Received: July 17, 2022; **Accepted:** June 02, 2023.
DOI. 10.30473/COAM.2023.64776.1208
Summer-Autumn (2023) Vol. 8, No. 2, (17-32)
Research Article

Open Access

Control and Optimization in Applied Mathematics - COAM

Optimal Control of Infectious Diseases Using the Artificial Neural Networks

Rasoul Heydari Dastjerdi*, Ghasem Ahmadi, Mahmood Dadkhah, Yari, Ayatollah

Department of Mathematics, Payame Noor University (PNU), P.O. Box 19395-4697, Tehran, Iran.

Correspondence:
Rasoul Heydari Dastjerdi
E-mail:
rasoul59heydari@pnu.ac.ir

How to Cite
Heydari Dastjerdi, R., Ahmadi, Gh., Dadkhah, M., Yari, A. (2023). "Optimal control of infectious diseases using the artificial neural networks", Control and Optimization in Applied Mathematics, 8(2): 17-32.

Abstract. This paper presents a novel approach that uses artificial neural networks to solve the SEIR (Susceptible, Exposed, Infected, and Recovered) model of infectious diseases based on dynamical systems. Optimal control techniques are used to find a vaccination schedule for a standard SEIR epidemic model. The multilayer perceptron is applied to approximate the state and co-state functions of the SEIR model and to solve the optimal control problem using a nonlinear programming approach. By applying Pontryagin's Minimum Principle (PMP) to the SEIR model and constructing a loss function, a minimization problem is defined, and the approximate solution of the Hamiltonian system is computed. This method is compared with the fourth-order Runge-Kutta method. Illustrative examples are used to demonstrate the usefulness of the proposed approach

Keywords. Optimal control, Pontryagin's minimum principle, Artificial neural network, Epidemic model.

MSC. 49N90; 68T07.

1 Introduction

The matter of disease as well as the reasons why the models (SI, SIR, SEIR, etc.) are studied, may dictate the need for different compartments to be included. For instance, SEIR models can be used in cases where infected individuals are not immediately infected. People under investigation are first exposed to the disease and become in compartment E, after a latent period, they become compartment I. Some infectious diseases, such as measles, HCV, AIDS, HBV, and mumps have a latent period, from the time of exposure to the occurrence of disease symptoms. The term SEIR epidemic model refers to infectious diseases that have a latent period. By dividing a population into several subpopulations, using mathematical models, we can study the evolution of a disease outbreak in different stages of disease outbreak. For example, consider the SEIR model. A person who is not currently a carrier of a disease but can develop the disease because of lack of immunity is called 'susceptible'. A person who has contracted the disease but is not yet able to transmit it to others is called 'Exposed'. 'Infected' is the part where the person is reasonably infected with the disease and his disease can be spread to other people, and in the 'Recovered', the person is no longer sick and is protected and will not be infected again.

In the literature, SEIR models have been discussed by researchers in different works. Moneim et al. [32] considered an SEIR model with a fixed vaccination rate and infection during the incubation period to investigate the behavior of HBV disease. Greenhalgh [10, 11] applied Hopf bifurcations to consider SEIR and SEIRS models. Li, et al. [23, 24], and Zhang et al. [49] studied the SEIRS models with time delay. Samples of SEIRS models were investigated by Avila et al. [3]. In these models, it has been shown that a person from the latent class can transmit the disease to susceptible people. Li et al. [21, 22] showed that diseases such as HIV/AIDS have two different infectious stages with different abilities to transmit the disease. A delayed epidemic model was studied by Samantha et al. [41] in which the contact rate was different. Moneim et al. [12, 31] found that some infectious diseases have fluctuations in the number of occurrences due to certain conditions, for example, school opening and closing days or changes in weather. Boni et al. [4] showed that in many countries, such as Vietnam, a significant peak of H1N1 influenza is detected every year. A SIR model with vaccination rates was studied by Shulgin et al. [45, 46]. A dengue disease model with a periodic contact rate was investigated by Jan and Xiao [14], in which a pulse vaccination strategy was used. Several simple and continuous vaccination-based control strategies were applied by Sen et al. [7, 8] in an SEIR model that considered the entire population as a deterrent to consider disease transmission. They concluded that the susceptible, infectious, and infected populations converge to zero asymptotically over time by applying these vaccination strategies. Moneim [30] studied a SIRS model to investigate the outbreak of influenza H1N1, in which the vaccination rate was periodic. Authors [38, 40, 42] have studied this disease. Moneim [28, 29, 30] concluded that no fixed periodic vaccination provides good compliance to the periodic matter of some infectious diseases.

This paper aims to use the ability of artificial neural networks (ANNs) to approximate the states, co-states, and control functions of the SEIR epidemic model. The main motivation for using neural networks is that the usage of neural networks provides differentiable solutions. In the last decade, ANNs play an essential role in solving many problems, and their results can be compared with other methods using mathematical algorithms. These methods have been used in solving ODEs and PDEs in [19]. Vrabie et al. [47] used ANNs to solve unknown nonlinear systems from a reinforcement learning scheme. There are many references in theory and applications, modeling, algorithms, design, and mathematics of

neural networks (see [13, 33]), in particular, the numerical solution of ordinary and partial differential equations [18, 44], optimal control problems [5, 9] and mathematical programming [34, 36]. Mohammadi and Mansoori [27] utilized ANN for identifying copy number variants that help the diagnosis of many diseases. Mansoori et al. [25] investigated the fuzzy-constrained matrix game problems using the concepts of recurrent neural networks. Yu et al. [48] employed a novel recurrent neural network to deal with a kind of nonsmooth nonconvex optimization problem in which the objective function may be nonsmooth and nonconvex, and the constraints include linear equations and convex inequations. Shi and Chui [43] utilized fuzzy neural networks (FNNs) algorithm for detecting cardiovascular diseases. Acar et al. [1], using the various types of ANN, applied the backpropagation (BP) algorithm for forecasting diabetes mellitus. Afshar et al. [2] used the Levenberg-Marquardt learning algorithm for the recognition and prediction of leukemia. Khemphila et al. [16] employed ANN for heart disease classification. Chowdhury et al. [6] applied MLP with a backpropagation learning algorithm for neonatal disease diagnosis. Recently, Effati and Pakdaman proposed the ANNs to solve optimal control problems [9]. They have used this approach for optimal control of some nonlinear systems with one state variable. In this work, we extend this approach for the optimal control of an important and complex real nonlinear system with many state variables. This system is the SEIR (Susceptible, Exposed, Infected, Recovered) model of infectious disease.

Today, people of the world are challenged with many infectious diseases, and their optimal control is the ultimate goal of the human being in this context. In the literature, some mathematical methods have been proposed for this problem, such as Runge-Kutta (R-K) method [20].

In the present paper, based on the proposed approach in [9], ANNs are used to solve this problem. In recent years, ANNs have been used to solve many nonlinear problems, successfully. This work proposes the ANNs for the optimal control of the SEIR model of infectious diseases with four state variables. The capabilities of this approach in the optimal control of the SEIR model have been shown in the simulation results. In the future, we try to utilize this approach for the optimal control of more complex models of infectious diseases, such as fuzzy and fractional models of infectious diseases.

The rest of the paper is organized as follows. Section 2 introduces the SEIR epidemic model. In Section 3, the structure of ANNs and the related necessary optimal conditions are presented, and the states and co-state functions in the SEIR epidemic model are approximated by ANN. Numerical simulations are presented in Section 4, and Section 5 contains concluding remarks.

2 Model Description

Let $S(t), I(t), E(t)$, and $R(t)$ respectively denote the number of susceptible, infectious, exposed, and recovered individuals, and $N(t)$ represents the total number of people at the time t, so that $N(t) = S(t)+E(t)+I(t)+R(t)$, and let $u(t)$, the control function, be the percentage of susceptible individuals being vaccinated per unit of time. Due to the fact that vaccination of the entire susceptible population is impossible, we bound the control with $0 \leq u(t) \leq 0.9$. The dynamical system for the SEIR epidemic model can be constructed as:

$$\begin{aligned} S^{'}(t) &= bN(t) - dS(t) - cS(t)I(t) - u(t)S(t), \\ E^{'}(t) &= cS(t)I(t) - (e+d)E(t), \\ I^{'}(t) &= eE(t) - (g+a+d)I(t), \\ R^{'}(t) &= gI(t) - dR(t) + u(t)S(t), \\ N^{'}(t) &= (b-d)N(t) - aI(t), \end{aligned} \tag{1}$$

where a is the death rate that occurs due to the disease in infected individuals, b is the natural exponential birth rate, d is the natural exponential death rate, the term $cS(t)I(t)$ is the frequency or incidence of the disease, e, the rate of exposed individuals who become infected, and g is the infectious individuals' recovery rate. Also, it is assumed that only susceptible people are vaccinated and that the vaccine is compelling so that all vaccinated susceptible individuals become immune. In this model, $S(0) = S_0$, $E(0) = E_0$, $I(0) = I_0$, $R(0) = R_0$, and $N(0) = N_0$ are the number of individuals susceptible, exposed, infected, recovered, and total population, respectively, in the first performance of the operation.
Assume that the objective functional of the optimal control problem can be formulated as follows:

$$\min_{u} \int_0^T \Big(AI(t) + u^2(t) \Big) dt,$$

where the control variable is Lebesgue measurable and $u : [0, T] \to [0, 0.9]$. On the other hand, we are going to minimize the total cost of vaccination and the number of infected people during T times. The value of A in the objective functional determines the relative importance of the variables I and u and is actually a balancing parameter. Since the variable R appears only in the $R^{'}$ differential equation and the other variables do not depend on R, it is better to consider the dynamic system without this variable. Finally, by obtaining approximate values $S(t)$, $E(t)$, $I(t)$, and $N(t)$, the value $R(t)$ is obtained using the formula $R(t) = N(t) - S(t) - E(t) - I(t)$.

3 Artificial Neural Network for Solving the SEIR Model

ANNs process the calculation that involves training the network with representative data. The network consists of several inputs and outputs, and between them, there are hidden layers consisting of some hidden nodes. The number of hidden nodes and layers is empirically determined to optimize the performance of a network and to obtain a better result. Before using ANNs in solving SEIR epidemic model, we first shortly introduce ANNs.

Consider the following optimal control problem:

$$\begin{aligned} \min \quad & \int_{t_0}^{t_f} f_0\Big(x(t), u(t), t \Big) dt, \\ \text{s.t.} \quad & \dot{x} = g\Big(x(t), u(t), t \Big), \\ & x(t_0) = x_0, \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the state and control variables, respectively and $t \in \mathbb{R}$. Assume that the integrand f_0, concerning all its arguments, has continuous first and second partial derivatives, also the values of t_0 and t_f are constant and function g has the property of Lipschitz continuity on a set $\Omega \subset \mathbb{R}^n$. The Hamiltonian can be defined as:

$$H\Big(x(t),u(t),\lambda(t),t\Big)=f_0\Big(x(t),u(t),t\Big)+\lambda(t).g\Big(x(t),u(t),t\Big),$$

where $\lambda(t)\in\mathbb{R}^n$ is the co-state vector, so one can have the first order necessary conditions by PMP [39].

Theorem 1. ([17]) If $u^*(t)$ and $x^*(t)$ are optimal control and state functions for problem (1), then there exists an adjoint variable $\lambda(t)$ such that

$$H\Big(x^*(t),u^*(t),\lambda(t),t\Big)\leq H\Big(x^*(t),u(t),\lambda(t),t\Big),$$

at each time.

PMP provides a necessary condition for optimal control. This theorem shows that if $x(t)$, $\lambda(t)$, and $u(t)$ are the optimal values of the state, co-state, and control, respectively, they must satisfy the following conditions:

$$\begin{cases}\dfrac{\partial H(x,u,t,\lambda)}{\partial x}=-\dot{\lambda}(t),\\ \dfrac{\partial H(x,u,t,\lambda)}{\partial \lambda}=\dot{x}(t),\\ \dfrac{\partial H(x,u,t,\lambda)}{\partial u(t)}=0.\end{cases}\tag{2}$$

From (2), a system of ODEs is obtained, and nowadays, there are various numerical methods to solve such problems. We are trying to propose an approximation scheme for solving (2). Now we define the trial solutions in such a way that the initial conditions apply to it, so we have:

$$\begin{cases}x_T=x_0+(t-t_0)n_x,\\ \lambda_T=(t-t_f)n_\lambda,\\ u_T=n_u,\end{cases}$$

where n_x, n_u, and n_λ are ANN for variables of the state $x(t)$, control $u(t)$, and co-state $\lambda(t)$, respectively, and each ANN model contains its particular adjustable parameters as following:

$$\begin{cases}n_x=\sum_{i=1}^{n}v_x^i\sigma(z_x^i), & z_x^i=w_x^it+b_x^i,\\ n_\lambda=\sum_{i=1}^{n}v_\lambda^i\sigma(z_\lambda^i), & z_\lambda^i=w_\lambda^it+b_\lambda^i,\\ n_u=\sum_{i=1}^{n}v_u^i\sigma(z_u^i), & z_u^i=w_u^it+b_u^i,\end{cases}$$

for $i=1,2,\cdots,n$, where for each ANN, the number of neurons is shown by n, which may be different, w is a weight vector of the input layer, b is a vector containing bias weight, v is output layer weights, z is a vector of the hidden layer, and σ is an arbitrary activation function, where in this work, the activation function is tanh function. Based on Kolmogorov's theorem, any continuous function can be implemented with an MLP [15]. Since the initial conditions must satisfy, then $x_T(t_0)=x_0$. Note that if $x(t_f)$ is free, then $\lambda(t_f)=0$. Appropriate trial functions should be defined for other initial/boundary conditions. The functions x, λ, and u can be replaced in the Hamiltonian with x_T, λ_T, and u_T respectively, as:

$$H_T\Big(x_T(t), u_T(t), \lambda_T(t), t\Big) = f_{0_T}\Big(x_T(t), u_T(t), t\Big) + \lambda_T(t).g_T\Big(x_T(t), u_T(t), t\Big).$$

By applying conditions (2) in the obtained Hamiltonian, the following relationships are obtained

$$\begin{cases} \dfrac{\partial H_T}{\partial x_T} + \dot{\lambda}_T = 0, \\ \dfrac{\partial H_T}{\partial \lambda_T} - \dot{x}_T = 0, \\ \dfrac{\partial H_T}{\partial u_T} = 0. \end{cases} \tag{3}$$

Hence, the optimization problem can be constructed as follows:

$$\min E(\phi) = \frac{1}{2}\{E_1(t,\phi) + E_2(t,\phi) + E_3(t,\phi)\},$$

where $\phi = (w_x, w_\lambda, w_u, b_x, b_\lambda, b_u, v_x, v_\lambda, v_u)$, and

$$\begin{cases} E_1(t,\phi) = \left[\dfrac{\partial H_T}{\partial x_T} + \dot{\lambda}_T\right]^2, \\ E_2(t,\phi) = \left[\dfrac{\partial H_T}{\partial \lambda_T} - \dot{x}_T\right]^2, \\ E_3(t,\phi) = \left[\dfrac{\partial H_T}{\partial u_T}\right]^2. \end{cases} \tag{4}$$

To solve (4), the interval $[t_0, t_f]$ can be discretized to m points and solve the unconstrained optimization problem.

Lemma 1. ([35]) If $\phi^* = (w_x^*, w_\lambda^*, w_u^*, b_x^*, b_\lambda^*, b_u^*, v_x^*, v_\lambda^*, v_u^*)$, satisfies the following relationship

$$\eta(\phi) = \Big[E_1(t_1,\phi), \cdots, E_1(t_m,\phi), E_2(t_1,\phi), \cdots, E_2(t_m,\phi), E_3(t_1,\phi), \cdots, E_3(t_m,\phi)\Big]^T = 0,$$

then ϕ^* is an optimal solution of (3).

Due to Lemma 1, the system in (4) can be considered equivalent to the following minimization problem.

$$\min_{\phi} E(\phi) = \frac{1}{2}||\eta(\phi)||^2. \tag{5}$$

The unconstrained optimization problem (5) can be solved using optimization algorithms, such as Newton, Quasi-Newton, or steepest descent methods, etc.

Now we consider the optimal control problem due to the SEIR epidemic model. We have:

$$\begin{aligned} &\min_u \int_0^T \Big(AI(t) + u^2(t)\Big)dt, \\ &S^{'}(t) = bN(t) - dS(t) - cS(t)I(t) - u(t)S(t), \\ &E^{'}(t) = cS(t)I(t) - (e+d)E(t), \\ &I^{'}(t) = eE(t) - (g+a+d)I(t), \\ &N^{'}(t) = (b-d)N(t) - aI(t), \\ &S(0) = S_0, E(0) = E_0, I(0) = I_0, N(0) = N_0, \end{aligned}$$

Note that we have removed the variable R. The Hamiltonian function can be constructed as the following:

$$\begin{aligned} H = AI + u^2 + \lambda_S(bN - dS - cSI - uS) + \lambda_E(cSI - (e+d)E) \\ + \lambda_I(eE - (g+a+d)I) + \lambda_N((b-d)N - aI), \end{aligned}$$

where the values $\lambda_S, \lambda_E, \lambda_I$, and λ_N are the associated adjoints for the state variables S, E, I, and N, respectively. Based on first part of (2), by deriving the Hamiltonian concerning all the state variables, we get the differential equations for the associated adjoints. Therefore, the adjoint system is stated as follows:

$$\begin{aligned} \lambda_S^{'} &= \lambda_S(d + cI + u) - \lambda_E cI, \\ \lambda_E^{'} &= \lambda_E(e+d) - \lambda_I e, \\ \lambda_I^{'} &= -A + \lambda_S cS - \lambda_E cS + \lambda_I(g+a+d) + \lambda_N a, \\ \lambda_N^{'} &= -\lambda_S b + \lambda_N(d-b). \end{aligned} \tag{6}$$

According to the second part of (2), the equation system (1) is obtained. Furthermore, the conclusion can be drawn from the third part of (2) as:

$$u(t) = \frac{S(t).\lambda_S(t)}{2}. \tag{7}$$

Considering the initial conditions in the equations of state and adjoint, the trial solutions can be constructed as follow:

$$\begin{aligned} S_T &= S_0 + t\Big(\sum_{i=1}^{n} v_S^i \times \tanh(w_S^i t + b_S^i)\Big), \\ E_T &= E_0 + t\Big(\sum_{i=1}^{n} v_E^i \times \tanh(w_E^i t + b_E^i)\Big), \\ I_T &= I_0 + t\Big(\sum_{i=1}^{n} v_I^i \times \tanh(w_I^i t + b_I^i)\Big), \\ N_T &= N_0 + t\Big(\sum_{i=1}^{n} v_N^i \times \tanh(w_N^i t + b_N^i)\Big). \end{aligned}$$

Additionally,

$$\begin{aligned} \lambda_{S_T} &= (t-T)\Big(\sum_{i=1}^{n} v_{\lambda_S}^i \times \tanh(w_{\lambda_S}^i t + b_{\lambda_S}^i)\Big), \\ \lambda_{E_T} &= (t-T)\Big(\sum_{i=1}^{n} v_{\lambda_E}^i \times \tanh(w_{\lambda_E}^i t + b_{\lambda_E}^i)\Big), \\ \lambda_{I_T} &= (t-T)\Big(\sum_{i=1}^{n} v_{\lambda_I}^i \times \tanh(w_{\lambda_I}^i t + b_{\lambda_I}^i)\Big), \\ \lambda_{N_T} &= (t-T)\Big(\sum_{i=1}^{n} v_{\lambda_N}^i \times \tanh(w_{\lambda_N}^i t + b_{\lambda_N}^i)\Big), \end{aligned}$$

such that $\lambda_S(T) = \lambda_E(T) = \lambda_I(T) = \lambda_N(T) = 0$. By placing the trial solutions in the (6) and (1), the first and second parts of (4) are formed.

To solve the optimal control problem, a variant of the forward-backward sweep method [26] is utilized. Using this approach, we first consider the initial guess for the control variable, which our proposal in this work is $u(t) = 0$, for $0 \leq t \leq T$. Using the equation system (1) and the initial value of the control variable, the new state variables are obtained. Then, we put the obtained values in (6) to get the new value of co-state variables. By obtaining the value of $S(t)$, and $\lambda_S(T)$, the new value of control can be obtained using (7). It should be noted that according to the assumption of the problem, if the control value exceeds 0.9, we set its value to 0.9. If the algorithm stop condition is confirmed, the obtained values for the state, co-state, and control variables are optimal; otherwise, we replace the new control variable with the initial value of the control variable and repeat the process. We continue this process until the algorithm stop condition is reached. In the MATLAB program related to this article, the algorithm stops when the distance between the variables in each step is less than 0.00001 compared to the previous step. At the same time, we also update the control value.

Remark 1. In [20], using the PMP, the optimal control of the SEIR model has been converted to some sets of differential equations, and then, the R-K method of order four is used to solve these sets of differential equations. Also, in the present work, the PMP is used to convert the optimal control of the SEIR model to two sets of differential equations where the neural networks are utilized to solve these sets of differential equations. In this work, we compared the graphs and the values of the objective function obtained from these methods.

4 Numerical Result

In this section, we apply ANN discussed in Section 3 for solving the SEIR model. The dynamics in the absence of vaccination are shown as the dashed curve in the infectious class. In this work, we approximate the objective functional using the left-point Reimann sum included in the program. Each neural network is a multilayer perceptron with five neurons in the hidden layer where the hyperbolic tangent is the activation function of them. Therefore, there are five parameters between the inputs and hidden neurons, five parameters between the hidden neurons and outputs, and five parameters for the biases. By replacing the trial solutions obtained from the neural networks in (1) and (6), the set of differential equations is made. Then, using the nonlinear optimization methods, the best values for the weights and biases have been computed. We test the performance of the proposed scheme on two test problems. Our criterion for the effectiveness of the discussed method compared to the R-K method is the obtained value of the objective functional. To clear up the ambiguity of this issue, we prepared two tables related to the examples. All numerical computations have been coded in Matlab R2017b with 4GB RAM.

Example 1. We consider the optimal control problem that is described in Section 3. By considering the parameters as in Table 1, Figure 1 shows the optimal vaccination schedule and the corresponding

population dynamics. The red, blue, and dashed lines in the infectious class show the R-K method, the presented method, and the dynamics in the absence of vaccination, respectively.

In Figure 1, the diagram of state and control functions that are found by R-K method of the fourth order in [37] are compared with the results that are found by the presented method. Furthermore, in Table 2, we compare the value of objective functional, which is given as follows, from both methods.

$$\min_{u} \int_0^T \Big(AI(t) + u^2(t) \Big) dt.$$

Table 1: Parameter values for Example 1

Value	Description
$S_0 = 1000$	initial susceptible population
$E_0 = 100$	initial exposed population
$I_0 = 50$	initial infected population
$R_0 = 15$	initial recovered population
$b = 0.525$	birth rate
$d = 0.5$	death rate
$c = 0.001$	incidence coefficient
$e = 0.5$	exposed to infectious rate
$g = 0.1$	recovery rate
$a = 0.2$	disease induced death rate
$A = 0.1$	weight parameter
$T = 20$	number of years

Table 2: Results for Example 1

Method	Description	
	Number of points	Objective function value
ANN	1000	24.1567
R-K	1000	24.2463

Table 3 compares the values obtained from both methods at several points.

Example 2. [20] Let us consider the same optimal control problem but with different objective functional as:

$$\max_{u} \int_0^T \big(AN(t) - u^2(t) dt \big).$$

In this example, we consider a case where the infection has been spreading unchecked for some time before intervention occurs. Figure 2 shows the optimal vaccination program and the dynamics of relevant population based on the data presented in Table 4. The red, blue, and dashed lines in the infectious class show the R-K method, the presented method, and the dynamics in the absence of vaccination, respectively. In Table 5, we compare the value of objective functional and the number of vaccinated individuals. We compare the values obtained from both methods at some points in Table 6.

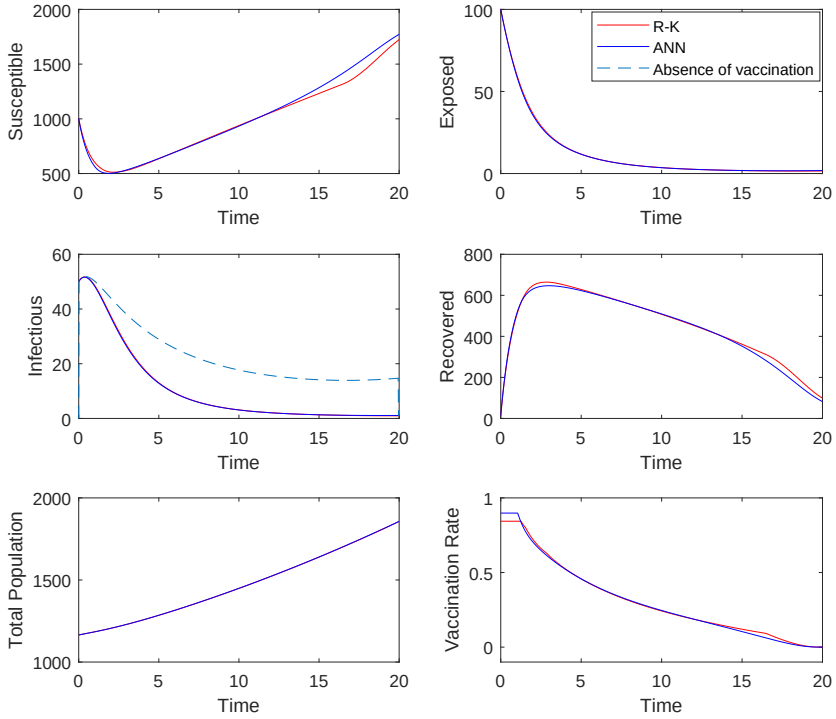

Figure 1: Optimal control schedule for Example 1.

Table 3: Values of functions related to Example 1

Time		0.1	2.4	5	10.3	13	15.5	17.9	19.8
Susceptible	ANN	937	510	636	951	1129	1334	1579	1762
	R-K	960	513	633	955	1114	1265	1473	1707
Exposed	ANN	96.07	29.85	11.79	3.35	2.24	1.78	1.69	1.82
	R-K	97.18	30.80	11.94	3.42	2.26	1.72	1.5	1.57
Infectious	ANN	50.67	32.63	12.95	2.89	1.74	1.26	1.09	1.1
	R-K	50.51	33.25	13.16	2.94	1.77	1.24	1.01	0.97
Recovered	ANN	82	641	623	502	426	326	188	87
	R-K	61	660	629	500	424	342	225	105
Population	ANN	1166	1214	1284	1460	1560	1664	1771	1852
	R-K	1166	1214	1283	1458	1558	1662	1768	1849
Vaccination	ANN	0.89	0.66	0.46	0.24	0.16	0.09	0.02	0
	R-K	0.84	0.67	0.46	0.23	0.16	0.11	0.03	0

Remark 2. In the comparison of the proposed method with the result obtained from the R-K method in [20], as described in the simulation results, we conclude that the precision of the proposed method is

Table 4: Parameter values for Example 2

Value	Description
$S_0 = 1000$	initial susceptible population
$E_0 = 1000$	initial exposed population
$I_0 = 2000$	initial infected population
$R_0 = 500$	initial recovered population
$b = 0.525$	birth rate
$d = 0.5$	death rate
$c = 0.001$	incidence coefficient
$e = 0.5$	exposed to infectious rate
$g = 0.1$	recovery rate
$a = 0.2$	disease induced death rate
$A = 0.1$	weight parameter
$T = 20$	number of years

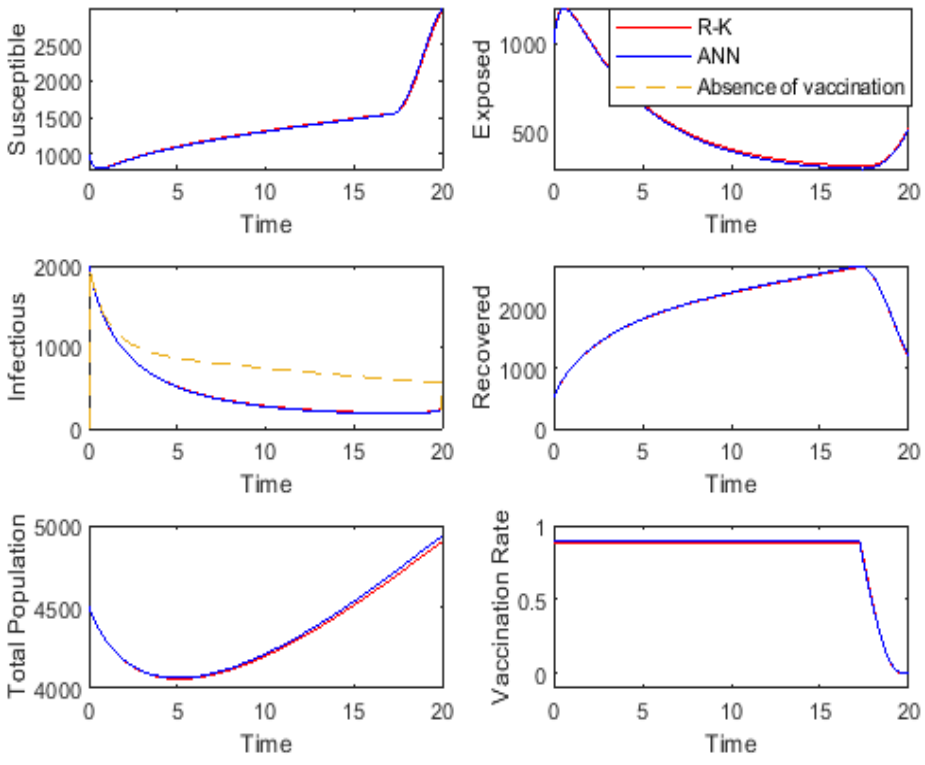

Figure 2: Optimal control schedule for Example 2.

Table 5: Results for Example 2

Method	Description	
	Number of points	Objective function value
ANN	1000	8692
R-K	1000	8687

Table 6: Values of functions related to Example 2

Time		0.4	3.1	5	7.52	13	15.6	18.1	19.9
Susceptible	ANN	815	975	1088	1205	1407	1492	1823	2949
	R-K	817	983	1099	1216	1418	1502	1765	2892
Exposed	ANN	1180	857	647	474	316	289	292	486
	R-K	1185	869	658	490	330	303	304	489
Infectious	ANN	1662	740	513	352	214	189	179	228
	R-K	1666	746	519	362	222	198	188	233
Recovered	ANN	747	1531	1815	2073	2451	2606	2482	1262
	R-K	741	1522	1808	2059	2437	2588	2490	1266
Population	ANN	4404	4103	4063	4108	4390	4577	4778	4926
	R-K	4406	4102	4059	4098	4370	4552	4745	4889
Vaccination	ANN	0.9	0.9	0.9	0.9	0.9	0.9	0.41	0
	R-K	0.89	0.89	0.89	0.89	0.89	0.89	0.43	0

better. However, it should be noted that, in the proposed method, a nonlinear optimization algorithm is used to adjust the parameters of the neural networks. As a result, the execution time of this process is relatively longer compared to the R-K method. But, it must be mentioned that in the optimal control of disease models, between precision and acceleration, the priority is precision. Because, we solve these problems one time, and often, repetition is not needed.

5 Conclusion

This paper proposes an approach that combines artificial neural networks (ANNs) and optimization techniques to determine an approximate solution for the SEIR epidemic model. Notably, in the method outlined in this study, the obtained weights from solving the unconstrained optimization problem are directly used in the trial solutions, eliminating the need for interpolation or fitting methods to plot the response curve continuously. ANNs possess excellent properties, such as their ability to adapt to nonlinear systems through training, making them useful for optimal control of the SEIR epidemic model and other related problems. In future studies, this approach could be applied to solve fuzzy and fractional SEIR models and other epidemic models.

Declarations

Availability of supporting data

All data generated or analyzed during this study are included in this published paper.

Funding

This study received no funds, grants, or other financial support.

Competing interests

The authors declare no competing interests are relevant to the content of this paper.

Authors' contributions

The main manuscript text is collectively written by all authors.

References

[1] Acar E., Ozerdem M.S., Akpolat V. (2001). "Diabetes mellitus forecast using various types of artificial neural network", International Advanced Technologies Symposium, 11.

[2] Afshar S., Abdolrahmani F. (2011). "Recognition and prediction of leukemia with artificial neural network (ANN)", Medical Journal of Islamic Republic of Iran, 25(1), 35-39.

[3] Avila E., Rivero E., Emilio G. (2017). "Global dynamics of a periodic SEIRS model with general incidence rate", International Journal of Differential Equations, 2017, 1-14.

[4] Boni M.F., Manh B.H., Thai P.Q., Farrar J., Hien N.T., Kinh N.V., Horby B. (2009). "Modelling the progression of pandemic influenza a (H1N1) in Vietnam and the opportunities for reassortment with other influenza viruses", BMC Medicine, 7(43).

[5] Cheng T., Lewis F.L., Abu-Khalaf M. (2007). "Fixed-final-time constrained optimal control of nonlinear systems using neural network HJB approach", IEEE Transaction Neural Network, 18, 1725-1737.

[6] Chowdhury D.R., Chatterjee M., Samanta R.K. (2011). "An artificial neural network model for neonatal disease diagnosis", International Journal of Artificial Intelligence and Expert Systems (IJAE), 2(3).

[7] De la Sen M., Alonso-Quesada S. (2011). "Vaccination strategies based on feedback control techniques for a general SEIR epidemic model", Applied Mathematics and Computation, 218(7), 2637-2658.

[8] De la Sen M., Ibeas A., Alonso-Quesada S. (2012). "On vaccination controls SEIR epidemic model", Communications in Nonlinear Science Numerical Simulation, 17(6), 3888-3904.

[9] Effati S., Pakdaman M. (2013). "Optimal control problem via neural networks", Neural Computing and Application, 23, 2093-2100.

[10] Greenhalgh D. (1992). "Some results for an SEIR epidemic model with density dependence in the death rate", IMA journal of mathematics applied in medicine and biology, 9(2), 67-106.

[11] Greenhalgh D. (1997). "Hopf bifurcation in epidemic models with a latent period and nonpermanent immunity", Mathematical and Computer Modelling, 25, 85-93.

[12] Greenhalgh D., Moneim I.A. (2003). "SIRS epidemic model and simulations using different types of seasonal contact rate", Systems Analysis Modelling Simulation, 43(5), 573-600.

[13] Haykin S. (2007). "Neural networks: A comprehensive foundation", 3rd editions Prentice-Hall, Upper Saddle River.

[14] Jan R., Xiao Y. (2019). "Effect of pulse vaccination on dynamics of dengue with periodic transmission functions", Advances in Difference Equations, 219(1), 368.

[15] Keckman V. (2001). "Learning and soft computing", MIT press, Cambridge, MA.

[16] Khemphila A., Boonjing V. (2011). "Heart disease classification using neural network and feature selection", 21st International Conference on Systems Engineering.

[17] Kirk D. (2004). "Optimal control theory an introduction", Dover Publications, Mineola, NY.

[18] Kumar M., Yadav N. (2011). "Multilayer perceptrons and radial basis function neural network methods for the solution of differential equations: a survey", Computers & Mathematics with Applications, 62, 3796-3811.

[19] Lagaris I.E., Likas A. (2012). "Hamilton–Jacobi theory over time scales and applications to linear-quadratic problems", IEEE Transactions on Neural Network, 9(5), 987-1000.

[20] Lenhart S., Workman J. (2007). "Optimal control applied to biological models", Boca Raton, Chapman Hall/CRC.

[21] Li X., Fang B. (2009). "Stability of an age-structured SEIR epidemic model with infectivity in latent period", Applications and Applied Mathematics, 4(1), 218-236.

[22] Li G., Jin Z. (2005). "Global stability of a SEIR epidemic model with infectious force in latent, infected and immune period", Chaos, Solitons and Fractals, 25(2), 1177-1184.

[23] Li M.Y., Muldowney J.S. (2001). "Global dynamics of a SEIR epidemic model with vertical transmission", SIAM Journal on Applied Mathematics, 62(1), 58-69.

[24] Li M.Y., Muldowney J.S., Wang L.C., Karsai J. (1999). "Global dynamics of an SEIR epidemic model with a varying total population size", Mathematical Biosciences, 160(2), 191-213.

[25] Mansoori A., Eshaghnezhad M., Effati S. (2019). "Recurrent neural network model: A new strategy to solve fuzzy matrix games", IEEE Transactions on Neural Networks and Learning Systems, 30(8), 2538-2547.

[26] McAsey M., Mou L., Weimin W. (2012). "Convergence of the forward-backward sweep method in optimal control", Computational Optimization and Applications, 53, 207-226.

[27] Mohammadi M., Mansoori A. (2018). "A projection neural network for identifying copy number variants", IEEE Journal of Biomedical and Health Informatics, 23(5), 2182-2188.

[28] Moneim I.A. (2011). "Different vaccination strategies for measles diseases: A simulation study", Journal of Informatics and Mathematical Sciences, 3, 227-236.

[29] Moneim I.A. (2013). "Efficiency of different vaccination strategies for childhood diseases: A simulation study", Advances in Bioscience and Biotechnology, 4(2), 193-205.

[30] Moneim I.A. (2016). "Modeling and simulation of the spread of H1N1 flu with periodic vaccination", International Journal of Biomathematics, 9(1).

[31] Moneim I.A., Greenhalgh D. (2005). "Threshold and stability results for an SIRS epidemic model with a general periodic vaccination strategy", Journal of Biological Systems, 13(2), 131-150.

[32] Moneim I.A., Khalil H.A. (2015). "Modelling and simulation of the spread of HBV disease with infectious latent", Applied Mathematics, 6(5), 745-753.

[33] Muller B., Reinhardt J., Strickland M.T. (2002). "Neural networks: An introduction", 2nd editions Springer, Berlin.

[34] Nazemi A.R., Effati S. (2013). "An application of a merit function for solving convex programming problems", Computers & Industrial Engineering, 66, 212-221.

[35] Nazemi A., Karami R. (2017). "A neural network approach for solving optimal control problems with inequality constraints and some applications", Neural Processing Letters, 45(3), 995-1023.

[36] Nazemi A.R., Omidi F. (2012). "A capable neural network model for solving the maximum flow problem", Journal of Computational and Applied Mathematics, 236, 3498-3513.

[37] Neilan R.M., Lenhart S. (2010). "An introduction to optimal control with an application in disease modeling", DIMACS Series in Discrete Mathematics, 78, 67-81.

[38] Nishiur H., Inaba H. (2011). "Estimation of the incubation period of influenza a (H1N1) among imported cases: Addressing censoring using out break data origin of importation", Journal of Theoretical Biology, 272(1), 123-130.

[39] Pontryagin L.S., Boltyanskii V.G., Gamkrelize R.V., Mishchenko E.F. (1962). "The mathematical theory of optimal processes", New York, Wiley.

[40] Rausanu S., Grosan C. (2014). "A hierarchical network model for epidemic spreading. Analysis of a H1N1 virus spreading in Romania", Applied Mathematics and Computation, 233, 39-54.

[41] Samanta G.P., Sen P., Maiti A. (2016). "A delayed epidemic model of diseases through droplet infection and direct contact with saturation incidence and pulse vaccination", Systems Science and Control Engineering: An Open Access Journal, 4(1), 320-333.

[42] Sasuzzoha M., Singh M., Lucy D. (2013). "Parameter estimation of influenza epidemic model", Applied Mathematics and Computation, 220, 616-629.

[43] Shi J., Chui D.M. (2012). "Extract knowledge from site-sampled data sets and fused hierarchical neural networks for detecting cardiovascular diseases", International Conference on Biomedical Engineering and Biotechnology.

[44] Shirvany Y., Hayati M., Moradian R. (2009). "Multilayer perceptron neural networks with novel unsupervised training method for numerical solution of the partial differential equations", Applied Soft Computing, 9, 20-29.

[45] Shulgin B., Stone L., Agur Z. (1998). "Pulse vaccination strategy in the SIR epidemic model", Bulletin of Mathematical Biology, 60, 1123-1148.

[46] Stone L., Shulgin B., Agur Z. (2000). "Theoretical examination of the pulse vaccination policy in the SIR epidemic model", Mathematical and Computer Modelling, 31(4-5), 207-215.

[47] Vrabie D., Lewis F.L. (2009). "Neural network approach to continuous-time direct adaptive optimal control for partially unknown nonlinear systems", Neural Network, 22, 237-246.

[48] Yu X., Wu L., Xu C., Hu Y., Ma C. (2019). "A novel neural network for solving nonsmooth non-convex optimization problems", IEEE Transactions on Neural Networks and Learning Systems, 31(5), 1475-1488.

[49] Zhang J., Ma Z. (2003). "Global stability of SEIR model with saturating contact rate", Mathematical Biosciences, 185(1), 15-32.