Control and Optimization in Applied Mathematics (COAM)
Vol. 1, No. 1, Spring-Summer 2016(21-30), ©2016 Payame Noor University, Iran

Regularity Conditions for Non-Differentiable Infinite Programming Problems using Michel-Penot Subdifferential

N. Kanzi*
Department of Mathematics, Payame Noor University,
P.O. Box. 19395-3697, Tehran, Iran

Received: May 23, 2013; Accepted: June 23, 2014.

Abstract. In this paper we study optimization problems with infinite many inequality constraints on a Banach space where the objective function and the binding constraints are locally Lipschitz. Necessary optimality conditions and regularity conditions are given. Our approach are based on the Michel-Penot subdifferential.

Keywords. Programming problem, Regularity conditions, Optimality condition, Michel-Penot subdifferential.
MSC. 90C34; 90C40.

* Corresponding author
nad.kanzi@gmail.com
http://mathco.journals.pnu.ac.ir

1 Introduction

Infinite Programming problem is an optimization problem on a feasible set described by infinitely many inequalities in a Banach space $\mathcal{B}$.

In the classical nonlinear programming, the necessary conditions of Fritz-John type can be viewed as being degenerate when the multiplier corresponding to the objective function vanishes, since then the function being minimized is not involved. Various supplementary conditions have been proposed under which it is possible to assert that the multiplier rule holds in the Karush-Kuhn-Tucker (KKT, briefly) form (i.e., the multiplier corresponding to objective function is equal to one). These conditions are called *regularity conditions*.

The theory of optimality conditions for infinite problems can be seen as a natural extension the classical KKT theory for ordinary nonlinear programming. Several papers studied infinite problems and gave the KKT necessary conditions (See e.g., [1, 6, 8] and their references). In these papers, two kinds of regularity conditions (RC in brief) are usually considered including " Farkas-Minkowski RC " and " closedness RC ", using basic/limiting subdifferential or convex ones.
This paper focuses mainly on some kinds of RCs for infinite problem which are based on Michel-Penot subdifferential, their interrelations, and their applications to KKT necessary optimality conditions.
The remainder of the present paper is organized as follows. In Section 2, basic notations and preliminary results are reviewed. In Section 3, we present our main results.

2 Notations and Preliminaries

Let $\mathcal{B}^*$ be the (continuous) dual space of $\mathcal{B}$ and let $\langle x^*, x\rangle$ denotes the value of the function $x^* \in \mathcal{B}^*$ at $x \in \mathcal{B}$. If $A^* \subseteq \mathcal{B}^*$, set $\langle A^*, x\rangle := \{\langle a^*, x\rangle \mid a^* \in A^*\}$. The symbols $\overline{A}$, $conv\,(A)$, and $cone(A)$ denote the closure, the convex hull, and the convex cone (containing zero) of $A \subseteq \mathcal{B}$. When we write $C \leq 0$ for some $C \subseteq \mathbb{R}$, means $c \leq 0$ for all $c \in C$.

Let $\hat{x} \in \mathcal{B}$ and let $\varphi : \mathcal{B} \to \mathbb{R}$ be a function. The Michel-Penot (M-P, briefly) directional derivative of φ at $\hat{x}$ in the direction $v \in \mathcal{B}$ introduced in [4] is given by

$$\varphi^{\diamondsuit}(\hat{x}; v) := \sup_{w \in \mathcal{B}} \limsup_{a \downarrow 0} \frac{\varphi(\hat{x} + av + aw) - \varphi(\hat{x} + aw)}{a}$$

and the M-P subdifferential of φ at $\hat{x}$ is given by the set

$$\partial^{\diamondsuit}\varphi(\hat{x}) := \left\{\xi \in \mathcal{B}^* \;\middle|\; \langle \xi, v\rangle \leq \varphi^{\diamondsuit}(\hat{x}; v) \;\; \forall\, v \in \mathcal{B}\right\}.$$

The M-P subdifferential is a natural generalization of the Gateaux derivative since it is known (See [4], Proposition 1.3]) that when function φ is Gateaux differentiable at $\hat{x}$,

$\partial^{\diamond}\varphi(\hat{x}) = \{D\varphi(\hat{x})\}$ and $\varphi^{\diamond}(\hat{x}; v) = \varphi'(\hat{x}, v)$, where $\varphi'(\hat{x}, v)$ denotes the usual directional derivative of φ at $\hat{x}$ in the direction v, i.e.,

$$\varphi'(\hat{x}, v) := \lim_{a \downarrow 0} \frac{\varphi(\hat{x} + av) - \varphi(\hat{x})}{a}$$

Moreover when a function φ is convex, the M-P subdifferential coincides with the subdifferential in the sense of convex analysis, denoted by ∂.

Given a locally Lipschitz function $\varphi : \mathcal{B} \to \mathbb{R}$ we say that φ is M-P regular at $\hat{x}$ if $\varphi'(\hat{x}; v)$ exists and

$$\varphi^{\diamond}(\hat{x}; v) = \varphi'(\hat{x}, v)$$

for all $v \in \mathcal{B}$. Observe that convex functions are examples for M-P regular functions.

In the following theorem we summarize some important properties of the M-P directional derivative and the M-P subdifferential from [4] and [5] which are widely used in what follows.

Theorem 1. Let φ and ψ be functions from $\mathcal{B}$ to $\mathbb{R}$ which are Lipschitz near $\hat{x}$. Then, the following assertions hold:

(i) One always has

$$\varphi^{\diamond}(\hat{x}; v) = \max\left\{\langle \xi, v\rangle \;\middle|\; \xi \in \partial^{\diamond}\varphi(\hat{x})\right\}, \tag{1}$$

$$\partial^{\diamond}(\varphi + \psi)(\hat{x}) \subseteq \partial^{\diamond}\varphi(\hat{x}) + \partial^{\diamond}\psi(\hat{x}). \tag{2}$$

(ii) The function $v \to \varphi^{\diamond}(\hat{x}; v)$ is finite, positively homogeneous and subadditive on $\mathcal{B}$ and

$$\partial\left(\varphi^{\diamond}(\hat{x}; .)\right)(0) = \partial^{\diamond}\varphi(\hat{x}). \tag{3}$$

(iii) $\partial^{\diamond}\varphi(\hat{x})$ is a nonempty, convex and weak star compact of $\mathcal{B}^*$.

3 Main Results

We consider the following optimization programming problem:

$$(P)\begin{cases} \quad minimize \quad f(x) \\ s.t. \quad f_t(x) \leq 0, \quad t \in T \end{cases}$$

where T is an arbitrary set and all emerging functions f and f_t for $t \in T$ are extended real-valued locally Lipschitz from the Banach space $\mathcal{B}$.

As a starting, we denote the feasible set of problem (P) with

$$\Omega := \{x \in \mathcal{B} \mid f_t(x) \leq 0 \qquad t \in T\}.$$

The contingent cone, the attainable directions cone and the feasible directions cone of Ω at $\hat{x} \in \Omega$ are respectively defined as:

$$K_{\Omega}(\hat{x}) := \left\{ z \in \mathcal{B} \;\middle|\; \exists a_k \downarrow 0,\ \exists z_k \overset{in\ \mathcal{B}}{\longrightarrow} z,\ \ such\ that\ \ \hat{x} + a_k z_k \in \Omega \ \ \forall\ k \in \mathbb{N} \right\},$$
$$A_{\Omega}(\hat{x}) := \left\{ z \in \mathcal{B} \;\middle|\; \forall a_k \downarrow 0,\ \exists z_k \overset{in\ \mathcal{B}}{\longrightarrow} z,\ \ such\ that\ \ \hat{x} + a_k z_k \in \Omega \ \ \forall\ k \in \mathbb{N} \right\},$$
$$D_{\Omega}(\hat{x}) := \{ z \in \mathcal{B} \mid \exists \varepsilon > 0\ \ such\ that\ \ \hat{x} + az \in \Omega \ \forall\ a \in (0, \varepsilon) \}.$$

For a given $\hat{x} \in \Omega$, let $T(\ \hat{x})$ denotes the index set of all active constraints at $\hat{x}$, i.e.,

$$T(\hat{x}) := \{ t \in T \mid f_t(\hat{x}) = 0 \}.$$

Based on the above notations and the Michel-Penot sub-differential, we extend the following RCs to non-differentiable infinite problem (P).

Definition 1. Let $\hat{x} \in \Omega$. We say that

1. The First regularity condition (RC_1, shortly) holds at $\hat{x}$ if

$$\left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}), v \right\rangle \leq 0 \right\} \subseteq K_{\Omega}(\hat{x}).$$

2. The Second regularity condition (RC_2, shortly) holds at $\hat{x}$ if

$$\left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}), v \right\rangle \leq 0 \right\} \subseteq \overline{D_{\Omega}(\hat{x})}.$$

3. The Threee regularity condition (RC_3, shortly) holds at $\hat{x}$ if

$$\left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}), v \right\rangle \leq 0 \right\} \subseteq A_{\Omega}(\hat{x}).$$

4. The Fourth regularity condition (RC_4, shortly) holds at $\hat{x}$ if

$$\left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}), v \right\rangle \leq 0 \right\} \subseteq \overline{conv}\ \left(K_{\Omega}(\hat{x})\right).$$

It is easy to see that $\overline{D_{\Omega}(\hat{x})} \subseteq A_{\Omega}(\hat{x}) \subseteq K_{\Omega}(\hat{x}) \subseteq \overline{conv}\ (K_{\Omega}(\hat{x}))$ and hence the following relationships among the above constraint qualifications are obvious:

$$RC_2 \Rightarrow RC_3 \Rightarrow RC_1 \Rightarrow RC_4. \tag{4}$$

Remark 1. In particular case where T is a finite set and $\mathcal{B}$ has finite dimension, the RC_1, RC_2, RC_3 and RC_4 are similar to the Abadie, Zangwill, Kuhn-Tucker, and Guignard RCs, respectively, which defined for non-differentiable problems (See e.g., [3]). It is worth mentioning that if f_ts are convex functions, these RCs are equivalent.

The following theorem shows that in M-P regular systems the converse inclusion of RC_1 is always true.

Theorem 2. If for each $t \in T(\hat{x})$, f_t is a regular function at $\hat{x} \in \Omega$, then

$$K_\Omega(\hat{x}) \subseteq \left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^\diamond f_t(\hat{x}), v \right\rangle \leq 0 \right\}. \tag{5}$$

Proof. Suppose that z is an arbitrary element of $K_\Omega(\hat{x})$. Then, there exist $a_k \to 0^+$ and $z_k \to z$ such that $\hat{x} + a_k z_k \in \Omega$ for all $k \in \mathbb{N}$. If ℓ_t denotes the Lipschitzian constant of f_t near $\hat{x}$, then

$$|f_t(\hat{x} + a_k z_k) - f_t(\hat{x} + a_k z)| \leq \ell_t a_k \parallel z - z_k \parallel \;\; (when\ a_k \to 0).$$

The above relation leads us to

$$\lim_{a_k \to 0} \left| \frac{f_t(\hat{x} + a_k z_k) - f_t(\hat{x} + a_k z)}{a_k} \right| = 0 \qquad \forall t \in T. \tag{6}$$

Let $\hat{t} \in T(\hat{x})$. With regard to $f_{\hat{t}}(\hat{x}) = 0$, feasibility of $\hat{x} + a_k z_k$, regularity of f_ts at $\hat{x}$, and virtue of (6) we conclude

$$\begin{aligned}
f_{\hat{t}}^\diamond(\hat{x};;z) &= \lim_{a_k \to 0} \frac{f_{\hat{t}}(\hat{x} + a_k z) - f_{\hat{t}}(\hat{x})}{a_k} \\
&= \lim_{a_k \to 0} \frac{f_{\hat{t}}(\hat{x} + a_k z) - f_{\hat{t}}(\hat{x} + a_k z_k)}{a_k} \\
&\quad + \lim_{a_k \to 0} \frac{f_{\hat{t}}(\hat{x} + a_k z_k) - f_{\hat{t}}(\hat{x})}{a_k} \\
&= \lim_{a_k \to 0} \frac{f_{\hat{t}}(\hat{x} + a_k z_k) - f_{\hat{t}}(\hat{x})}{a_k} \\
&= \lim_{a_k \to 0} \frac{f_{\hat{t}}(\hat{x} + a_k z_k)}{a_k} \leq 0.
\end{aligned}$$

Since $\hat{t}$ is an arbitrary index in $T(\hat{x})$, the last inequality and (1) imply that

$$\left\langle \bigcup_{t \in T(\hat{x})} \partial^\diamond f_t(\hat{x}), z \right\rangle \leq 0,$$

and hence

$$z \in \left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^\diamond f_t(\hat{x}), v \right\rangle \leq 0 \right\}.$$

□

Remark 2. If $\mathcal{B} = \mathbb{R}^n$ then the convexity and closedness of

$$\left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}), v \right\rangle \leq 0 \right\}$$

implies that the inclusion (5) can be rewritten as

$$\overline{conv}\;(K_{\Omega}(\hat{x})) \subseteq \left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}), v \right\rangle \leq 0 \right\}.$$

Therefore, if $\mathcal{B}$ has finite dimension and f_ts for $t \in T(\hat{x})$ are regular at $\hat{x}$ then the following implications are fulfilled at $\hat{x}$:

$$RC_4 \Longleftrightarrow \overline{conv}\;(K_{\Omega}(\hat{x})) = \left\{ v \in \mathcal{B} \;\middle|\; \left\langle \bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}), v \right\rangle \leq 0 \right\}.$$

Before proving the next theorems, we give a lemma, which shall be useful.

Lemma 1. Let $\hat{x}$ be an optimal solution of problem(P), and the mapping $v \to f^{\diamondsuit}(\hat{x}, v)$ be linear. Then,

$$\left\{ v \in \mathcal{B} \;\middle|\; f^{\diamondsuit}(\hat{x}; v) < 0 \right\} \cap \overline{conv}\;(K_{\Omega}(\hat{x})) = \emptyset.$$

Proof. The proof falls naturally into two parts:
Part one. We first establish that

$$\left\{ v \in \mathcal{B} \;\middle|\; f^{\diamondsuit}(\hat{x}; v) < 0 \right\} \cap K_{\Omega}(\hat{x}) = \emptyset. \tag{7}$$

On the contrary, suppose that there exists $\hat{v} \in K_{\Omega}(\hat{x})$ such that $f^{\diamondsuit}(\hat{x}; \hat{v}) < 0$. Then

$$\limsup_{a \downarrow 0} \frac{f(\hat{x} + a\hat{v}) - f(\hat{x})}{a} \leq f^{\diamondsuit}(\hat{x}; \hat{v}) < 0,$$

which implies that there exist $\delta > 0$ and $\varepsilon > 0$ such that

$$f(\hat{x} + a\hat{v}) - f(\hat{x}) \leq -\delta a \quad \forall a \in (0,\ \varepsilon). \tag{8}$$

Since f is Lipschitzian near $\hat{x}$, there exists $r > 0$ such that

$$f(\hat{x} + a\hat{u}) - f(\hat{x} + a\hat{v}) \leq \ell_f a \parallel \hat{u} - \hat{v} \parallel, \quad \forall\ \hat{u} \in \mathbb{B}(\hat{x}, r), \tag{9}$$

where ℓ_f is the Lipschitz constant of f near $\hat{x}$ and $\mathbb{B}(\hat{x}, r)$ denotes the open ball with center $\hat{x}$. By the definition of $K_{\Omega}(\hat{x})$, there exist sequences $a_k \to 0^+$ in $\mathbb{R}$ and $\hat{v}_k \to \hat{v}$ in Ω such that $\hat{x} + a_k\hat{v}_k \in \Omega$.

Therefore for k large enough, one has

$$\parallel \hat{v}_k - \hat{v} \parallel \leq \frac{\delta}{2\ell_f} \qquad and \qquad a_k \in (0,\ \varepsilon).$$

Now from (8), (9) and above inequality we can deduce that

$$\begin{aligned} f(\hat{x}+a\hat{v}_k)-f(\hat{x}) &= f(\hat{x}+a\hat{v}_k)-f(\hat{x}+a\hat{v}) \\ &+ f(\hat{x}+a\hat{v})-f(\hat{x}) \leq \\ &\ell_f a_k \parallel \hat{v}_k-\hat{v} \parallel -\delta a_k < -\frac{\delta}{2}a_k < 0. \end{aligned}$$

But this contradicts the fact that $\hat{x}$ is an optimal solution of (P) and hence (7) holds.

Part two. Let $v^* \in conv\ (K_\Omega(\hat{x}))$. Then, there exist non-negative scalars $\beta_1,\ \beta_2,\ldots,\ \beta_m \in \mathbb{R}$ and vectors $\hat{v}_1,\ \hat{v}_2,\ldots,\ \hat{v}_m \in K_\Omega(\hat{x})$, such that

$$\sum_{l=1}^{m}\beta_l = 1, \qquad v^* = \sum_{l=1}^{m}\beta_l\ \hat{v}_l.$$

Using the linearity of $f^\diamondsuit(\hat{x};.)$ and part one, we get

$$f^\diamondsuit(\hat{x};v^*) = f^\diamondsuit\left(\hat{x};\sum_{l=1}^{m}\beta_l\ \hat{v}_l\right) = \sum_{l=1}^{m}\beta_l f^\diamondsuit(\hat{x};\hat{v}_l) \geq 0.$$

Taking into consideration the continuity of $f^\diamondsuit(\hat{x},.)$ and above inequality, it follows that

$$f^\diamondsuit(\hat{x};\tilde{v}) \geq 0 \qquad for\ all \quad \tilde{v} \in \overline{conv}\ \ (K_\Omega(\hat{x}))\,.$$

The proof is complete. □

Theorem 3. Suppose that $\hat{x}$ is an optimal solution of problem (P) and the RC_4 is satisfied at $\hat{x}$. If the mapping $v \to f^\diamondsuit(\hat{x};v)$ is linear, then the following inclusion holds:

$$0 \in \partial^\diamondsuit f(\hat{x}) + \overline{conv}\left(\bigcup_{t\in T(\hat{x})}\partial^\diamondsuit f_t(\hat{x})\right).$$

Proof. Let $\hat{v}$ is an element of $\mathcal{B}$ satisfying

$$\left\langle \bigcup_{t\in T(\hat{x})}\partial^\diamondsuit f_t(\hat{x}),\hat{v}\right\rangle \leq 0.$$

Owning to RC_4 and Lemma 1 we conclude

$$f^\diamondsuit(\hat{x};\hat{v}) \geqslant 0.$$

Thus, we can obtain

$$f^\diamondsuit(\hat{x},\ \hat{v}) \geq 0 \qquad \forall\,\hat{v} \in \left\{v\in\mathcal{B}\ \middle|\ \left\langle \overline{conv}\left(\bigcup_{t\in T(\hat{x})}\partial^\diamondsuit f_t(\hat{x})\right),v\right\rangle \leq 0\right\},$$

in view of

$$\begin{aligned} &\left\{v\in\mathcal{B}\ \middle|\ \left\langle \overline{conv}\left(\bigcup_{t\in T(\hat{x})}\partial^\diamondsuit f_t(\hat{x})\right),v\right\rangle \leq 0\right\} \\ &= \left\{v\in\mathcal{B}\ \middle|\ \left\langle \bigcup_{t\in T(\hat{x})}\partial^\diamondsuit f_t(\hat{x}),v\right\rangle \leq 0\right\}. \end{aligned}$$

The above result implies that $v^* = 0$ is a global minimizer of the convex function

$$v \to H(v) := f^{\diamondsuit}(\hat{x};;v) + \Theta(v),$$

where $\Theta(.)$ denotes the indicator function of set

$$\left\{ v \in \mathcal{B} \;\middle|\; \left\langle \overline{conv}\left(\bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}) \right), v \right\rangle \leq 0 \right\};$$

i.e.
$\Theta(v) = 0$ if $\left\langle \overline{conv}\left(\bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}) \right), v \right\rangle \leq 0$ and $\Theta(v) = +\infty$ otherwise.

Now, by necessary condition for convex optimization problems (See e.g. [2]) and by the sum rule formula (2) (which equality holds there for convex functions), one has

$$0 \in \partial \left(f^{\diamondsuit}(\hat{x};.) \right)(0) + \partial\Theta(0),$$

where $\partial\varphi$ denotes the subdifferential of convex function φ in the sense of convex analysis. Finally, the virtue of (3) and the fact that $\Theta(0) = \overline{conv}\left(\bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}) \right)$, one can conclude that

$$0 \in \partial^{\diamondsuit} f(\hat{x}) + \overline{conv}\left(\bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}) \right).$$

The proof is complete. □

Theorem 4. Suppose that $\hat{x}$ is an optimal solution of problem (P) and the RC_1 is satisfied at $\hat{x}$. Then the following inclusion holds:

$$0 \in \partial^{\diamondsuit} f(\hat{x}) + \overline{conv}\left(\bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}) \right).$$

Proof. Owning to the equality (7), the proof is similar to Theorem 3. □

Theorem 5 (KKT Necessary Condition)**.** Suppose that $\hat{x}$ is an optimal solution of problem (P) and one of the following conditions holds:

- RC_4 at $\hat{x}$ and linearity of $f^{\diamondsuit}(\hat{x};.)$.
- RC_1 at $\hat{x}$.
- RC_2 at $\hat{x}$.
- RC_3 at $\hat{x}$.

If $cone\left(\bigcup_{t \in T(\hat{x})} \partial^{\diamondsuit} f_t(\hat{x}) \right)$ is a closed cone, then, there exist $\lambda_t \geq 0,\ \ t \in T(\hat{x})$, where $\lambda_t \neq 0$ for finitely many $t \in T(\hat{x})$, such that

$$0 \in \partial^{\diamondsuit} f(\hat{x}) + \sum_{t \in T(\hat{x})} \lambda_t \partial^{\diamondsuit} f_t(\hat{x}).$$

Proof. Owning to the Theorems 3 and 4, virtue of (4) and the following fact for convex sets A_γ, $\gamma \in \Gamma$ (See e.g., [2]):

$$cone\left(\bigcup_{\gamma\in\Gamma} A_\gamma\right) = \left\{ \sum_{\gamma\in\Gamma_0} \tau_\gamma a_\gamma \middle| \Gamma_0 \ is\ a\ finite\ subset\ of\ \Gamma, a_\gamma \in A_\gamma, \quad a_\gamma \geq 0 \right\},$$

the result is immediate. □

Note that $cone\left(\bigcup_{t\in T(\hat{x})} \partial^\diamond f_t(\hat{x})\right)$ is assumed to be closed in Theorem 5. The following example shows that this assumption can not be waived, even when $\mathcal{B}$ has finite dimension and f_ts are convex.

Example 1. For all $t \in T := \mathbb{N}$, take

$$A_t := \left\{(a_1, a_2) \in \mathbb{R}^2 \mid a_1^2 + a_2^2 - 2ta_2 \leq 0\right\}.$$

Set $f(x_1, x_2) := -x_1$ and

$$f_t(x_1, x_2) := \sup_{(a_1,a_2)\in A_t} (a_1x_1 + a_2x_2).$$

It is easy to see that $\Omega = (-\infty, 0] \times (-\infty, 0]$ and $\hat{x} = (0, 0)$ are respectively the feasible solution set and the optimal solution of the following problem:

$$\inf\{f(x_1, x_2) \mid f_t(x_1, x_2) \leq 0, \quad t \in T\}.$$

We observe that $T(\hat{x}) = T$. Since f_t is support function of A_t, we obtain $\partial^\diamond f_t(\hat{x}) = A_t$, and hence

$$cone\Big(\bigcup_{t\in T(\hat{x})} \partial^\diamond f_t(\hat{x})\Big) = ((0, + \in fty) \times (0, + \in fty)) \cup \{(0, 0)\},$$

$$\{v \in \mathcal{B} \mid \langle \bigcup_{t\in T(\hat{x})} \partial^\diamond f_t(\hat{x}), v\rangle \leq 0\} = \Omega.$$

Owning to $K_\Omega(\hat{x}) = \Omega$ and convexity of Ω we conclude that the RC_i holds at $\hat{x}$ for $i = 1, 2, 3, 4$. Note that $cone\left(\bigcup_{t\in T(\hat{x})} \partial^\diamond f_t(\hat{x})\right)$ is not closed. It is easy to see that there is no sequence of scalars satisfying Theorem 5. Moreover, it can show that

$$0 \in \partial^\diamond f(\hat{x}) + \overline{conv}\left(\bigcup_{t\in T(\hat{x})} \partial^\diamond f_t(\hat{x})\right).$$

References

[1] Borwein J. M., Zhu Q. J. (1999). “ A Survey of subdifferential calculus with applications ”, Nonlinear Analysis, 38, 687-773.

[2] Hiriart-Urruty J. B., Lemarechal C. (1991). " Convex analysis and minimization algorithms, I & II ", Springer, Berlin, Heidelberg.

[3] Giorgi J., Gwirraggio A., Thierselder J. (2004). " Mathematics of optimization; smooth and non-smooth cases ", Elsivier.

[4] Michel P., Penot J. P. (1984). " Calculsous differentiel pour des fonctions lipschitziennes et non-lipschitziennes ", Academic Sciences Paris (I); Mathematics, 12, 269-272.

[5] Michel P., Penot J. P. (1992). " A generalized derivative for calm and stable functions ", Differential and Integral Equations, 5, 433- 454.

[6] Mordukhovich B. S., Nghia T. T. A. (2013). " Constraint qualification and optimality conditions in semi-infinite and infinite programming ", Mathematical Programming, 139, 271-300.

[7] Mordukhovich B. S., Nghia T. T. A. (2012). " Nonsmooth cone-constrained optimization with applications to semi-infinite programming ", Optimization, online, 3/3396.

[8] Mordukhovich B. S., Nghia T. T. A. (2011). " Sub-differentials of non-convex supremum functions and their applications to semi-infinite and infinite programs with Lipschitzian data ", Optimization, online, 12/3261.

شرایط نظم‌پذیری برای مسائل برنامه‌ریزی نامتناهی غیر مشتق پذیرتوسط زیرمشتق میشل-پینت

کنزی ن.
استادیار ریاضی کاربردی
ایران، تهران، گروه ریاضی، دانشگاه پیام نور، صندوق پستی ۳۶۹۷-۱۹۳۹۵.
nad.kanzi@gmail.com

چکیده
در این مقاله، مسائل بهینه‌سازی شامل تعداد نامتناهی قید نامساوی در یک فضای باناخ مورد بررسی و مطالعه قرار گرفته است. این دسته مسائل به گونه‌ای است که تابع هدف و تمامی توابع فید در نزدیکی نقطه بهینه به طور موضعی لیپ شیتز هستند. هدف، ارائه شرایط لازم بهینگی و بررسی شرایط نظم‌پذیری برای مسائل فوق، توسط زیرمشتق میشل-پینت است.

کلمات کلیدی
مسئله برنامه‌ریزی نامتناهی، شرط نظم‌پذیری، شرایط بهینگی، زیرمشتق میشل – پینت.