباً با مباحث انتگرال مشکل داشته، به‌ویژه در حل مسائل انتگرال‌های مثلثاتی بسیار ضعیف عمل می‌کنند، در واقع آنها به جاي مفهوم‌سازی کامل و بی‌عیب انتگرال بیشتر به دنبال یادگیری انتگرال محاسبه‌ای هستند. بیشترین خطای انجام شده توسط دانشجویان خطای رویه‌ای بوده است که غالباً این نوع خطاها ناشی از استفاده مشتق به‌جای انتگرال می‌باشد. همچنین اکثر اشتباهات دانشجویان در حل انتگرال‌های معین و محاسبه مساحت‌های محدود بین دو منحنی می‌باشد که این هم ناشی از عدم درک کافی انتگرال و هم ناشی از نداشتن اطلاعات لازم در سایر زمینه‌های ریاضی می‌باشد.

کلمات کلیدی

انتگرال، درک دانشجو، درک آموزش ریاضی، مفهوم تابع اولیه، مفهوم ریمان.