Received: xxx **Accepted:** xxx.
DOI. xxxxxxx
xxx Vol. xxx, No. xxx, (1-23)
Research Article

Open Access

Control and Optimization in Applied Mathematics - COAM

A Multi-Objective Model for Humanitarian Logistics Model During an Earthquake Crisis: A Case Study of Iran

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How to Cite
Yaghoubi, M., Dadmand, F. (2024). "A multi-objective model for humanitarian logistics model during an earthquake crisis: A case study of Iran", Control and Optimization in Applied Mathematics, 9(): 1-23.

Abstract. Natural disasters, such as earthquakes, result in significant financial and human losses. Rescue operations play a crucial role in managing such crises. However, the lack of precise information and the damage or destruction of urban transportation routes following earthquakes introduces uncertainty into these operations. This study presents a multi-objective humanitarian logistics model that utilizes a mixed-integer nonlinear programming (MINLP) approach. The model considers the reliability of transportation routes after an earthquake, the standard response time for allocating personnel and relief equipment, and the coverage maximization. This model incorporates various uncertainties, including the reliability of the transportation network. Real data from the city of Gonabad, Iran, was used to evaluate the proposed model. The results and sensitivity analysis demonstrated that the model exhibits desirable performance.

Keywords. Disaster management, Humanitarian logistics, Allocation, Scenario planning.

MSC. 90B06.

https://mathco.journals.pnu.ac.ir

1 Introduction

Natural disasters result in substantial economic and societal losses, stemming from various factors such as infrastructure damage, loss of life and livelihoods, displacement of people, and disruption of economic activity [27]. The economic losses can be significant, encompassing damage to critical infrastructure like buildings and transportation systems, as well as loss of crops, livestock, and other assets [28]. Societal losses can manifest as psychological trauma, social disruption, and the erosion of cultural heritage [13]. Furthermore, natural disasters can also have enduring effects, including increased poverty and vulnerability, reduced economic growth, and an elevated risk of future disasters [10]. Effective disaster preparedness, risk reduction, and response measures are crucial in minimizing these losses and fostering resilience in affected communities [13, 10]. Therefore, managing crises such as earthquakes requires attention to relief-chain management and humanitarian logistics [19]. Given the limited warning time for disasters like earthquakes [26], effective response activities are vital for minimizing human casualties. The timely transportation of relief goods, such as food, tents, blankets, and hygiene kits, is particularly critical in such situations and relies on the availability of transportation resources, rapid mobilization of resources, and efficient beneficiary visit planning [4].

Disaster management requires a systematic approach to address both natural and man-made disasters, comprising four main stages: mitigation, preparation, response, and recovery. During the mitigation stage, long-term endeavors aim to prevent disasters and minimize their impact. In the preparation stage, long-term strategic decisions and procedures are developed in advance, including determining the number and location of primary distribution centers (DCs), in anticipation of a disaster. The response stage involves operational decisions concerning transportation routes, personnel, equipment, and the distribution of relief goods to affected areas following a disaster. The recovery stage entails restoring affected areas to their pre-disaster state, with the government and non-governmental organizations playing a central role in disaster management [1]. Additionally, new trends like infrastructure design and management engineering issues should be taken into account for disaster planning and operations [3]. Humanitarian logistics, particularly logistics planning, has garnered significant attention in recent years as the core of every relief operation. The timely distribution of adequate supplies is crucial for minimizing human casualties after a disaster. However, after an earthquake, the destruction of certain parts of the transportation network presents significant challenges in delivering relief supplies to affected areas. Hence, this issue should be considered during the preparedness stage when developing a practical approach to disaster relief operations [1]. Furthermore, understanding the vulnerability of different areas can provide essential data regarding the extent of transportation network damage before implementing relief operations.

In the realm of humanitarian logistics, it is crucial to receive the first response within the initial 72 hours following an earthquake [16]. The first 12 hours, known as the Standard Relief

Time (SRT), are particularly critical. During this period, governments and non-governmental organizations must swiftly assess the situation and initiate the dispatch of relief supplies from local warehouses to areas of demand. An effective humanitarian logistics system should aim to minimize human casualties by delivering essential relief goods such as food, water, medical equipment, etc., within the SRT. Standards outlined by [24] require the consideration of a specific amount of relief goods per person. Any failure to adhere to the SRT limit or the inability to deliver relief goods can cause significant suffering and harm to those affected. It is imperative that those responsible be held accountable during the planning and decision-making process.

Iran is located in a highly earthquake-prone region, positioned along several major fault lines [9]. Consequently, Iran experiences a relatively high frequency of earthquakes compared to other parts of the world. Notably, Iran has encountered 524 earthquakes with magnitudes below 3,65 earthquakes ranging from 3 to 4, 13 earthquakes between 4 and 5, and 2 earthquakes between 5 and 6, making it one of the most seismically active countries globally [9]. Among the provinces, West Azarbaijan recorded the highest number of earthquakes with 89, followed by South Khorasan, Razavi Khorasan, and Semnan with 84, 79, and 78 earthquakes, respectively. Table 1 provides a comprehensive list of the most severe earthquakes that have occurred in Iran over the past 50 years, including the number of casualties and their magnitude on the Richter scale.

The significant number of casualties resulting from these earthquakes underscores the importance of effective crisis management in earthquake-prone countries like Iran. Consequently, questions arise regarding the minimization of equipment transportation costs and delivery time in the event of earthquakes of varying magnitudes. Additionally, consideration must be given to maximizing relief coverage in affected areas, minimizing the number of relief bases, and general support required for relief operations. Furthermore, there is a need to prioritize social justice alongside cost optimization and coverage. To address these inquiries, this study focuses on designing and solving a logistics model for earthquake relief operations in Iran, placing particular emphasis on the efficient and effective allocation of resources, while considering the challenges and constraints associated with such operations. This model can aid in ensuring the timely and effective delivery of relief supplies and resources to affected areas, thereby, reducing human casualties and mitigating the impact of earthquakes in Iran.

This paper is organized as follows: Section 2 provides a comprehensive review of the literature concerning the multi-objective transportation problem. Section 3 delineates the multi-objective transportation problem, including its associated parameters. In Section 4, we discuss the application of this model. In Section 5, we present the results of the model. Finally, in Section 6, we provide conclusions and recommendations along with for future research.

Table 1: The most severe earthquakes in Iran in the past 50 years.

Date	Location	Casualtiesc people	Magnitude on the Richter scale
Apr 10, 1972	Ghir Karzin	5,374	7.1
Mar 22, 1977	Bandar Abbas	167	7
Apr 6/7, 1977	Isfahan	352	6.5
Dec 21, 1977	Zarand	521	6.2
Sept 16, 1978	Tabas	15,000	between 7.5 and 7.9
Jan 16, 1979	Khorasan	199	7
Nov 14, 1979	Qaen and Khaf	385	5.6
Jun 11, 1981	Golbaf	1,027	6.8
Jun 21, 1990	Gilan and Zanjan	35,000	7.7
Jun 22, 2002	Qazvin	229	6.3
Dec 26, 2003	Bam	2000	6.3

2 Review of the Related Literature

In recent years, the utilization of multi-objective optimization models has gained significant popularity in the field of disaster relief logistics and resource allocation. These models serve as valuable tools to address the complex decision-making process associated with the distribution of resources to areas affected by disaster. For instance, in [6], a fuzzy multi-objective model was developed that considers various criteria, including satisfaction degree, transportation cost, and delivery time.

In [25], a mixed-integer nonlinear open location-routing model was proposed to address multiple conflicting objectives and factors, in the allocation of relief following an earthquake. This model considers factors such as travel time, total costs, and reliability. The study suggests the simultaneous utilization of terrestrial and aerial transportation modes to enhance route reliability and reduce travel time. The effectiveness of the proposed model and solution approach was evaluated through a case study focused on the earthquake in East Azerbaijan, Iran.

A bi-objective robust optimization model was introduced for strategic and operational response aimed at determining facility location, emergency resource allocation, and casualty transportation plans within a three-level rescue chain. The model incorporates the Injury Severity Score (ISS) to categorize casualties and accounts for the dynamic deterioration of injuries over time. The ε-constraint method is employed to solve the model, and case studies based on the Yushu earthquake are utilized to demonstrate its feasibility and validity. Sensitivity analyses are conducted to assess the impact of uncertainties on the model results.

Furthermore, other studies have proposed multi-objective mathematical models to address design problems in the humanitarian supply chain. For example, in [5] a multi-objective mathematical model was presented that aims to minimize the total number of injured individuals not transferred to hospitals, the total number of homeless individuals not evacuated from the affected area, and the total unmet relief commodity needs. The model considers uncertain demand and travel times, and a robust counterpart model utilizing "box and polyhedral" uncertainty sets is developed to handle these uncertainties. The model is solved for both deterministic and robust scenarios to generate Pareto optimal solutions, and the results indicate that the robust model performs worse than the deterministic model.

In [14], a multi-period multi-objective model with multi-sourcing was proposed to determine the location of temporary logistics hubs. The model incorporates a fuzzy factor rating system (FFRS) under group decision-making (GDM) conditions to assign weights to the objectives when multiple decision-makers are involved. By considering the trade-off between two non-commensurable objectives, the optimization results offer valuable managerial insights for decision-makers.

In [17] a multi-objective mathematical model was proposed under uncertainty conditions to find the optimal facility location and allocation of goods between the facility and the allocation of injured to hospitals. The model also aims to optimize the routing of human resources to disaster-stricken areas to achieve goals such as reducing costs, distributing goods and fair medical assistance between areas, and minimizing the time required for aid troops to arrive. The model employs robust optimization and single-purpose methods, and its accuracy and effectiveness are investigated through a case study.

In [29], a mathematical optimization model is presented that minimizes the overall expected costs of constructing a multi-period emergency relief system. This model takes into account allocation, location, and distribution decisions as well as transportation of injured individuals and medical supplies. Moreover, the model considers the vulnerability of medical supply distribution centers and roads.

In [21], a deterministic multi-objective mathematical programming formulation is developed to represent the design of a disaster relief distribution network. The primary objective of the model is to minimize the total unmet demand across all demand nodes, while the secondary objective is to minimize the total transportation time. This model is solved using the lexicographic method, and it is applied to a problem instance involving an earthquake in the Cascadia Subduction Zone (CSZ) in Oregon.

Lastly, [8] presented a network design model for humanitarian logistics that assists in making location and allocation decisions across multiple disaster periods. The model is formulated as a single-objective optimization problem, focusing on the response phase of disaster management and incorporating emergency tents as temporary medical centers. The multi-period robust model considers critical input data values under various scenarios and utilizes Monte Carlo simulation to generate related random numbers and different scenarios. A case study of a potential earthquake in Region 1 of Tehran is conducted to evaluate the proposed multi-period robust optimization approach, and sensitivity analyses are performed to explore the effects of various problem parameters.

Overall, the findings of these studies indicate that multi-objective optimization models are effective in addressing the complex decision-making process involved in disaster relief logistics and resource allocation. These models consider various objectives, uncertainties, and risks and incorporate different optimization techniques to enhance the efficiency and effectiveness of disaster relief operations.

Within this context, the present research has developed a multi-objective optimization model for humanitarian logistics, specifically focusing on the equitable distribution of supply and demand. The study aims to minimize overall logistics costs while maximizing the coverage of affected areas, minimizing the time taken to deliver services to those areas, and minimizing the utilization of relief bases. The authors incorporated a vulnerability assessment into the model to identify the most susceptible regions and allocate relief resources accordingly, while also considering social justice aspects.

The inclusion of a vulnerability assessment in models for disaster relief procurement and allocation models ensures a fair and efficient distribution of resources among affected regions and populations. By allocating resources based on vulnerability, relief efforts can be concentrated on regions and populations with the greatest needs, thereby enhancing the effectiveness of relief operations. This approach facilitates the timely and effective delivery of relief supplies and resources to affected areas, reducing human casualties and mitigating the impact of disasters.

Based on the studies and research in the literature, it is evident that the presented models are rarely multi-objective. In most studies, the emphasis has been primarily on cost minimization. Factors such as the reliability of access routes to the target area, the vulnerability level of the affected areas, and the maximization of the satisfaction of the injured individuals have received less attention. To address these gaps highlighted in the research background, a multi-objective location allocation planning for relief is proposed. This paper puts forth a multi-objective model that encompasses the following objectives:

i. Minimization of transportation and equipment costs.

ii. Maximization of coverage in affected areas subsequent to ensuring the delivery of relief goods to the people in those areas.

iii. Minimization of the duration of relief operations through the optimization of resource deployment of resources from aid bases to affected areas.

iv. Finally, minimization of the number of relief bases and the overall support required for relief operations.

The aforementioned objectives are examined in different scenarios, taking into account the reliability of the routes from the relief bases to demand nodes and the vulnerability levels of affected areas. In addition to optimizing costs and coverage, this model also places importance on social justice considerations.

3 Multi-Objective Optimization Problems and the Weighted Metric Method (Lp Method)

The optimization process allows us to find the optimal value or the best solution. Optimization problems can involve searching for maximum or minimum values, and they can have one objective or multi-objectives. This method is frequently used, as seen in e.g., [11] and [20]. Multi-objective optimization (MOO) refers to finding the optimal solution values of multiple desired goals. The MOO is motivated by the fact that it simplifies the problem by not requiring complex equations. Decision-making in MOOs allows for compromises (trade-offs) on some contradictory issues. In MOO, there is a vector of the objective functions, and each vector is a function of the solution vector. There is no single best solution for all purposes in MOO; instead, there are multiple solutions.

Mathematically, the equations of the MOO problem can be written as follows [7]:

$$\text{min / max} \quad \big(f_1(x), f_2(x), \ldots, f_p(x)\big) \tag{1}$$

$$\text{s.t}: \quad x \in U, \tag{2}$$

where x represents the solution, p is the number of objective functions, U is the feasible set, $f_j(x)$ is the jth objective function and min / max represents combined object operations.

Next, we will explain the weighted metric method as defined by [18]. The weighted metric method is an optimization technique used to solve MOO problems by combining all objectives into a single function. In this method, each objective function is assigned a weight $[w]$ based on its importance to the overall objective. A global minimal value result may not be the ideal solution for all the individual objective functions, so multiple iterations of weight substitution may be necessary to find the most optimal solution for a given application.

The set of solutions generated by this process is known as Pareto Optimal Solution (POS). POS represents the solutions where one objective function cannot be improved without sacrificing the performance of other objective functions. In other words, according to the definition of Pareto optimality, moving from one Pareto optimal solution to another requires a trade-off. Thus, improving one criterion comes at the expense of at least one other criterion.

Setting equal weights for the individual objective functions is a good starting point. The final assigned weights will depend on the decision maker's perspective and the deviation of the ideal solution. The decision maker needs to determine these weights based on the results observed at each weight assignment's iteration.

In the weighted metric method, the objective functions are combined into a single objective function, denoted as $L_p(x)$:

$$\min \quad L_p(x) = \left(\sum_{j=1}^{k} (w_j \left| F_j(x) - F_j^*(x) \right|)^p \right)^{\frac{1}{p}} \tag{3}$$

$$\text{s.t}: \quad x \in U, \tag{4}$$

where w_j represents a non-negative weight assigned to each objective function by DM, and p indicates the importance of the deviation of each objective function deviation from its ideal value. When $p=1$, the resulting problem simplifies to a weighted sum of the deviations.

When $p=2$, the problem aims to minimize the weighted Euclidean distance between any point in the objective space and the ideal point. On the other hand, when $p=\infty$, the problem minimizes the largest deviation $w_j \left| F_j(x) - F_j^*(x) \right|$ defined as:

$$\min \quad L_\infty(x) = \max_{j=1}^{k} (w_j \left| F_j(x) - F_j^*(x) \right|) \tag{5}$$

$$\text{s.t}: \quad x \in U. \tag{6}$$

In Equation (7), it is assumed that objective functions have the same scale. If the objective functions, $F_j(x)$ do not have the same scale, they can be made scale-less using either of the following equations:

$$\min \quad L_p(x) = \left(\sum_{j=1}^{k} (w_j \left| \frac{F_j(x) - F_j^*(x)}{F_j^*(x)} \right|^p) \right)^{\frac{1}{p}} \tag{7}$$

$$\text{s.t}: \quad x \in U. \tag{8}$$

4 Problem Statement

Let us consider a region that is prone to earthquakes. Although the uncertainty surrounds the timing and intensity of earthquakes, careful planning can significantly reduce casualties and hardships. The primary objective of this paper is to present a crisis management model that facilitates swift and optimal relief operations. These operations involve the allocation of relief bases and multiple support facilities taking into account the vulnerability of demand nodes and the reliability of routes from bases to demand nodes. Given the similarities between the present problem and supply chain planning and location problems in affected areas, we employ a mathematical modeling approach that combines a multi-level supply chain and location to formulate the problem [12]. Consequently, the problem assumptions consider three levels: the first level comprises affected areas, the second level consists of critical bases located near the affected regions, and the third level includes support bases in neighboring cities to the affected areas. When an incident occurs, the dispatch of equipment and relief forces to specific areas are determined based on the severity of the incident and expert opinions in the field. Relief operations progress from critical bases and then support bases to the affected areas, taking into account the time and capacity of each base. Given the problem definitions, we utilize a mixed-integer non-linear model. The developed model has three primary objectives: minimizing the impact of delays on crisis severity, minimizing the costs of relief operations, including personnel and equipment, and ensuring social justice based on distance considerations.

4.1 The Main Mathematical Model

The mathematical model presented in this study offers a comprehensive logistics framework for disaster relief operations. This model takes into account various aspects such as time management, reliability, and multiple objectives. It incorporates decision variables, parameters, and constraints to optimize the allocation of equipment and resources, thereby ensuring efficient and effective relief operations. Detailed descriptions of parameters and variables used in this model have been provided in Table 2.

4.2 Definition of the Mathematical Model

The logistics model in disaster relief is defined as follows:

$$
\begin{aligned}
\min \quad F_1 = & \sum_{i\in I}\sum_{j\in J}\sum_{s\in S} Q^s_{ij} T^s_{ij} r^s_{ij} v_j \left(CM^s_j + CT^s_{ij}\right) \\
& + \sum_{i\in I}\sum_{j\in J}\sum_{s\in S} Q^s_{ij} T^s_{ij} q^s (1 - r^s_{ij}) v_j
\end{aligned} \tag{9}
$$

$$
\max \quad F_2 = \sum_{i\in I}\sum_{j\in J}\sum_{s\in S} Q^s_{ij} T^s_{ij} r^s_{ij} v_j \tag{10}
$$

$$
\min \quad F_3 = max \sum_{i\in I}\sum_{j\in J} T^s_{ij} d_{ij} r^s_{ij} \tag{11}
$$

$$
\min \quad F_4 = \sum_{i\in I}\sum_{j\in J}\sum_{s\in S} T^s_{ij} r^s_{ij} \left(TU^s_i + TR^s_{ij} + TD^s_j\right) \tag{12}
$$

$$
\min \quad F_5 = \sum_{i\in I}\sum_{s\in S} w^s_i \tag{13}
$$

s.t:

$$\sum_{i} Q_{ij}^{s} T_{ij}^{s} = dg_{j}^{s} \qquad \forall j \in J, s \in S \tag{14}$$

$$\sum_{j} Q_{ij}^{s} T_{ij}^{s} \leq cap_{i}^{s} \qquad \forall i \in I, s \in S \tag{15}$$

$$\sum_{j \in J} a_{ij}^{s} w_{i}^{s} \geq 1 \qquad \forall i \in I, s \in S \tag{16}$$

$$T_{ij}^{s} \leq a_{ij}^{s} \qquad \forall i \in I, j \in J, s \in S \tag{17}$$

$$T_{ij}^{s} \leq w_{i}^{s} \qquad \forall i \in I, j \in J, s \in S \tag{18}$$

$$w_{i}^{s} \leq \sum_{j \in J} T_{ij}^{s} \qquad \forall i \in I, s \in S \tag{19}$$

$$\sum_{i \in I} T_{ij}^{s} \left(TU_{i}^{s} + TR_{ij}^{s} + TD_{j}^{s}\right) \leq st_{j} \qquad \forall j \in J, s \in S \tag{20}$$

$$Q_{ij}^{s} \geq 0 \;, T_{ij}^{s}, w_{i}^{s} \in \{0,1\} \qquad \forall i \in I, j \in J, s \in S \tag{21}$$

In the proposed model, the primary objective function (9) has two parts. The first part focuses on minimizing the overall cost associated with procuring and transporting equipment across various scenarios, considering both the reliability of routes from relief base nodes to demand nodes and the vulnerability levels of those points. The second part calculates the penalty cost for the unfulfilled demand in cases where the path to the damaged points is blocked, making it impossible to transfer the goods and equipment from the base nodes to the demand nodes. The second objective function (10) aims to maximize the possible services at the demand nodes. This means that as much as possible, the correct service should not be sacrificed to minimize the cost.

In addition to cost and coverage, the model also prioritizes social justice. The third objective function (11) focuses on balancing the allocation of resources, ensuring that all regions in need of assistance receive adequate support and relief. This is achieved by selecting the base node in such a way that damaged areas are not created far from the base. To accomplish this, we minimize the maximum distance between the damaged nodes and the bases.

Furthermore, the model addresses the critical aspect of time in disaster response. The fourth objective function (12) is designed to minimize the duration of rescue and relief operations by optimizing the deployment of resources from relief bases to the damaged nodes.

Lastly, the fifth objective function (13) aims to minimize the number of required relief bases and the overall support needed for relief operations. This promotes an efficient allocation of resources, ensuring that the available relief bases are strategically located and equipped to meet the demands of various scenarios.

Constraint (14) indicates that the demand node j receives the necessary equipment from the base nodes. Meanwhile, constraint (15) ensures that the supply from base node i to the demand node j, which it covers, does not exceed the capacity of the base node. Constraint (16) ensures

Table 2: Description of parameters and decision variables in the model.

Symbol	Sets
I	Set of base nodes
J	Set of demand nodes
S	Set of earthquake magnitude scenarios
Symbol	Parameter
r_{ij}^s	Reliability of the path from base node i to demand node j in scenario s
v_j	Vulnerability number of demand node
d_{ij}	The distance of demand node j from base node i
TU_i^s	The duration of loading equipment and relief forces at relief base node i in scenario s
TR_{ij}^s	The duration of the transfer of equipment and relief forces from the relief base node i to the demand node j in scenario s
TD_j^s	The duration of the deployment of equipment and relief forces in the demand node j in scenario s
CM_j^s	The cost of maintaining tools and equipment in the demand node j in scenario s
CT_{ij}^s	The cost of providing tools and equipment and moving from the relief base node i to the demand node j in scenario s
dg_j^s	The amount of demand for the equipment of demand node j in scenario s
cap_i^s	Storage capacity of base node i in scenario s
a_{ij}^s	$a_{ij}^s = 1$, when the demand node j is located in the coverage of base node i and otherwise $a_{ij}^s = 0$. According to experts' opinion.
q^s	The penalty cost per unit of unsatisfied demand under the scenario s
st_j	The standard maximum time for the equipment to arrive at demand node j, after which the delay occurs
M	A relatively large number used to linearize the model
Symbol	Decision Variables
Q_{ij}^s	The number of tools and equipment packages
T_{ij}^s	Variable 0 and 1, Allocation or not allocation of demand node j to safe warehouse base node i in scenario s
w_i^s	Variable 0 and 1, Allocating or not allocating a base node to provide services
k	$k = \max \sum_{i\in I} \sum_{j\in J} T_{ij}^s d_{ij} r_{ij}^s$
X_{ij}^s	Auxiliary variable for linearization

that all demand nodes are adequately covered. Constraints (17) guarantee that demand node j receives service from base node i if it falls within the coverage radius of the base. Constraint (18) ensures that demand node j receives service from base node i, provided that base node i is established. Constraint (19) requires the selection of at least one that covers a demand node. Constraint (20) guarantees that equipment arrives at the demand nodes within the standard time frame. Constraint (21) represents the range of decision variables.

4.3 Linear Equivalent Model of the Problem

The first and second objective functions (9) and (10), along with constraints (14) and (15), involve the multiplication of two decision variables T^s_{ij}, Q^s_{ij}, which renders the model non-linear. To linearize it, we can utilize a linearization technique outlined in [15]. By replacing the expression $Q^s_{ij}.T^s_{ij}$ with X^s_{ij}, we introduce constraints (34), (35), and (36) to the problem model.

Furthermore, the third objective function (11) exhibits non-linearity. To linearize it, we define k as the maximum value of the sum of $T^s_{ij} d_{ij} r^s_{ij}$ for all $i \in I, j \in J$. We then incorporate constraint (37) into the problem model.

The linear equivalent model of the problem is presented below:

$$\min \quad F_1 = \sum_{i\in I}\sum_{j\in J}\sum_{s\in S} X^s_{ij} v_j \left(CM^s_j + CT^s_{ij}\right) + \sum_{i\in I}\sum_{j\in J}\sum_{s\in S} X^s_{ij} q^s (1 - r^s_{ij}) v_j \tag{22}$$

$$\max \quad F_2 = \sum_{i\in I}\sum_{j\in J}\sum_{s\in S} X^s_{ij} r^s_{ij} v_j \tag{23}$$

$$\min \quad F_3 = k \tag{24}$$

$$\min \quad F_4 = \sum_{i\in I}\sum_{j\in J}\sum_{s\in S} T^s_{ij} r^s_{ij} \left(TU^s_i + TR^s_{ij} + TD^s_j\right) \tag{25}$$

$$\min \quad F_5 = \sum_{i\in I}\sum_{s\in S} w^s_i \tag{26}$$

s.t:

$$\sum_{i} Q^s_{ij} T^s_{ij} = dg^s_j \qquad \forall j \in J, s \in S \tag{27}$$

$$\sum_{j} Q^s_{ij} T^s_{ij} \leq cap^s_i \qquad \forall i \in I, s \in S \tag{28}$$

$$\sum_{j\in J} a^s_{ij} w^s_i \geq 1 \qquad \forall i \in I, s \in S \tag{29}$$

$$T^s_{ij} \leq a^s_{ij} \qquad \forall i \in I, j \in J, s \in S \tag{30}$$

$$T^s_{ij} \leq w^s_i \qquad \forall i \in I, j \in J, s \in S \tag{31}$$

$$w_i^s \leq \sum_{j\in J} T_{ij}^s \qquad \forall i \in I, s \in S \tag{32}$$

$$\sum_{i\in I} T_{ij}^s \left(TU_i^s + TR_{ij}^s + TD_j^s\right) \leq st_j \quad \forall j \in J, s \in S \tag{33}$$

$$X_{ij}^s \leq T_{ij}^s * M \qquad \forall i \in I, j \in J, s \in S \tag{34}$$

$$X_{ij}^s \leq Q_{ij}^s \qquad \forall i \in I, j \in J, s \in S \tag{35}$$

$$X_{ij}^s \geq Q_{ij}^s - (1 - T_{ij}^s) * M \qquad \forall i \in I, j \in J, s \in S \tag{36}$$

$$\sum_{i\in I} \sum_{j\in J} T_{ij}^s d_{ij} r_{ij}^s \leq k \qquad \forall s \in S \tag{37}$$

$$X_{ij}^s \geq 0, Q_{ij}^s \geq 0\ ,\ T_{ij}^s, w_i^s \in \{0,1\} \qquad \forall i \in I, j \in J, s \in S \tag{38}$$

4.4 Model Characteristics and Assumptions

This model identifies critical bases in several affected regions and designates neighboring cities as support bases. The transportation routes from these bases to the affected points are assumed to have a reliable probability of being secure. Relief operations are conducted based on the distance between the critical bases, support bases, and affected points as minimizing rescue time is prioritized. The journey commences at the critical and support bases and concludes at the affected regions. Each critical base and support base possesses the capability to dispatch equipment and forces to multiple affected regions based on their capacity. The vulnerability level of the regions is determined through expert opinions and various criteria. The demand is assessed according to the severity of vulnerability under different scenarios, and failure to meet the demand results in penalties, thereby emphasizing the importance of social justice in providing relief to the affected region.

The proposed model is a mixed-integer non-linear multi-objective model. Given its multiple objectives, the comprehensive criterion method weighted metric method (Lp method) is employed for its solution.

5 Case Study

This section presents a case study that utilizes real data to assess the accuracy of the proposed mathematical model. To this end, the city of Gonabad has been chosen as the case study, as illustrated in Figure 1.

In this study, the city of Gonabad is divided into nine regions based on population and area, considering them as vulnerable points. Seven potential relief bases are designated to assist

Figure 1: Location of Gonabad in the country [23].

affected regions. The distances between the relief bases and different regions are obtained using Google Maps. Subsequently, the arrival time and transportation cost for each region are calculated based on those distances. The demand for rescue equipment and personnel in each region is determined by the vulnerability score in different scenarios and the region's population. A penalty is imposed for each unmet demand unit, set at 50, 20, and 10 times the maximum transportation cost per relief package for each assumed scenario. Three scenarios are considered in this study based on earthquake intensity and the vulnerability score of the targeted regions. The reliability level of the routes between the affected points and the bases is determined through expert opinions and various criteria.

5.1 Definition of Vulnerability and It's Impact on Crisis Severity

Vulnerability refers to the combined level of risk and socio-economic capacity to handle a hazardous event. In [2], various types of vulnerability are introduced, including economic, social, organizational, educational and attitudinal, political, cultural, and physical vulnerability. This study primarily focuses on physical and economic vulnerability. As the focus of this study is on dealing with an already occurring crisis rather than predicting its onset, the severity and the affected points are known. Therefore, the intensity of vulnerability in each region becomes a crucial parameter to consider in resource allocation. Significant earthquakes typically result in land deformation, involving the depletion and dissipation of energy due to earthquake intensity. One consequence of an earthquake is the movement of fault lines caused by seismic activity, along with the deformation of various layers of the earth and the occurrence of fractures and structural failures in buildings. These factors contribute to the creation of areas with lower resistance and higher vulnerability. The location of settlements and human-made structures is strongly influenced by environmental and geological factors. With the rapid growth of population and inevitable development and construction, the pressure of human needs on land is increasing daily. Additionally, the exploitation of surrounding areas of cities and villages for housing, economic, and industrial purposes exacerbates this pressure. The extent of damage inflicted on buildings in different cities due to seismic vibrations depends on multiple factors. Table 3 presents the influential criteria used for calculating vulnerability based on expert opinions. After determining the criteria, the weights and vulnerability levels for different regions are assessed based on expert opinions. Subsequently, the average vulnerability score is calculated and utilized in the applied model. The calculations provided here are solely presented as an example for Region 1 in Table 3.

Table 3: Results of vulnerability calculation for Region 1.

Criterion	Sub-criterion	Level of vulnerability	Weight	Weightlevel of vulnerability
Natural factors	Distance from the fault	0.580	1	0.580
	Soil resistance	0.158	1	0.158
	Static level of water	0.057	2	0.114
	Slope	0.205	2	0.140
Human Factors	Population density	0.154	4	0.616
	Distance from dangerous centers	0.231	4	0.924
	Quality of the buildings	0.174	4	0.696
	The size of the pieces	0.076	1	0.076
	Land use	0.033	4	0.132
	Access to vital arteries	0.107	4	0.428
	The ability to move in critical situations	0.117	3	0.351
	Access to urban open spaces	0.06	4	0.240
	Access to relief centers	0.049	4	0.196
Average				0.378

We conducted a case study to determine the optimal disaster relief operations and guide the allocation of relief equipment in Gonabad, Iran. The symbols used in the study are as follows: I1 represents the urban area of Gonabad, I2 represents Najmabad, I3 represents Bajestan, I4 represents Torbat-e Heydariyeh, I5 represents Kashmar, I6 represents Ghayenat, and I7 represents Khaf. Symbol J1 represents Region 1 of Gonabad, J2 represents Region 2 of Gonabad, J3 represents Region 3 of Gonabad, J4 represents Region 4 of Gonabad, J5 represents Region 5 of Gonabad, J6 represents Region 1 of Kakhk, J7 represents Region 2 of Kakhk, J8 represents Region 1 of Beydokht, and J9 represents Region 2 of Beydokht.

Three different scenarios are considered: Scenario 1 (S1) corresponds to a moderate earthquake with a magnitude ranging from 5.0 to 5.9 on the Richter scale, Scenario 2 (S2) indicates the occurrence of a strong earthquake with a magnitude ranging from 6 to 6.9 on the Richter scale, and scenario 3 (S3) indicates the occurrence of a major earthquake with a magnitude ranging from 7 to 7.9 on the Richter scale. The data corresponding to these scenarios can be found in Tables 4-8.

Table 4: Data for v_j, dg_j^s and cap_i^s.

		J1	J2	j3	j4	j5	j6	j7	j8	j9
dg_j^s	S3	8662	5773	7021	10000	6813	2401	2224	2337	3164
	S2	4331	2886	3510	5000	3406	1200	1112	1168	1582
	S1	1732	1154	1404	2000	1362	480	444	467	632
v_j		0.378	0.329	0.344	0.39	0.386	0.35	0.42	0.36	0.41
	I1	I2	I3	I4	I5	I6	I7			
cap_i^s	S3	10000	10000	10000	10000	10000	10000	10000		
	S2	5000	5000	5000	5000	5000	5000	5000		
	S1	2000	2000	2000	2000	2000	2000	2000		

Table 5 presents the data regarding the distance between base and demand nodes is presented. Based on these distances, the data for the parameter TR_{ij}^s is estimated.

Table 5: Data for distance between set of base nodes and set of demand nodes.

	J1	J2	j3	j4	j5	j6	j7	j8	j9
I1	10	15	22	20	40	45	15	25	30
I2	20	40	35	40	20	20	25	10	35
I3	53	53	53	53	53	73	73	60	60
I4	138	138	138	138	138	158	165	145	150
I5	127	127	127	127	127	150	155	135	140
I6	105	105	105	105	105	130	140	100	95
I7	190	190	190	190	190	215	220	200	205

Table 6 displays the reliability data of the different paths from base I_1, I_2, and I_3 to the damage point J in all scenarios. It should be noted that the parameter r_{ij}^s data is determined based on expert opinions.

The allocation for assigning base i to the affected region j is presented in Table 7. This data is also determined based on expert opinions.

6 Results

The multi-objective optimization model was coded using GAMS in this study. The allocation results for different scenarios are presented in Table 8.

Table 6: Data for r^s_{ij}.

r^s_{ij}	s1	s2	s3	r^s_{ij}	s1	s2	s3	r^s_{ij}	s1	s2	s3
$I1{\rightarrow}J1$	0.6	0.5	0.4	$I2{\rightarrow}J1$	0.7	0.6	0.5	$I3{\rightarrow}J1$	0.6	0.5	0.4
$I1{\rightarrow}J2$	0.8	0.7	0.6	$I2{\rightarrow}J2$	0.9	0.8	0.7	$I3{\rightarrow}J2$	0.8	0.7	0.6
$I1{\rightarrow}J3$	0.5	0.4	0.3	$I2{\rightarrow}J3$	0.6	0.5	0.4	$I3{\rightarrow}J3$	0.5	0.4	0.3
$I1{\rightarrow}J4$	0.4	0.3	0.2	$I2{\rightarrow}J4$	0.5	0.4	0.3	$I3{\rightarrow}J4$	0.4	0.3	0.2
$I1{\rightarrow}J5$	0.7	0.6	0.5	$I2{\rightarrow}J5$	0.8	0.7	0.6	$I3{\rightarrow}J5$	0.7	0.6	0.5
$I1{\rightarrow}J6$	0.5	0.4	0.3	$I2{\rightarrow}J6$	0.6	0.5	0.4	$I3{\rightarrow}J6$	0.5	0.4	0.3
$I1{\rightarrow}J7$	0.4	0.3	0.2	$I2{\rightarrow}J7$	0.5	0.4	0.3	$I3{\rightarrow}J7$	0.5	0.4	0.3
$I1{\rightarrow}J8$	0.7	0.6	0.5	$I2{\rightarrow}J8$	0.7	0.6	0.5	$I3{\rightarrow}J8$	0.7	0.6	0.5
$I1{\rightarrow}J9$	0.5	0.4	0.3	$I2{\rightarrow}J9$	0.6	0.5	0.4	$I3{\rightarrow}J9$	0.6	0.5	0.4

Table 7: Data for as^s_{ij} in Scenario 1.

a^1_{ij}	J1	J2	j3	j4	j5	j6	j7	j8	j9
I1	1	1	1	1	1	1	1	1	1
I2	1	1	1	1	1	1	1	1	1
I3	0	0	0	0	0	1	1	1	1
I4	0	1	1	0	1	0	0	0	0
I5	0	0	0	0	1	0	0	1	1
I6	1	0	0	1	0	0	0	0	0
I7	0	0	0	0	0	1	1	1	1

Table 8: Allocation results.

	S1	S2	S3		S1	S2	S3
$I1{\rightarrow}J1$	886	4331	0	$I3{\rightarrow}J3$	1404	0	0
$I1{\rightarrow}J2$	0	292	0	$I3{\rightarrow}J5$	596	0	2201
$I1{\rightarrow}J3$	0	0	5262	$I3{\rightarrow}J6$	0	404	0
$I1{\rightarrow}J6$	480	0	2401	$I4{\rightarrow}J1$	846	0	4227
$I1{\rightarrow}J8$	309	1168	2337	$I4{\rightarrow}J2$	1154	2594	5773
$I2{\rightarrow}J4$	0	4418	2224	$I4{\rightarrow}J5$	0	3406	3960
$I2{\rightarrow}J5$	766	0	4612	$I6{\rightarrow}J3$	0	3510	1759
$I2{\rightarrow}J7$	444	0	0	$I6{\rightarrow}J4$	2000	582	7776
$I2{\rightarrow}J8$	158	0	0	$I6{\rightarrow}J6$	0	796	0
$I2{\rightarrow}J9$	632	1582	3164	$I6{\rightarrow}J7$	0	1112	2224
$I3{\rightarrow}J1$	0	0	4435	$I7{\rightarrow}J3$	0	0	1759

The allocation of assistance in different earthquake scenarios for various regions is summarized in Table 8. In the case of a moderate earthquake, assistance will be provided from the base of Region 1 in Gonabad (I1) to the affected area (1) in Gonabad (J1). If a strong earthquake occurs, support will be extended not only from the Region 1 base (I1) but also from the base in Bajestan County (I3). Furthermore, in the event of a major earthquake, support will be dispatched from the Torbat-e Heydariyeh base (14) in addition to the Region 1 base (I1). For Region J2, in the event of a moderate earthquake, support will be provided from the Torbat-e Heydariyeh base (I4). In Scenario 2, support will be required not only from the Region 1 base (I1) but also from the Torbat-e Heydariyeh base (I4). In the case of a large-scale earthquake in this region, assistance will be dispatched from the Khaf base (I7).

In Region J3, if Scenario 1 occurs, the optimal response is to send assistance from the Bajestan base (I3). In the case of a strong earthquake, the most effective assistance would be provided from Ghayenat base (I6). Finally, if Scenario 3 takes place in this area, support from both the Gonabad urban base (I1) and the Ghayenat base (I6) would be required. In Region J4, for Scenario 1, the optimal solution suggests receiving assistance from the Ghayenat base (I6). In the event of Scenario 2, the recommended approach is to utilize assistance from both the Najmabad base (I2) and also the Ghayenat base (I6). In the case of a large-scale earthquake in this area, assistance will be provided from both the Ghayenat base (I6) and the Najmabad base (I2).

To manage a moderate-scale earthquake in Region J5, assistance needs to be dispatched from Najmabad and Bajestan bases, i.e., (I2) and (I3), respectively. If the earthquake intensity reaches a strong level (Scenario 2), assistance from the Torbat-e Heydariyeh base (I4) should be utilized. Finally, in the case of Scenario 3 in this area, assistance from Najmabad base (I2) and Torbat-e Heydariyeh base (I4) would be used. In Region J6, the recommended approach for managing a moderate earthquake is to utilize assistance from the Gonabad urban base (I1). If Scenario 2 occurs, assistance will be needed from both the Bajestan base (I3) and Ghayenat base (I6). In case of the most severe earthquake in this area, assistance from Gonabad urban base (I1) is required.

For managing a moderate earthquake in Region Kakhk (J7), it is sufficient to send assistance from the Najmabad base (I2). If Scenarios 2 and 3 occur in this region, assistance will be dispatched from the Ghayenat base (I6). In Scenario 1 in Region Kakhk (J8), assistance is required from both the Gonabad urban base (I1) and the Najmabad base (I2). If Scenarios 2 and 3 occur in this region, assistance will be dispatched from the Gonabad urban base (I1). The model's recommended response for crisis management in the Beydokht region in all three scenarios involves sending assistance from the Najmabad base (I2).

6.1 Sensitivity Analysis of the Model

In this section, we have assessed the impact of increasing and decreasing the reliability of the path from base i to the affected point j in Scenario (r^s_{ij}) by 0.2. The effects of these changes have been analyzed in three scenarios. Figure 2 presents the results of the sensitivity analysis in Scenario 1.

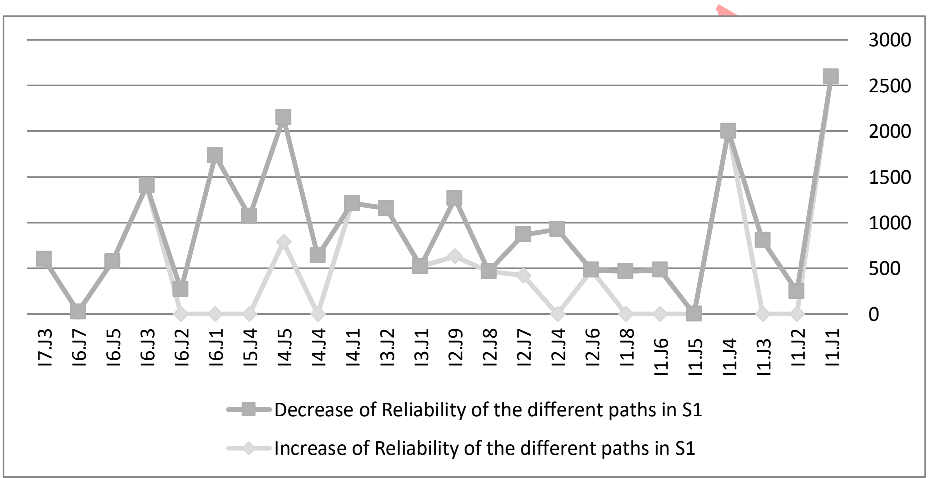

Figure 2: Sensitivity analysis results of the model in Scenario 1.

The graph illustrates how the model's response varies with the changes in the reliability of the traffic paths. The sensitivity analysis results in Scenarios 2 and 3 are depicted in Figures 3 and 4, respectively.

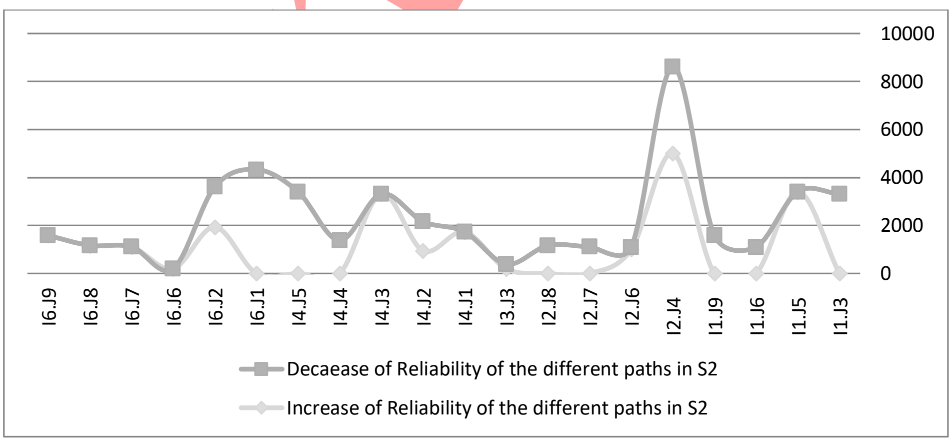

Figure 3: Sensitivity analysis results of the model in Scenario 2.

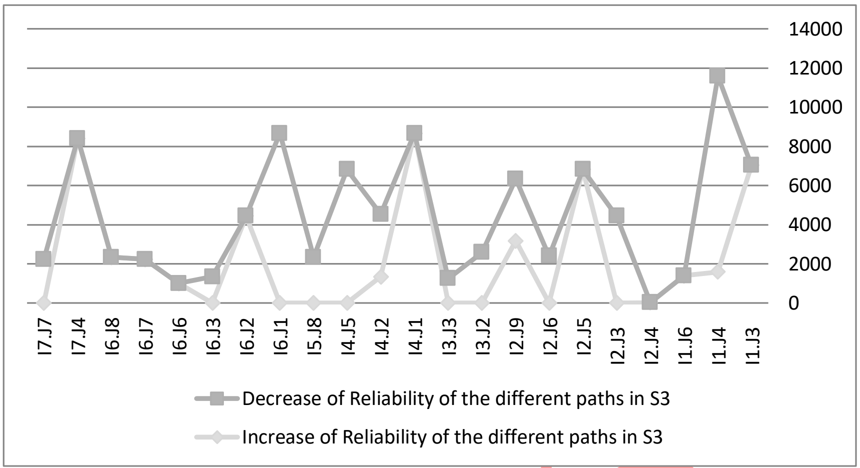

Figure 4: Sensitivity analysis results of the model in Scenario 3.

According to Figures 2 to 4, we can conclude that the behavior of the model is consistent and reliable.

7 Conclusion

The present study focuses on the logistics of earthquake relief operations, with the primary objective of minimizing human casualties by ensuring the timely delivery of essential relief supplies to the affected populations. It is also crucial to consider the quantity and volume of these supplies. The analysis incorporates the standard relief time (SRT) as a factor in evaluating timely delivery. Another challenge addressed is the destruction of communication routes during earthquakes, which has a detrimental impact on relief operations. Therefore, the study incorporates the reliability of different paths into the modeling process. Additionally, vulnerability coefficients are used to account for varying vulnerability levels in different locations. Furthermore, the proposed model includes penalties for unmet demands to minimize the number of unfulfilled requests. The model is based on real data collected from the city of Gonabad in Razavi Khorasan Province. The results indicate that the model demonstrates satisfactory computational capabilities across various earthquake scenarios. Consequently, decision-makers can utilize the findings of this study to enhance their planning for earthquake crisis management and formulate appropriate responses to such emergencies. Future researchers are encouraged to explore other aspects of logistics in relief operations and consider alternative methods to address these challenges.

Declarations

Availability of supporting data

All data generated or analyzed during this study are included in this published paper.

Funding

No funds, grants, or other support were received for conducting this study.

Competing interests

The authors have no competing interests to declare that are relevant to the content of this paper.

Authors' contributions

The main manuscript text is written collectively by the authors.

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