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Research Article

A Hybrid Heuristic Algorithm to Provide a Multi-Objective Fuzzy Supply Chain Model with a Passive Defense Approach

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Abstract. In this paper, a stable multi-objective model of location, inventory, and supply chain routing is presented under conditions of uncertainty and using a passive defense approach. Parameters such as demand, cost of setting up the facility and cost of maintaining inventory are considered uncertain and in the form of triangular fuzzy numbers. Also, in order to increase supply chain resilience, the characteristics and capabilities of passive defense in the supply chain, such as "ready flow rate", "security of backup routes", "possibility of deployment of resources and equipment", and "the principle of dispersion for location" are considered. Multipurpose, multipartite algorithms, based on the Pareto archive and genetic algorithm, are used to solve the model. The results of validation show that the proposed model is valid and feasible, and the proposed algorithm is also valid and converges to the optimal solution. Sample problems, in three groups of small, medium and large, are solved by two algorithms, and the results are compared based on quality, dispersion, uniformity and execution time. The results of this section show that in all cases, the multi-objective particle mass algorithm has a higher ability than the GA to produce solutions of higher quality and to explore and extract the scalable area of the solution. Also, the comparison of the execution times of the algorithms indicates that the multi-objective particle mass algorithm has a higher solution time.

Keywords. Supply chain, Sustainability, Passive defense, Multi-objective fuzzy optimization, Meta-heuristic algorithm.

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1 Introduction

Sustainable supply chain management is defined as the management of materials, information, and investment to make harmony between companies during the supply chain. This topic has been at the center of attention of managers and researchers over the recent two decades [10]. The sustainability of supply chain and its relevant environmental-social aspects have become substantial issues over recent years ([3] and [12]). Environmental and social sustainability are relatively complex issues that affect the empowerment of different parts of supply change by adopting technologies, creating a friendly environment, and attention to environmental factors ([6] and [9]). Passive defense, or resilience, is a paradigm that boots tolerance power and resistance of organizations against the possible incidents and risks. It includes all projections and measures that reduce vulnerabilities, increase national sustainability and endurance of public institutions against external threats without employing any weapon [1].

The present study aims to provide a sustainable, multi-objective model for supply chain location, inventory, and routing under uncertainty by using a passive defense approach. In this research, an integrated model is proposed for the location-routing-inventory problem in a four-echelon supply chain as uncertain and triangular fuzzy numbers by consideration of some parameters, such as demand, facility cost, and inventory cost. In the studied model, the characteristics and capabilities of passive defense in the supply chain are used to increase the resilience of the supply chain. Accordingly, the potential locations of the facility are weighed and ranked based on the criteria of “logistical flow rate”, “backup path security”, and “the possibility of resource and equipment deployment”, and then the model finds the location of the facility based on the generated weights.

This paper is organized as follows. First, the literature on the subject is reviewed, which includes the expression of the novelty of the problem, and the objectives of our research. In the next section, the research background is stated. Then, the structure of the mathematical model of the problem is described by introducing the involved variables, parameters, objective functions, and constraints. Afterward, the method of solving the model using a meta-heuristic mechanism is described. Finally, the model is measured and validated in several issues, small and large samples.

2 Literature Review

Some studies have been carried out on supply chain based on the passive defense approach. Now, we briefly describe some of them.

Peng et al. in 2016 studied the design problem of supply chain network including supply, transfer, and demand (bi-level supply chain) nodes by considering the disruption risks of facilities, such as disruption in the commodity transfer node [12]. They solved the problem as a mixed-integer programming model (that includes robustness criteria in the constraints) to minimize total costs, such as transfer cost and fixed location cost within different scenarios. Golpira et al. in 2017 and 2019 studied green chain management by consideration of retailers' risk (see [8] and [7] respectively). They designed a multi-objective model for a multi-echelon green supply chain. They considered some non-deterministic parameters, such as demand, pollution created by facilities, and gas emission resulting in environmental pollution. Also, they took into account the product return rate that was evaluated using the CVAR method.

In 2016, Shishebori provided a reliable logistics network of multi-car, multi-road and three types of communication roads considering the disruption of facilities and communication roads of the vehicle [14]. In 2019, Tavakoli Moghaddam et al. considered a multi-cycle routing, location, and inventory problem with a heterogeneous transportation fleet. In this paper, uncertainty in customer demand was considered fuzzy [15].

The optimal raw material supply chain for elementary and secondary school uniforms was obtained via an improved GA to resolve the model (see [17] for more details). Yixin and Zhen in 2021, proposed an enhanced genetic operator algorithm to assess the optimization of the service supply chain network [16].

Gao et al. [5] proposed a supply chain network model based on the comparison of traditional supply chain and the modern supply chain to solve the poor communication effect, uncirculated information, and unbalanced supply and demand in enterprises. After three algorithms and three commodity predication models were compared, a model combining with the network neural commodity demand predication method and the particle swarm optimization (PSO) algorithm was used to comprehensively evaluate the predication effect and algorithm performance by using the supply chain data. The supply chain network model constructed in this study can provide enterprises with a good commodity demand predication method and improve their ability to respond to risks in the supply chain. Salehi and Jabarpour in 2020 discussed a multi-objective model for multi-period location-distribution-routing problems considering the evacuation of casualties and homeless people and fuzzy paths in relief logistics. Since their problem was NP-hard and GAMS software was not able to solve the model in larger sizes, meta-heuristic algorithms of NSGA-II and MOPSO were used to solve the problem [13].

The hybrid model proposed in this study seeks to strike a balance between the concepts of efficiency, cost and dispersion. To evaluate the performance of the model, a set of different weights is assigned to the objective functions to observe the way the centers are selected on the basis of different management views. As is known, by increasing the weight of one of the objective functions by the decision-maker, the hybrid model tries to apply the management opinion and bring the obtained solutions closer to the independent optimal solution of that function. In general, the selection of points is done according to the following seven conditions.

- Maximize the efficiency of selected centers.
- Minimize the inventory system costs.
- Maximize the dispersion of selected centers.
- Satisfy the management opinion (the set of weights of objective functions).

- Meet the customers' demand.
- Do not exceed the sales capacity of candidate centers.
- Prevent the construction of an additional distribution center and impose additional costs.
- Improve the dimensions of stability.

Given the conditions above, the aim of this article is to solution the following important question. *How can we model and solve the problem of location-inventory-supply chain routing under conditions of uncertainty and using a passive defense approach?*
The passive defense approach to provide a mathematical model for supply chain has been used in a limited number of research papers. However, in the present study and for the first time, the model of a stable multilevel supply chain has been studied under conditions of uncertainty and using a passive defense approach. In general, the innovation of the present study can be described as follows.

- Presenting a stable, three-objective mathematical model for the routing-location-inventory problem in the supply chain under conditions of uncertainty and using a passive defense approach.
- Finding the location of facilities based on passive defense and dispersal measures.
- Considering the concepts of agility, flexibility and dispersion in the mathematical model as passive defense measures and concepts.
- Combining expert opinions and supply chain management to select optimal planning based on passive defense concepts and supply chain policies.
- Providing appropriate solutions to achieve sustainability goals by considering passive defense policies.

3 The Mathematical Model

The problem examined in this study includes four echelons, namely, the supplier, manufacturer, distributor, and sales center with limited capacity. This problem is solved for the location of facilities, distribution, and delivery of commodities. Some parameters are considered in the fuzzy mode to design a real problem.

According to the points mentioned above, some assumptions have been considered for modeling:

- The model is designed as a multi-period and multi-commodities model.
- The number of facilities is not predetermined; potential locations are considered for distributors and sales centers (customers). All facilities have limited capacities.
- Customers' demand is fuzzy; so, some demands of customers may remain unmet.

- Retailers have attraction, so that the more attractive the retailers, the more chance they have to send commodities. Moreover, commodity transfer between retail centers is allowed. In other words, it is assumed in this research that a retailer could be a distribution center to send commodities to other retailers under specific conditions.

- Maintenance cost depends on the ending inventory, and the shortage is not permitted. The cost of construction of centers is considered as a fuzzy number. The cost of transportation and displacement of each commodity from supplier centers to manufacturer centers is considered as the cost of raw materials.

- The minimum flexibility rate of the company is predetermined.

Table 1: Continued

$VARF_{it}$: risk of liquidity for supplier i in period t	α_l: number job opportunities created in sales center l
α_k: number job opportunities created in sales center k	sp_{js}: average rate of wastes produced in manufacturing center j to manufacture per unit of product s
dp_{js}: average rate of hazardous materials used in manufacturing center j to manufacture per unit of product s	dl_j: average lost dates caused by work injury in manufacturing center j to produce per unit of product s
θ_w: weighted factor of produced wastes (weight of produced wastes in the objective function)	θ_h: weighted factor of hazardous materials (weight of hazardous materials in the objective function)
θ_l: weighted factor of a work injury (weight of work injury in the objective function)	w_i: weight of supplier i
w_k: weight of point k to establish distributor	w_l: weight of point l to establish sales center
DL: the minimum distance between established sales centers	DK: the minimum distance between established distribution centers

The variables of the model are described as follows:
$y_l = 1$, if sales center is located in site l, and $y_l = 0$, otherwise,
y_k: 1, if the distribution center is located in site $k, 0$, otherwise,
x_{ijt}^{sv}: The amount of components flow of required product s from supplier center i to manufacturer center j in period t by vehicle v,
x_{jkt}^{sv}: The amount of product flow from manufacturer center j to distribution center k in period t by vehicle v,
Q_{jjt}^{sv}: The flow of product s from manufacturer center j to its warehouse in period t by vehicle v,
Q_{jkt}^{sv}: The product flow from manufacturer warehouse j to distribution center k in period t by vehicle v,

Table 1: Indices and parameters of Mathematical Model

I: including the points with coordinates (c_i, d_i) and the set of actual points for suppliers centers $(i = 1, 2, \ldots, I)$	J: including the points with (a_j, b_j) coordinates and the set of actual points for manufacturers centers $(j = 1, 2, \ldots, J)$
K: including the points with (x_k, y_k) coordinates and the set of actual points for distribution centers $(k = 1, 2, \ldots, K)$	L: including the points with (x'_l, y'_l) coordinates and the set of actual points for retail centers $(l = 1, 2, \ldots, L)$
S: set of products $(s = 1, 2, \ldots, S)$	V: the set of vehicles
T: set of planning times $(t = 1, 2, \ldots, T)$	$\tilde{d}^s_{lt}$: demand of product s by retailer l in period t
$\tilde{f}_k$: cost of construction of distribution center in site k	$\tilde{f}_l$: cost of construction of the retail center in site l
$\mathrm{B0_l}$: maximum attraction of customer l	B_l: coefficient of attraction of customer l $B_l = B0_l e^{-\gamma d^2} + \alpha$ where d indicates Euclidean distance and γ represents attraction coefficient
d_{ij}: the distance between supplier i and manufacturer j that is measured as Euclidean distance	d_{jk}: the distance between manufacturer j and distributor k that is measured as Euclidean distance
dj_j: The distance between manufacturer j and its warehouse that is measured as Euclidean distance	dj_{jk}: The distance between manufacturer j and distributor k that is measured as Euclidean distance
d_{kl}: the distance between distributor k and retailer l that is measured as Euclidean distance	dk_k: the distance between distributor k and its warehouse that is measured as Euclidean distance
dk_{kl}: the distance between warehouse of distributor k and retailer l that is measured as Euclidean distance	$dl_{ll'}$: the distance between retailer l and retailer l' that is measured as Euclidean distance
$dk_{kk'}$: the distance between distributors k and k' that is measured as Euclidean distance	c^s_{ij}: total transportation cost of the product s from supplier center i to manufacturing center j

x^{sv}_{klt}: The flow of product s from distribution center k to customer l in period t by vehicle v,
Q^{sv}_{kkt}: The product flow from distributer k to its warehouse in period t by vehicle v,
Q^{sv}_{klt}: The flow of product s from warehouse of distribution center k to customer l in period t by vehicle v,
$x^{sv}_{ll't}$: The flow of product s from customer l to customer l' in period t by vehicle v,
U^s_{jt}: The remained inventory of the product "s" in the warehouse of manufacturer center j in period t,

Table 1: Continued

c_{jk}^{s}: total transportation cost of the product s from manufacturing center j to distribution center k	cq_{jk}^{s}: total transportation cost of the product s from manufacturing center j to distribution center k
cq_{kl}^{s}: total transportation cost of the product s from distribution center k to retail center l	c_{kl}^{s}: total transportation cost of the product s from distribution center k to retail center l
$c_{ll'}^{s}$: total transportation cost of the product s from retail center l to retail center l'	ca_i: delivery capacity of supplier center in site I
ca_j: delivery capacity of manufacturing center in site j	caj_j: warehouse capacity of the manufacturer in site j
ca_k: delivery capacity pf distributor center in site k	cak_k: warehouse capacity of distributor center in site k
cap_v: capacity of vehicle v	h_j^s: maintenance cost of the product s in manufacturer warehouse in site j
h_k^s: maintenance cost of the product s in distributor warehouse in site k	LDC_{ist}: delay cost of supplier i to provide the product s in period t
O_{ist}: cost of ordering the product s to supplier i in period t	$F0$: the flexibility considered by the factory or organization
F_{ist}: flexibility of supplier i to provide product s in period t	R_{ist}: percentage of returned items s from supplier i in period t
$R0$: maximum acceptable percent of returned items within the planning horizon	$VARD_{ist}$: the value of the risk of delay in delivery of components related to product s by supplier i in period t
$VARD_{kst}$: the value of the risk of delay in delivering product s by distributor k in period t	$VARQ_{ist}$: risk of quality of product s received from supplier i in period t
$VARQ_{Kst}$: risk of quality of product s received from distributor k in period t	$VARND_{it}$: risk of natural disaster for supplier i in period t

U_{kt}^{s}: The remained inventory of the product "s" in the warehouse or distribution center k in period t,

$z_{ijt}^{v} = 1$, if vehicle v goes from supplier center i to manufacturer center j in period t, and $z_{ijt}^{v} = 0$, otherwise,

$z_{jkt}^{v} = 1$, if vehicle v goes from manufacturer center j to the distribution center k in period t, and $z_{jkt}^{v} = 0$, otherwise,

$z_{klt}^{v} = 1$, if vehicle v goes from distribution center k to retailer l in period t, and $z_{klt}^{v} = 0$, otherwise,

$z^v_{jjt} = 1$, if vehicle v goes from manufacturer center j to its warehouse in period t, and $z^v_{jjt} = 0$, otherwise,
$z^v_{kkt} = 1$, if vehicle v goes from distribution center k to its warehouse in period t, and $z^v_{kkt} = 0$, otherwise,
$zj^v_{jkt} = 1$, if vehicle v goes from warehouse of manufacturer j to distribution center k in period t, and $zj^v_{jkt} = 0$, otherwise,
$zk^v_{klt} = 1$, if vehicle v goes from the warehouse of distribution center k to retail center l in period t, and $zk^v_{klt} = 0$, otherwise,
$zl^v_{ll't} = 1$, if vehicle v goes from retail center l to retail center l' in period t, and $zl^v_{ll't} = 0$, otherwise,
q^s_{lt}: The unmet demand of product s in retail center l in period t.

3.1 Structure of the Mathematical Model

The symbols mentioned above are used in this section to present the mixed-integer linear fuzzy programming model for the design of a multi-objective integrated logistic network. This model consists of objective functions and constraints that have been described herein.

Objective Functions: The first objective function (1) minimizes the costs of supply chain (costs of establishment, transportation, inventory maintenance and ordering)

$$\begin{aligned}
\min \quad z1 = & \sum_{k\in K} \tilde{f}_k w_k y_k + \sum_{l\in L} \tilde{f}_l w_l y_l \\
& + \sum_{t\in T} \sum_{v\in V} \Big(\sum_{s\in S} \sum_{i\in I} \sum_{j\in J} c^s_{ij} d_{ij} x^{sv}_{ijt} \\
& + \sum_{s\in S} \sum_{j\in J} \sum_{k\in K} (c^s_{jk} d_{jk} x^{sv}_{jkt} + cq^s_{jk} dj_{jk} Q^{sv}_{jkt}) \\
& + \sum_{s\in S} \sum_{k\in K} (1 - B_l) \sum_{l\in L} (c^s_{kl} d_{kl} x^{sv}_{klt} + cq^s_{kl} dk_{kl} Q^{sv}_{klt}) \\
& + \sum_{s\in S} \sum_{j\in J} cq^s_{jj} dj_j Q^{sv}_{jjt} \\
& + \sum_{s\in S} \sum_{k\in K} cq^s_{kk} dk_k Q^s_{kkt} \Big) \\
& + \sum_{s\in S} \sum_{l\in L} \sum_{l'\in L} c^s_{ll'} dl_{ll'} x^{sv}_{ll't} \\
& + \sum_{t\in T} \Big(\sum_{s\in S} \sum_{j\in J} h^s_j U^s_{jt} \\
& + \sum_{s\in S} \sum_{k\in K} h^s_k U^s_{kt} \Big) \\
& + \sum_{i\in I} (1 - w_i) \sum_{t\in T} \sum_{j\in J} \sum_{s\in S} (O_{ist} + LDC_{ist}) \sum_{v\in V} x^{sv}_{ijt}.
\end{aligned} \tag{1}$$

The second objective function (2) minimizes the unmet demand.

$$\min \quad z2 = \sum_t \sum_l \sum_s \frac{q^s_{lt}}{\tilde{d}^s_{lt}}. \tag{2}$$

The third objective function of the model that is indicated in (3) maximizes the quality level of products purchased from suppliers.

$$\max \quad z3 = \sum_{i\in I} w_i \sum_{t\in T} \sum_{j\in J} \sum_{s\in S} (1 - R_{ist}) \sum_{v\in V} x^{sv}_{ijt}. \tag{3}$$

The objective function (4) maximizes social responsibility or social benefits; all items are on average that their weighted sum amount is considered regarding weight factors.

$$\max \quad z4 = \sum_{t\in T} \Big(\sum_{k\in K} \alpha_k w_k y_k + \sum_{l\in L} \alpha_l w_l y_l \Big). \tag{4}$$

The fifth objective function minimizes the environmental effects. It is given in (5).

$$\begin{aligned} \min \quad z5 = {} & \theta_w \sum_{t\in T} \sum_{j\in J} \sum_{s\in S} sp_{js} \Big(\sum_{v\in V} Q^{sv}_{jjt} \\ & + \sum_{k\in K} x^{sv}_{jkt} \\ & + \theta_h \sum_{t\in T} \sum_{j\in J} \sum_{s\in S} sp_{js} \Big(\sum_{v\in V} Q^{sv}_{jjt} \\ & + \sum_{k\in K} x^{sv}_{jkt} \Big) \\ & + \theta_l \sum_{t\in T} \sum_{j\in J} dl_j \sum_{s\in S} \Big(\sum_{v\in V} Q^{sv}_{jjt} + \sum_{k\in K} x^{sv}_{jkt} \Big). \end{aligned} \tag{5}$$

The sixth and the last objective function (6) minimizes the risk.

$$\begin{aligned} \min \quad z6 = {} & \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{i=1}^{I} VARD_{ist} \Big(\sum_{j=1}^{J} \sum_{v=1}^{V} x^{svt}_{ij} \Big) \\ & + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{k=1}^{K} VARD_{kst} \Big(\sum_{l=1}^{L} \sum_{v=1}^{V} x^{svt}_{kl} \Big) \\ & + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{i=1}^{I} VARQ_{ist} \Big(\sum_{j=1}^{J} \sum_{v=1}^{V} x^{svt}_{ij} \Big) \\ & + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{k=1}^{K} VARQ_{kst} \Big(\sum_{l=1}^{L} \sum_{v=1}^{V} x^{svt}_{kl} \Big) \\ & + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{i=1}^{I} VARND_{it} \Big(\sum_{j=1}^{J} \sum_{v=1}^{V} x^{svt}_{ij} \Big) \\ & + \sum_{t=1}^{T} \sum_{s=1}^{S} \sum_{i=1}^{I} VARF^{sen}_{it} \Big(\sum_{j=1}^{J} \sum_{v=1}^{V} x^{svt}_{ij} \Big). \end{aligned} \tag{6}$$

The constraints of the model are given as follows:

$$\sum_{v} \Big(\sum_{k} \left(x^{sv}_{klt} + Q^{sv}_{klt} \right) + \sum_{l'\in L} x^{sv}_{ll't} \Big) + q^{s}_{lt} = \tilde{d}^{s}_{lt} \quad \forall l, t, s, \tag{7}$$

Constraint (7) calculates the unmet demand.

$$\sum_{v} \sum_{j} \left(x^{sv}_{jkt} + Q^{sv}_{jkt} \right) = \sum_{v} \left(\sum_{l} x^{sv}_{klt} + Q^{sv}_{kkt} \right) \quad \forall k, s, t \tag{8}$$

$$\sum_{v} \sum_{i} x^{sv}_{ijt} = \sum_{v} \left(\sum_{k} x^{sv}_{jkt} + Q^{sv}_{jjt} \right) \quad \forall j, s, t \tag{9}$$

$$U_{jt}^{s}=\sum_{v}Q_{jjt}^{sv}+U_{jt-1}^{s}-\sum_{v}\sum_{k}Q_{jkt}^{sv}\quad\forall j,s,t \tag{10}$$

$$U_{j1}^{s}=\sum_{v}Q_{jj1}^{sv}-\sum_{v}\sum_{k}Q_{jk1}^{sv}\quad\forall j,s \tag{11}$$

$$U_{kt}^{s}=\sum_{v}Q_{kkt}^{sv}+U_{kt-1}^{s}-\sum_{v}\sum_{k}Q_{klt}^{sv}\quad\forall k,s,t \tag{12}$$

$$U_{k1}^{s}=\sum_{v}Q_{kk1}^{sv}-\sum_{v}\sum_{k}Q_{kl1}^{sv}\quad\forall k,s, \tag{13}$$

Constraints (8)-(13) represented the constraints of product flow balance in the nodes.

$$\sum_{v}\sum_{k}Q_{jkt}^{sv}\le\sum_{v}Q_{jjt}^{sv}\quad\forall j,s,t, \tag{14}$$

$$\sum_{v}\sum_{l}Q_{klt}^{sv}\le\sum_{v}Q_{kkt}^{sv}\quad\forall k,s,t, \tag{15}$$

Constraints (14) and (15) ensure that the outflow of manufactures and distributors' warehouses is less than the sum of inflow of their warehouses.

$$\sum_{v}\sum_{s}\sum_{j}x_{ijt}^{sv}\le ca_i\quad\forall i,t, \tag{16}$$

$$\sum_{v}\left(\sum_{s}\sum_{k}x_{jkt}^{sv}+\sum_{s}Q_{jjt}^{sv}\right)\le ca_j\quad\forall j,t, \tag{17}$$

$$\sum_{v}\left(\sum_{s}\sum_{l}x_{klt}^{sv}+\sum_{s}Q_{kkt}^{sv}\right)\le ca_k y_k\quad\forall k,t, \tag{18}$$

$$\sum_{s}U_{jt}^{s}\le caj_j\quad\forall j,t, \tag{19}$$

$$\sum_{s}U_{kt}^{s}\le cak_k y_k\quad\forall k,t, \tag{20}$$

Constraints (16)- (20) ensure that the flow moves only between those sites in which facilities are located. Also, these constraints ensure that sum of flow does not exceed the capacity of the facility.

$$\sum_{k}y_k\ge 1, \tag{21}$$

$$\sum_{l}y_l\ge 1, \tag{22}$$

Constraints (21) and (22) ensure that at least one potential center is working.

$$\sum_{v}\sum_{i}z_{ijt}^{v}=\sum_{v}z_{jjt}^{v}+\sum_{v}\sum_{k}z_{jkt}^{v}\quad\forall j,t, \tag{23}$$

$$\sum_v \sum_j (z^v_{jkt} + zj^v_{jkt}) = \sum_v \sum_l z^v_{klt} + \sum_v z^v_{kkt} \quad \forall k, t, \tag{24}$$

$$\sum_v z^v_{jjt} = \sum_v \sum_k zj^v_{jkt} \quad \forall j, t, \tag{25}$$

Constraints (23) and (25) indicate that vehicles entered the centers and their warehouses exit from these sites.

$$\sum_v \sum_i z^v_{ijt} \geq 1 \quad \forall j, t \tag{26}$$

$$\sum_v \sum_j (z^v_{jkt} + zj^v_{jkt}) \geq 1 \quad \forall k, t \tag{27}$$

$$\sum_v \sum_k (z^v_{klt} + zk^v_{klt}) \geq 1 \quad \forall l, t \tag{28}$$

Constraints (26), (27) and (28) indicate that distribution centers, markets, and customers are met at least by one vehicle.

$$x^{sv}_{ijt} \leq M \times z^v_{ijt} \quad \forall i, j, t, s, v \tag{29}$$

$$x^{sv}_{jkt} \leq M \times z^v_{jkt} \quad \forall k, j, t, s, v \tag{30}$$

$$x^{sv}_{klt} \leq M \times z^v_{klt} \quad \forall k, l, t, s, v \tag{31}$$

$$Q^{sv}_{jjt} \leq M \times z^v_{jjt} \quad \forall j, t, s, v \tag{32}$$

$$Q^{sv}_{kkt} \leq M \times z^v_{kkt} \quad \forall k, t, s, v \tag{33}$$

$$Q^{sv}_{jkt} \leq M \times zj^v_{jkt} \quad \forall k, j, t, s, v \tag{34}$$

$$Q^{sv}_{klt} \leq M \times zk^v_{klt} \quad \forall k, l, t, s, v \tag{35}$$

$$x^{sv}_{ll't} \leq M \times zl^v_{ll't} \quad \forall i, j, t, s, v \tag{36}$$

Constraints (29)-(36) ensure that a product is delivered from one center to another by a vehicle if the journey is done by those two centers by the same vehicle.

$$\sum_{t \in T} \sum_{i \in I} R_{ist} \sum_{v \in V} \sum_{i \in I} x^{sv}_{ijt} \leq R_0 \sum_{t \in T} \sum_{l \in L} \tilde{d}^t_{ls} \quad \forall s, \tag{37}$$

Constraint (37) ensures the sum of returned products does not exceed the maximum permitted rate.

$$F_{ist} \sum_{v \in V} \sum_{j \in J} x^{sv}_{ijt} \geq F_0 \quad \forall i, s, t, \tag{38}$$

Constraint (38) associates with the flexibility level of the supplier that must be greater than the level determined by the organization or factory.

$$\sum_{k=1}^{K} \min(dk_{k'k}) \geq DK \quad \forall k' \in K, \tag{39}$$

$$\sum_{l=1}^{L} \min(dl_{l'l}) \geq DL \quad \forall l' \in L, \tag{40}$$

Constraints (39) and (40) are related to the dispersion principle ensuring the minimum distance between distribution and sales centers.

$$y_l, y_k \in \{0,1\} \quad \forall l,k, \tag{41}$$

$$x^{sv}_{ijt},\ x^{sv}_{jkt}, Q^{sv}_{jjt}, Q^{sv}_{jkt},\ x^{sv}_{klt}, Q^{sv}_{kkt}, Q^{sv}_{klt}, U^s_{jt}, U^s_{kt} \geq 0 \quad \forall i,j,k,l,s,t, \tag{42}$$

Constraints (41) and (42) are rational and obvious constraints related to the decision variables of the problem.

3.2 Defuzzification of the Model

In the model developed in the previous section, some coefficients of the objective function and the right-hand values are fuzzy numbers. Various methods have been recommended to solve the mathematical fuzzy programming problems. In this research, the ranking method designed by Jimenez (2007) was used. Jimenez proposed a ranking method for fuzzy numbers based on their waiting scale comparison. The proposed fuzzy programming method was converted into an equivalent deterministic model according to the method developed by Jimenez (2007).

The deterministic form of first objective function is given in (43).

$$\begin{aligned}
\min \quad z1 = & \sum_{k\in K} \frac{f_k^1+2f_k^2+f_k^3}{4} w_k y_k \\
& + \sum_{l\in L} \frac{f_l^1+2f_l^2+f_l^3}{4} w_l y_l \\
& + \sum_{t\in T} \sum_{v\in V} (\sum_{s\in S} \sum_{i\in I} \sum_{j\in J} c^s_{ij} d_{ij} x^{sv}_{ijt} \\
& + \sum_{s\in S} \sum_{j\in J} \sum_{k\in K} (c^s_{jk} d_{jk} x^{sv}_{jkt} \\
& + cq^s_{jk} dj_{jk} Q^{sv}_{jkt}) \\
& + \sum_{s\in S} \sum_{k\in K} (1-B_l) \sum_{l\in L} (c^s_{kl} d_{kl} x^{sv}_{klt} \\
& + cq^s_{kl} dk_{kl} Q^{sv}_{klt}) \\
& + \sum_{s\in S} \sum_{l\in L} \sum_{l'\in L} c^s_{ll'} dl_{ll'} x^{sv}_{ll't} \\
& + \sum_{t\in T} (\sum_{s\in S} \sum_{j\in J} h^s_j U^s_{jt} \\
& + \sum_{s\in S} \sum_{k\in K} h^s_k U^s_{kt}) \\
& + \sum_{i\in I} (1-w_i) \\
& \sum_{t\in T} \sum_{j\in J} \sum_{s\in S} (O_{ist} + LDC_{ist}) \sum_{v\in V} x^{sv}_{ijt}.
\end{aligned} \tag{43}$$

The deterministic form of second objective function is indicated in (44).

$$\min \quad z2 = \sum_t \sum_l \sum_s \frac{q_{lt}^s}{\frac{d_{lt}^{s1} + 2d_{lt}^{s2} + d_{lt}^{s3}}{4}}. \tag{44}$$

The deterministic form of constraints (6) and (37) are given in (45) and (46) respectively.

$$\sum_v \Big(\sum_k \left(x_{klt}^{sv} + Q_{klt}^{sv} \right) + \sum_{l' \in L} x_{ll't}^{sv} \Big) + q_{lt}^s = \alpha \frac{d_{lt}^{s1} + d_{lt}^{s2}}{2} + (1-\alpha) \frac{d_{lt}^{s2} + d_{lt}^{s3}}{2} \quad \forall l, t, s, \tag{45}$$

$$\sum_{t \in T} \sum_{i \in I} R_{ist} \sum_{v \in V} \sum_{i \in I} x_{ijt}^{sv} \le R_0 \sum_{t \in T} \sum_{l \in L} \left[\alpha \frac{d_{lt}^{s1} + d_{lt}^{s2}}{2} + (1-\alpha) \frac{d_{lt}^{s2} + d_{lt}^{s3}}{2} \right] \quad \forall s. \tag{46}$$

4 The Method of Solution

Regarding the structure of the problem and its NP-Hard nature, in this research, Multi-objective particle swarm optimization (MOPSO) based on Pareto archive was generalized to solve the model. Some sample problems were solved by using the generalized algorithm to evaluate the performance of the proposed algorithms. In the next step, the obtained results were compared with the results of the NSGA-II algorithm.

4.1 The Method of Displaying the Solutions

Matrices are used in this study to display each solution; every solution includes several matrices that are designed on the basis of the model outputs. For instance, a linear one-dimensional matrix whose elements are equal to 1 is defined for each variable yl; on the other hand, a three-dimensional matrix of dimension $I * J * T * S$ is defined for variable x_{ijt}^s. This is done for all outputs.

4.2 How to Generate the Initial Solutions?

A parallel neighborhood search method was used in this study to generate the initial solutions. This method was applied to produce some initial solutions with appropriate quality, solution, and diversity.

In this research, two operators were used in parallel. In each neighborhood search function, a solution was sent as the initial solution to the relevant function, and then the considered operator was applied to the solution, and the solution neighborhood was obtained in the first step. After all neighborhood solutions were generated, the solution with the highest quality and diversity was chosen and added to the initial solutions based on the Deb Rule [4].

The next section describes the solution search operators and their parallel integration as the parallel neighborhood search procedure.

Operator 1: An index l is randomly produced in the uniform interval $[l \ldots L]$ (L represents the number of sales centers). If $y_l(l)$ equals 1 and there are some other points with value 1, then $y_l(l)$ converts to 0, and a corrective process is applied to the matrices related to this solution to correct them based on the constraints considered in the model. If the value $y_l(l)$ equals 0, this value converts to 1, and the corrective process is applied to other matrices to change them based on the constraints.

Operator 2: An index k is randomly produced in the uniform interval $[l \ldots K]$ (K represents the number of distribution centers). If $y_k(k)$ equals 1 and there are some other points with value 1, then $y_k(k)$ converts to 0, and a corrective process is applied to the matrices related to this solution to correct them based on the constraints considered in the model. If the value $y_k(k)$ equals 0, this value converts to 1, and the corrective process is applied to other matrices to change them based on the constraints.

The two neighborhood operators described above are applied to the produced iteration in parallel. Several neighborhood solutions that are local optimums in the neighborhood of the solution are reported for that solution. The reported solutions are chosen based on the high quality and diversity based on the Deb Rule [4], and then are added to the solution population provided that they are not repetitive.

Assume that the number of solutions existing per iteration of the PSO algorithm is indicated by N; this value remains constant during optimization. To produce N feasible, initial solutions, the designed parallel neighborhood search algorithm must generate N feasible non-repetitive solutions. This algorithm uses a predetermined feasible solution as the initial solution. The proposed method inserts the existing solution as the input to the parallel neighborhood search structure, and then selects a solution as output and adds it to the solution population if it is non-repetitive. This process continues until the algorithm reaches the ending condition. The complete structure of the parallel neighborhood search method is as follows.

0. Set the counter equal to 0.

1. Give the input solution (s) to the first neighborhood operator and name the output $s1$.

2. Give the input solution (s) to the second neighborhood operator and name the output $s2$.

3. Select the solution with the highest quality and diversity among the solutions $s, s1,$ and $s2$ using the Deb Rule.

4. Name the selected solution s.

5. Add one unit to the counter.

6. Go to Step 1 if the counter does not exceed the maximum permitted limit, and go to Step 9, otherwise.

7. End.

4.3 The Improvement Procedure

The improvement procedure in this research was designed on the basis of variable neighborhood search (VNS). Two neighborhood search structures (NSS) were designed and combined with the VNS structure. The NSS structures used to create VNS procedures were the same as the two NSSs (solution) described above that had been combined as VNS structures.

This combination can be described for each input solution s as follows.

```
BEGIN
   K:=1
   WHILE the stopping criterion is met DO
         s1=Apply Mutation type k.
         s=Acceptance_Method(s,s1).
         IF   s is improved   THEN   K=1   ELSE    K=k+1.
         IF    k=3   THEN    K=1.
   END WHILE
END
```

As can be seen in the structure above, after the application of the neighborhood structure to the solution, the acceptance procedure is applied to the obtained and previous solutions, and one of them is selected as the next repetitive solution of VNS. The acceptance procedure determines and selects the dominated solution from the two solutions by using non-dominated relations.

4.4 Updating Particles

The GA's operators were used to update particles. Particles were updated using the following process.

$$x_i^{t+1} = (x_i^t - p_i^t) + (x_i^t - p_g^t) + \overline{x_i^t}. \tag{47}$$

The symbols used in this equation can be described as follows:
x_i^{t+1}: particle i in iteration (generation) $t+1$,
x_i^t: particle i in iteration t,
p_i^t: the best solution that particle i has reached until this generation,
p_g^t: the best solution that has been found,
$\overline{x_i^t}$: a neighborhood of x_i^t that has been generated using the mutation operator,
'-': the crossover operator,
'+': represents a selection.
To obtain solution i in iteration $t+1$, five solutions are produced; among these solutions, two are obtained from the crossover operator between x_i^t and p_i^t, two are obtained from the crossover operator on x_i^t and p_g^t, and one solution is obtained from the mutation operator on x_i^t. Finally, the solution with the highest quality and diversity is selected as x_i^{t+1}. In the formula, p_g^t and p_i^t are used as guides to achieve the solutions of the next iteration.

Crossover operator. The crossover operator in this algorithm is a one-point crossover operator. After two parents were inserted as inputs to the crossover operator, two location matrices of each parent will be selected as peer-to-peer for the crossover operation.
Mutation operator. The mutation operator used for updating particles in equation (47) is the same VNS described in the previous section.

4.5 Updating p_i^t and p_g^t

For every particle i, if there exists any neighborhood better than p_I among the neighborhoods found for this solution, then p_i will be replaced; otherwise, it will remain unchanged.
If the best solution is better than $p_g$among all solutions, p_g will be replaced; otherwise, it will remain unchanged.

4.6 Updating the Pareto Archive

A set called *Pareto archive* is considered in the recommended algorithm. This set includes non-dominated solutions produced by the algorithm. It will be updated per iteration of the algorithm. In the update process, the solutions generated in that iteration and the solutions existing in the Pareto archive are placed in a solution pool and leveled. In the next step, the solutions existing in the first level of non-dominated solutions are selected and considered as a new Pareto archive.

4.7 Solution selection

The algorithm needs a solution population per iteration. To select the population of the next iteration, the solutions existing in the population are repeated, and the new solutions generated by the algorithm are inserted into a solution pool. After the leveling step and calculation of the crowding distance for each solution, the Deb Rule [4] is used, and N solutions with the highest quality and diversity are selected as the population of the next iteration of the algorithm.

5 Computational Results

The computational results were obtained via coding in $R2015a\ MATLAB$ software in a PC (with 10Gb RAM and 7-Core CPU). Several sample problems were designed to solve the model by algorithms. After the parameters of the model and algorithm were adjusted, these problems were solved using the proposed algorithms. Moreover, the proposed model was implemented and solved in the GAMS software. Then, the results were compared with those of the solution algorithms to examine the validity of the model and algorithm.

5.1 Comparative Indicators

Various indicators can be used to evaluate the quality and diversity (dispersion) of multi-objective, meta-heuristic algorithms. Three indicators were used in this research for the comparison process.

The quality index. This index compares the quality levels of the Pareto solutions obtained from each method. This index indeed levels all Pareto solutions obtained from both methods and determines the percentage of surface solutions that belong to each method. The higher the percentage, the higher the quality of the algorithm.

The spacing index. This criterion tests the distribution uniformity of the Pareto solutions in the border of solutions. It can be defined as follows.

$$s = \frac{\sum_{i=1}^{N-1} |d_{\text{mean}} - d_i|}{(N-1) \times d_{\text{mean}}}. \tag{48}$$

Herein, d_i represents the Euclidean distance between two non-dominated, adjacent solutions, and d_{mean} indicates the mean of the values d_i.

The dispersion index. This is defined by

$$D = \sqrt{\sum_{i=1}^{N} \max(\left\| x_t^i - y_t^i \right\|)}, \tag{49}$$

where $\left\| x_t^i - y_t^i \right\|$ indicates the Euclidean distance between the two adjacent solutions x_t^i and y_t^i on the optimal boundary.

5.2 Sample Problems

The problems have been organized in small, medium, and large categories based on the area existing in the previous studies. These problems have been shown in Tables 2-4.

Table 2: Small problems (number of vehicles 2)

Problem	**Number of products**	**Periods**	**Number of supply centers**	**number of manufacturing centers**	**Number of distribution centers**	**Number of customers**
1	1	1	2	2	2	2
2	1	1	2	2	2	2
3	2	1	2	2	2	2
4	2	1	2	2	2	2

Table 3: Medium-sized problems (number of vehicles 10)

Problem	Number of products	Periods	Number of supply centers	number of manufacturing centers	Number of distribution centers	Number of customers
1	1	4	3	3	7	7
2	2	8	3	3	7	7
3	3	12	3	3	7	7
4	1	4	6	6	8	10
5	2	8	6	6	8	10
6	3	12	6	6	8	10
7	1	4	7	7	9	15
8	2	8	7	7	9	15

Table 4: Large problems (number of vehicles 20)

Problem	Number of products	Periods	Number of supply centers	number of manufacturing centers	Number of distribution centers	Number of customers
1	1	4	10	10	20	30
2	2	8	10	10	20	30
3	3	12	10	10	20	30
4	1	4	15	15	40	70
5	2	8	15	15	40	70
6	3	12	15	15	40	70
7	1	4	15	15	45	90
8	2	8	15	15	45	90
9	3	12	15	15	45	90

5.3 Setting of the Parameters

This part of the study sets the solution parameters, including the model and algorithm parameters.

5.3.1 Setting the algorithm parameters

- In PSO, population size equals 150 with 15 VNS iterations and 300 algorithm iterations.
- In the NSGA-II algorithm, population size equals 200 by considering 300 algorithm iterations, 0.01-mutation rate, and 0.85-crossover rate.

5.4 Setting the model parameters

As mentioned before, some parameters of the model were considered as fuzzy numbers in the proposed model. A triangular fuzzy number was used to produce fuzzy rates. To generate triangular numbers related to each fuzzy parameter $(m1, m2, m3)$, $m2$ was generated and then, a stochastic number r was produced in the interval $[0, 1]$. In the next step, $m1$ and $m3$ were generated using $m2 * (1 - r)$ and $m2 * (1 + r)$, respectively.

To quantify fuzzy parameters, $m2$ was determined randomly and, $m1$ and $m3$ were determined through MATLAB software. Therefore, only the value of $m2$ is mentioned in the set parameters.

The following values have been considered to produce the sample problems.

- In each period, the customer demand of l for product s is considered as a triangular fuzzy number, and the amount or returned commodity is taken as a triangular fuzzy number $(m1, 30, m3)$.
- The capacity of all distribution centers was equal to 4000.
- The costs of the establishment of disposal, collection and rehabilitation, and recycling centers were considered as fuzzy numbers, namely, $(m1, 5000, m3)$, $(m1, 10000, m3)$, and $(m1, 15000, m3)$, respectively. Moreover, the cost of the establishment of distribution centers in the uniform interval was produced as fuzzy numbers: $(m1, 6000, m3)$.
- All distances between facilities have been randomly produced in the uniform interval $[1 \ldots 50]$.
- The percent of defective components was produced in the uniform interval $[0.05 \ldots 0.09]$.
- The cost of delay was produced in the uniform interval $[0 \ldots 1]$.
- The minimum flexibility level was set at the interval $[200 \ldots 500]$.
- Supplier flexibility values were considered at the uniform interval $[0 \ldots 1]$.
- The α values were considered to be 0.8 to rank fuzzy numbers.
- The average waste was considered 10% out of the production.
- The average rate of hazardous materials was considered 15% out of the production.
- The average number of lost working days was considered at the uniform interval $[5, 10]$.
- The values of weighted factors of produced wastes, hazardous materials, and work injury were measured based on the average values defined for producing wastes, hazardous materials, and work injuries. For instance, the sum value of the average parameter of produced wastes, and hazardous materials, as well as the average lost working days, were separately calculated. Moreover, the total rate of the mentioned variables was measured, and then this value of each parameter was divided by the sum of all three parameters and so on to calculate each factor.
- It was assumed in this study that retailers could create value-added for the supply chain by using innovative marketing. The attraction coefficients of retailers were measured based on their value-added rates. In the existing research, gray relations theory was used to determine the attraction coefficients of retailers based

on some variables, including pricing, branding and advertisement, market orientation, customer future value, and culture. Attraction scores given to retailers were performed on the basis of the innovative marketing and applying the term (1-B_l) to the objective functions.

- The weights of suppliers and potential sites were determined by using gray relations theory. The weight of suppliers in the objective function of the designed model was determined based on some indicators, including delivery velocity, minimum delay time, maximum quality, IT techniques, price, minimum instability, logistic, customer satisfaction, and information accuracy, by using gray relations theory and interval fuzzy numbers.
- Potential sites for the location of facilities were ranked by using the multi-criteria decision-making method of gray relations theory with interval fuzzy numbers based on some resilience criteria, including logistical flow rate, backup path security, and the possibility of resource and equipment deployment.
- The VAR method was used to determine the risks. Therefore, the following parameters were defined.

D_{ist}: Distribution function of delay in the delivery of components related to product s by supplier i in period t
D_{kst}: Distribution function of delay in the delivery of product s by distributor k in period t
Q_{ist}: Distribution function of defective product "s" received from supplier i in period t
Q_{Kst}: Distribution function of defective product s received from distributor k in period t
ND_i: The number of natural disasters that have disturbed the activity of supplier i
F_{it}: The fixed cost of purchase from supplier i in period t
VF_{ist}: The variable cost of purchasing product "s" from supplier i in period t

Ferin Theory was used to estimate VAR; a general form of generalized distribution of the Ferin value can be shown as follows.

$$\varphi_{\gamma,\delta,k}(x) = \begin{cases} \exp\left(-\left[1-k\left(\frac{x-\gamma}{\delta}\right)\right]^{\frac{1}{k}}\right) 1-k\left(\frac{x-\gamma}{\delta}\right) \geq 0, & k \neq 0 \\ \exp\left(-\exp\left(\frac{x-\gamma}{\delta}\right)\right) & k=0, \quad -\infty \leq x \leq \infty. \end{cases} \tag{50}$$

Herein, $f_{\gamma,\delta,k}(x)$ indicates the cumulative distribution function of the maximum variable (Ferin values), γ is related to the distribution situation, δ represents the distribution criterion parameter, and k is the sequence index indicating the shape or density of the distribution sequence.

5.5 Validation of the Model and Algorithm

The multi-objective model was converted to a single-objective model by using the LP-metric method, and then the single-objective model was solved for small problems through the GAMS software to validate the model.

In the LP-metric method, the individual solutions are first calculated for optimality of the objective functions. Then, the following objective function is minimized.

$$\begin{aligned}\min \quad z = & [w1^*(f_1(x) - f_1(x^*))/f_1(x^*) \\ & + [(w2^*)(f_2(x) - f_2(x^*))/f_2(x^*)] \\ & + [(w3^*)(f_3(x) - f_3(x^*))/f_3(x^*)] \\ & + [(w4^*)(f_4(x) - f_4(x^*))/f_4(x^*)] \\ & + [(w5^*)(f_5(x) - f_5(x^*))/f_5(x^*)] \\ & + [(w6^*)(f_6(x) - f_6(x^*))/f_6(x^*)].\end{aligned} \tag{51}$$

Here, $f_1(x^*)$ indicates the optimal value obtained from the model solution considering the first objective function, $f_2(x)$ indicates the value of the second objective function based on the optimal solution by solving the model only based on the first objective function, $f_2(x^*)$ indicates the optimal value obtained from the model solution considering the second objective function, and $f_1(x)$ indicates the value of the first objective function based on the optimal solution by solving the model only based on the second objective function. Moreover, wi^*represents the weight of the objective function.

The proposed model was coded by the GAMS software and solved by a BARON solver. In this method, the value $P = 1$ was considered, and the weights of the objectives were similar.

The gap between PSO and GAMS algorithms is measured based on the equation (52):

$$\text{GAP} = \frac{\text{Ob\_value}_{\text{PSO}} - \text{Ob\_value}_{\text{GAMS}}}{\text{Ob\_value}_{\text{PSO}}}, \tag{52}$$

where $\text{Ob\_value}_{\text{PSO}}$ represents the objective function of the LP-Metric model for PSO and $\text{Ob\_value}_{\text{GAMS}}$ represents the objective function of LP-Metric GAMS software.

Table 5: Comparative results of PSO and GAMS algorithms

Problem	PSO algorithm	GAMS	The gap between values
1	1155210000	1155210000	0
2	1134960000	1082488000	0.046
3	12377340000	11700220000	0.055
4	17412300000	15699798000	0.098

As seen in Table 5, the gap between PSO and GAMS algorithms equaled zero for the first problem while this gap was minor in other small problems. Since the accurate GAMS software is used to solve the model, the proposed model is valid and feasible. On the other hand, the proposed algorithm was valid and convergent towards optimal solution regarding the minor gap between results of GAMS and PSO algorithms.

5.6 Results of the Solved Problems

Tables 6, 7 and 8 report the comparative results to solve the problems based on the considered indicators.

Table 6: Results of solved small problems

Prob.	MOPSO					NSGA-II				
	Quality metric	Spacing metric	Diversity metric	CPU time	Number of Pareto solutions	Quality metric	Spacing metric	Diversity metric	CPU time	Number of Pareto solutions
1	96.96	1.09	1101.8	31.6	30	3.04	0.62	664.8	10.1	24
2	79.50	0.81	1257.5	49.3	31	20.50	0.74	754.7	16.8	33
3	98.64	0.95	1508.2	70.9	31	1.36	0.71	770.8	17.6	37
4	70.15	0.99	1857.1	71.5	36	29.85	0.76	1054.9	33.8	36

Table 7: Results of solved medium-sized problems

Prob.	MOPSO					NSGA-II				
	Quality metric	Spacing metric	Diversity metric	CPU time	Number of Pareto solutions	Quality metric	Spacing metric	Diversity metric	CPU time	Number of Pareto solutions
1	89.16	0.86	2051.4	100.6	50	10.84	0.64	1065.6	53.8	67
2	94.89	0.98	2105.4	101.3	75	5.11	0.63	1086.3	58.9	47
3	89.29	1.05	2500.1	117.7	71	10.71	0.67	1333.6	63.5	72
4	99.53	0.83	2535.5	122.5	60	0.47	0.62	1348.3	89.7	73
5	86.54	1.12	2567.9	136.7	79	13.46	0.63	1358.7	99.7	59
6	87.85	0.89	2631.1	153.2	58	12.15	0.61	1387.9	108.3	60
7	94.09	0.90	2644.5	170.5	54	5.91	0.72	1445.7	115.7	74
8	82.58	1.12	2927.4	177.6	65	17.42	0.61	1472.1	118.2	62
9	88.70	0.85	2993.3	180.2	65	11.30	0.61	1527.5	127.5	51

Table 8: Results of solved large problems

Prob.	MOPSO					NSGA-II				
	Quality metric	Spacing metric	Diversity metric	CPU time	Number of Pareto solutions	Quality metric	Spacing metric	Diversity metric	CPU time	Number of Pareto solutions
1	100	0.82	4535.9	215.5	118	0	0.79	2669.9	183.4	91
2	83.67	1.14	4900.1	226.4	102	16.33	0.60	2675.2	190.7	77
3	100	0.70	5006.7	232.6	104	0	0.67	2686.7	193.5	79
4	75.28	1.63	5047.4	239.7	84	24.72	0.97	2941.2	193.6	100
5	83.08	0.93	5149.8	307.7	82	16.92	0.78	3009.2	196.1	90
6	83.54	1.18	5518.6	323.1	110	16.46	0.79	3092.01	202.1	92
7	95.01	1.17	5606.4	356.6	113	4.99	0.82	3165.9	238.2	99
8	97.40	1.11	5663.3	369.7	91	2.60	0.54	3366.8	254.3	73
9	91.49	0.76	6472.6	395.5	111	8.51	0.66	3509.4	278.5	78

According to the comparative results reported in Tables 6, 7 and 8, the multi-objective PSO algorithm had a higher ability to produce high-quality solutions in all cases, compared to NSGA-II. In addition, the PSO algorithm produced solutions with higher diversity than NSGA-II; in other words, MOPSO had a higher ability to explore and extract the feasible region compared to NSGA-II. As can be seen in the tables above, the NSGA-II algorithm produced solutions with higher uniformity compared to the PSO algorithm. In terms of the implementation time, the MOPSO algorithm had a higher CPU time.

According to the structure of the proposed method, this method searches numerous points in the solution space per iteration. It takes, accordingly, a higher computational time compared to the NSGA-II algorithm.

As already mentioned, small, medium-sized, and large problems were solved by the two algorithms based on the comparative indicators of quality, diversity, and uniformity.

Some hypotheses have been designed to examine the difference between the results of these two algorithms based on analytical analysis.

The t-student test was used to find the difference between comparative indicators. It is worth noting that all hypotheses were tested for small, medium-sized, and large problems.

- **Hypothesis 1.** There is a significant difference between the quality indices of solutions produced by the two algorithms, namely, MOPSO and NSGA-II.
- **Hypothesis 2.** There is a significant difference between the diversity indices of solutions produced by the MOPSO and NSGA-II algorithms.
- **Hypothesis 3.** There is a significant difference between the uniformity indices of solutions produced by the MOPSO and NSGA-II algorithms.
- **Hypothesis 4.** There is a significant difference between the CPU times of the MOPSO and NSGA-II algorithms.

Table 9 reports the results of the evaluated hypotheses. As can be seen in Table 9, the t-values of quality, diversity, uniformity, and solution (implementation) time were equal to 16.603, 7.747, 5.043, and 7.437 at the significance level of 0.000 (< 0.05), respectively. The t-values of all indicators were out of the confidence interval; hence, all hypotheses were confirmed.

Therefore, there is a significant difference between quality, diversity, uniformity, and CPU time indicators of the solutions produced by the PSO and NSGA-II algorithms.

Table 9: Results of t-student test

	Mean difference	**SD**	**Sig.**	**df**	**t-value**	**The confidence interval of 95%**	
						Lower bound	Upper bound
Quality	67.28	19.01	0.000	21	16.603	58.85	75.71
Diversity	513.37	310.83	0.000	21	7.747	375.55	651.18
Uniformity	0.351	0.326	0.000	21	5.043	0.206	0.496
CPU time	102.84	64.86	0.000	21	7.437	74.09	131.60

6 Conclusion

The present study was conducted to propose a multi-objective model for supply chain location, inventory, and routing under uncertainty, by using passive defense in "Ministry of Defense and Armed Forces Logistics." To do so, an integrated model was designed for the location-routing-inventory problem in a four-echelon supply chain. This model comprised some parameters, such as demand, cost of deployment of facilities, and cost of inventory that were considered as non-deterministic and triangular fuzzy numbers.

In the designed model, characteristics and capabilities of passive defense, including logistical flow rate, backup path security, and the possibility of resource and equipment deployment were used to enhance supply chain resilience. In the next step, the potential sites were weighed or ranked according to the characteristics mentioned above for the location of facilities by using fuzzy multi-criteria decision-making methods. The model found locations of facilities based on the generated weights. In the first step, a six-echelon model was designed and some methods including LP-Metric, and a peer-to-peer single-objective model were produced through the GAMS software after proposing a multi-objective sustainable model for supply chain based on the passive defense. In the next step, the model was solved for small problems, and the proposed model was validated. Then, the MOPSO algorithm were used to solve the model based on the Pareto archive and NSGA-II algorithm.

According to validation results, a minor gap (equal to zero) was found between the MOPSO algorithm and the GAMS software. Since the GAMS was an accurate instrument that could solve the model, the proposed model had validity and feasibility. On the other hand, the minor gap between GAMS and PSO algorithm proved the validity of the proposed algorithm that produced an optimal solution. After the validity of the model was confirmed, sample problems were solved in three small, medium, and large groups using two MOPSO and NSGA-II algorithms. The obtained results were compared in terms of the quality, diversity, uniformity, and solution time indices. According to the obtained results, the MOPSO algorithm was more capable of producing high-quality solutions rather than the NSGA-II algorithm. The MOPSO algorithm could produce solutions with higher diversity compared to the NSGA-II algorithm; in other words, the MOPSO algorithm had a higher ability to explore and extract the feasible region rather than the NSGA-II algorithm. The MOPOS had a higher solution time of algorithms. According to the designed structure of the proposed method, this method searched numerous points in the solution space per iteration. This method took a higher computational time compared to the NSGA-II algorithm.

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