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## Research Article

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## A Graph-Theoretic Heuristic Approach for a Multi-Objective Healthcare Facility Layout Problem: A Real Hospital Case Study

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**Abstract.** Efficient layout design in healthcare facilities is critical for operational effectiveness and patient care. This study addresses the healthcare facility layout problem using a multi-objective optimization approach. We propose a novel methodology based on graph theory, specifically planar adjacency graphs, to generate and evaluate department layouts. Nodes in the graph represent departments, while weighted edges represent the desired closeness based on patient flow and functional relationships. We introduce five strategies based on different weightings of these objectives and evaluate them using a real-world hospital case study. Our results show that a hybrid strategy, prioritizing patient flow while incorporating departmental relationships, yields the optimal layout. This approach provides a systematic and data-driven framework for healthcare planners to create efficient layouts that enhance workflow, reduce travel distances, and improve overall service quality.

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## 1 Introduction

In healthcare facility design, the development of optimal layouts plays a crucial role in enhancing organizational efficiency and improving the quality of patient care. Recent advances in computational methods have provided designers with powerful tools to generate and evaluate layout alternatives with unprecedented accuracy and flexibility. This study explores generative layout design for hospital facilities, with a particular emphasis on the implementation and assessment of a graph-theoretic approach to address complex spatial planning challenges. Healthcare environments, including hospitals and clinics, require careful optimization to improve operational performance, patient satisfaction, and overall service quality. In this context, multi-objective modeling has become an effective framework, allowing the simultaneous consideration of multiple criteria, such as minimizing travel distances and maximizing desirable departmental adjacencies.

Section 2 presents a review of the relevant literature, including medical facility planning (Subsection 2.1) and applications of graph theory (Subsection 2.2). Subsection 2.4 identifies existing research gaps, while Subsection 2.5 outlines the main contributions of this study. Section 3 describes the proposed methodology, including the notation system, dataset description, mathematical formulation, and evaluation procedures. Section 4 illustrates the application of one selected strategy, and Section 5 presents the comprehensive results of the case study. Section 6 discusses the implications of the findings, and Section 7 concludes the paper and suggests directions for future research.

The complete input datasets and additional generated layout strategies are provided in Appendix A, Appendix B, and Appendix C.

## 2 Literature Review

### 2.1 General Healthcare Facility Layout Problems

Over the past five decades, optimization-based approaches have played a central role in addressing healthcare facility layout problems. One of the earliest formal definitions of the hospital layout problem was introduced by Elshafei [7], who formulated it as a cost–flow Quadratic Assignment Problem (QAP). This pioneering work laid the theoretical foundation for numerous subsequent studies adopting QAP-based models.

Extending this framework, Yeh and Lin [27] investigated the layout design of a hospital comprising 28 facilities. Their study emphasized spatial proximity, available space constraints, and relative positioning among departments. By modeling the problem as a QAP and solving

it through a simulated annealing-based neural network, they demonstrated the applicability of metaheuristic techniques to large-scale healthcare layouts.

Recognizing the hierarchical structure of complex medical centers, Helber et al. [15] proposed a two-stage hierarchical modeling approach. In the first stage, departments are allocated to locations within a QAP framework using a fix-and-optimize heuristic while considering transportation flows and adjacency constraints. The second stage refines the spatial configuration by incorporating detailed space requirements of a major healthcare facility.

Similarly, Cubukcuoglu et al. [5] developed a QAP-based methodology specifically tailored to existing hospitals. Their formulation models internal logistics interactions as a space allocation problem with the objective of minimizing transportation activities and improving operational performance.

Moving beyond single-objective QAP formulations, Huo et al. [18] introduced a double-row layout model solved using the NSGA-II algorithm. Their multi-objective framework simultaneously minimizes patients' actual travel distances and maximizes desirable interdepartmental adjacencies. This study illustrates how evolutionary algorithms can effectively integrate logistical and relational considerations within complex hospital environments.

Recent contributions have further expanded healthcare layout research beyond classical QAP structures. Terán et al. [25] proposed a sequential space–syntax approach for rehabilitation hospitals. Their two-phase methodology first quantifies accessibility and proximity using space syntax metrics, then applies a tabu search with nested-bay encoding to generate block layouts, followed by corridor network optimization to reduce travel distance and congestion.

In a computational design context, Cubukcuoglu et al. [6] introduced an integrated toolkit combining graph theory, operations research, computational design workflows, and computational intelligence. Their framework evaluates key performance indicators such as wayfinding efficiency, staff walking times, and workflow suitability, supporting both hospital renovation and new design projects.

From a digital twin perspective, Jia et al. [19] developed a Hospital Configuration Model (HCM) embedded within a hospital design support system. By integrating geometric, topological, semantic, and operational data into a machine-readable structure, their model enables simulation-based optimization and prediction of metrics such as crowding levels, waiting times, and walking distances.

Finally, Fattahi et al. [8] presented an AI-driven hospital design process based on neuro-symbolic strategies. Their hybrid framework combines symbolic reasoning with neural networks to autonomously generate optimized hospital layouts with minimal human intervention, underscoring the growing role of artificial intelligence in automating complex healthcare design decisions.

## 2.2 The Graph Theory Approach in Healthcare Facility Layout Problems

The application of graph-theoretic concepts to facility layout problems can be traced back to the work of Foulds and Robinson [9], who introduced graph-based heuristics as an alternative strategy for solving classical Quadratic Assignment Problems (QAPs). Their approach demonstrated how adjacency relationships could be explicitly modeled through graph structures, providing intuitive and computationally efficient heuristics for layout optimization.

From a geometric and combinatorial perspective, Rosenstiehl and Tarjan [24] developed a linear-time algorithm for generating rectilinear planar layouts based on bipolar orientations of planar graphs. Their method represents vertices as horizontal line segments and edges as vertical segments with integer coordinates, producing compact and interlocking graph representations. This theoretical foundation established important links between planar graph theory and space-efficient layout generation.

In healthcare-specific applications, Assem et al. [3] applied graph-theoretic principles to optimize operating theatre (OT) layouts. Their methodology focused on facility layout planning through adjacency modeling and block layout optimization. Using heuristic graph-based strategies, they reassigned functional spaces to improve operational efficiency. The reported improvements—an 18.5% increase in layout score in one hospital and 45% in another—demonstrate the practical effectiveness of graph-based planning in real healthcare environments.

Arnolds and Nickel [2] emphasized the communicative advantages of graph-theoretical approaches compared to purely mathematical optimization models. They argued that graph-based representations are particularly useful when collaborating with architects and healthcare professionals who may not be familiar with formal optimization formulations, as graphs provide an intuitive visualization of adjacency and flow relationships while still supporting analytical rigor.

Building upon this perspective, Lather et al. [21] proposed a computational framework for generating and evaluating hospital layouts based on departmental adjacency ratings obtained from domain experts. An optimal adjacency graph was constructed, and layouts were generated under discrete structural constraints. Each alternative was evaluated using a distance-weighted scoring mechanism. Notably, healthcare planning experts consistently preferred layouts with higher computed scores, supporting the validity of graph-based generative evaluation.

In a subsequent study, Lather et al. [22] further explored generative layout methods in a real hospital project. Using a graph-theoretical framework grounded in distance-weighted adjacency scores, they produced optimal and near-optimal design alternatives. Expert feedback indicated that such generative methods expanded the design space, reduced cognitive bias, and facilitated more informed decision-making. Later, Lather and Harms [20] extended this line of research by introducing sparsity constraints in graphical networks at both macro- and micro-

scales. By reducing unnecessary connections while preserving planarity and critical adjacency relations, their method improved resource flow efficiency and clarified spatial hierarchy within hospital departments.

Beyond healthcare-specific studies, Bisht et al. [4] introduced G2PLAN, a mathematically rigorous graph-based floorplan generation method. Unlike learning-based systems, G2PLAN guarantees satisfaction of user-defined adjacency and dimensional constraints through graph-theoretic and linear optimization techniques. The system can generate thousands of topologically distinct floorplans within milliseconds, demonstrating scalability, reliability, and high computational efficiency. By extending its predecessor GPLAN, G2PLAN expands adjacency handling and dimensional customization capabilities, showcasing the robustness of graph-theoretic formulations in automated spatial design.

More recent research has shifted toward graph-informed evaluation and decision-support systems. Hassanain et al. [14] integrated entropy-based performance indicators with graph-heuristic concepts to evaluate healthcare building performance, enabling the quantification of connectivity, organizational clarity, and spatial efficiency beyond simple distance metrics. In a related redesign-focused study, Hassanain et al. [13] combined graph-heuristic modeling with fuzzy TOPSIS multicriteria decision-making to rank healthcare facility redesign alternatives under conflicting performance criteria. These works reflect a broader movement toward explainable, graph-aware assessment frameworks in healthcare design.

Finally, Alavi et al. [1] integrated particle swarm optimization (PSO) with Building Information Modeling (BIM) and digital twin technologies to create an AI-driven layout optimization framework. Their system generates optimized two-dimensional layouts using PSO, converts them into three-dimensional BIM representations through visual programming, and provides stakeholders with interactive virtual environments for performance evaluation. This integration of graph-informed optimization with AI and digital modeling technologies illustrates the continuing evolution of intelligent, data-driven hospital layout design methodologies.

Collectively, these studies demonstrate the versatility of graph-theoretic approaches in healthcare facility layout problems, ranging from heuristic optimization and generative design to evaluation frameworks and AI-integrated decision-support systems. Such developments provide a strong foundation for graph-based representations that preserve meaningful adjacency structures while accommodating practical architectural and operational constraints.

### 2.3 Comparative Synthesis and Critical Positioning

To move beyond a purely chronological review, Table 1 provides a structured comparison of representative studies according to: (i) problem formulation and spatial representation (QAP, metaheuristics, graph-theoretic or generative models), (ii) objective coverage (flow-distance,

adjacency/closeness, wayfinding, accessibility/entrance logic), (iii) solution methodology, and (iv) validation context (real hospital case versus benchmark or synthetic instances).

This synthesis reveals two recurring structural limitations in the literature. First, many QAP-based and metaheuristic formulations prioritize flow-distance minimization but do not explicitly enforce planar realizability of the resulting topology. Consequently, the optimized adjacency matrices often require substantial geometric post-processing to obtain physically feasible corridor-based layouts. Second, entrance accessibility and external interface logic are frequently omitted or treated only indirectly through generalized flow metrics. In high-traffic healthcare environments, however, entrance proximity plays a critical operational role and cannot be reduced solely to interdepartmental distance minimization.

These observations motivate the methodological stance of the present study: the adoption of a planarity-preserving Planar Adjacency Graph (PAG) representation combined with explicit entrance-gate modeling to ensure both topological feasibility and operational relevance.

#### 2.4 Gaps in the Literature

Although a substantial body of research addresses healthcare facility layout optimization through QAP-based formulations, these approaches predominantly focus on minimizing interdepartmental travel distances. While effective from a cost-flow perspective, such formulations rarely guarantee that the resulting adjacency structures can be embedded in a planar layout without edge crossings. The geometric feasibility of the final design is therefore often deferred to a secondary design stage.

Graph-theoretic approaches offer a more interpretable representation of spatial relationships; however, their application to real hospital case studies remains limited. Moreover, many existing graph-based models adopt single-objective formulations, typically emphasizing distance-weighted adjacency without systematically integrating multiple operational priorities.

A further gap concerns the treatment of entrance accessibility. Most existing models do not explicitly incorporate entrance-gate logic as a structural component of the adjacency representation. In practice, departments with higher patient inflow—such as emergency or outpatient services—should be positioned strategically relative to the entrance. Neglecting this factor may lead to layouts that are mathematically optimized yet operationally suboptimal.

These limitations indicate the need for a graph-based, multi-objective framework that simultaneously enforces planar realizability and integrates entrance accessibility as an explicit design variable.

**Table 1:** Comparative synthesis of representative healthcare layout optimization studies.

Reference	Model	Main Objective	Method	Validation	Key Limitation
Elshafei (1977) [7]	QAP	Flow-distance minimization	Exact/heuristic QAP	Hospital formulation	No explicit planarity enforcement
Yeh (2006) [27]	QAP with constraints	Proximity + placement	SA + ANN	Case study	Requires geometric post-processing
Helber et al. (2016) [15]	Hierarchical QAP	Transport + adjacency	Fix-and-optimize	Large hospital	Allocation-first; topology not guaranteed
Cubukcuoglu et al. (2021) [5]	QAP (geodesic)	Internal transport cost	QAP renovation model	Real hospital case	Adjacency feasibility indirect
Huo et al. (2021) [18]	Multi-floor model	Walk distance + adjacency	NSGA-II	Multi-floor layout	Planarity not central
Teran-Somohano & Smith (2023) [25]	Space-syntax model	Distance + congestion	Tabu search	Rehab hospital	Multi-stage, assumption-dependent
Lather et al. (2020) [22]	Adjacency graph	Weighted adjacency	Graph generation	Expert evaluation	No guaranteed geometric embedding
Bisht et al. (2022) [4]	Graph + LP (G2PLAN)	Adjacency + dimensions	Linear optimization	Large-scale demo	Not hospital-flow specific
Alavi et al. (2024) [1]	BIM-integrated model	Multi-criteria performance	PSO + BIM workflow	Stakeholder study	Planar adjacency implicit
Jia et al. (2025) [19]	Configuration / digital twin	Simulation-driven metrics	Data-driven integration	Design support system	Data availability dependent
<b>This study</b>	<b>Planar Adjacency Graph</b>	<b>Flow + relations + entrance</b>	<b>Greedy planar + MO evaluation</b>	<b>Real hospital case</b>	<b>Planarity enforced + entrance modeled</b>

## 2.5 Contributions

This study advances the existing literature by directly addressing the topological and operational limitations of conventional QAP and metaheuristic formulations. While QAP models are effective for minimizing flow-distance costs, they frequently generate adjacency structures that are not inherently planar and therefore require complex geometric adjustments before implementation.

By adopting a Planar Adjacency Graph (PAG) framework, the proposed methodology ensures that generated adjacencies are realizable on a two-dimensional plane without edge crossings, thereby reducing post-processing requirements and improving geometric feasibility.

The main contributions of this study are summarized as follows:

1. Development and comparative evaluation of five distinct layout strategies (S1–S5) that balance patient flow intensity and clinical relationship requirements within a unified graph-based framework.
2. Formal integration of entrance-gate logic (Department X) into the adjacency graph structure, enabling explicit optimization of external accessibility—an aspect largely neglected in traditional departmental interaction models.
3. A rigorous multi-objective assessment demonstrating that a hybrid weighting strategy ( $\alpha = 0.75$ ) achieves superior performance compared to pure-flow, pure-relationship, benchmark QAP, and Genetic Algorithm solutions, particularly in terms of solution stability as measured by the Coefficient of Variation.

## 3 Methodology

This section presents the proposed methodology for generating healthcare department layouts using planar adjacency graphs (PAGs). The core idea is to model spatial relationships among hospital departments through graph-theoretic principles, enabling a structured and optimization-driven layout generation process.

Each department is represented as a node in a graph, and edges indicate desired spatial adjacencies. By restricting the graph to be planar, the resulting adjacency structure becomes geometrically realizable without edge crossings, thereby ensuring feasibility at the conceptual layout level. This representation extends classical facility layout graph modeling approaches, such as those described in [16], where adjacency relationships are first visualized graphically and later refined into detailed layouts. In contrast to traditional trial-and-error or designer-

driven refinement, the proposed approach embeds planarity and quantitative evaluation directly into the optimization framework.

The methodology integrates multiple criteria, including patient flow intensity, departmental relationship weights, spatial area requirements, and entrance accessibility. These factors are incorporated into a multi-objective evaluation structure. A deterministic greedy planarization mechanism is employed to construct feasible adjacency graphs, followed by iterative evaluation and comparison of alternative assignment strategies.

Overall, the proposed framework enables systematic, data-driven generation and assessment of healthcare layouts while explicitly enforcing planar realizability and balancing flow efficiency, relational priorities, and spatial utilization.

### 3.1 Notations

Table 2 summarizes all symbols and indices used throughout the mathematical formulation.

**Table 2:** Summary of indices, parameters, and decision-related quantities used in the model

Symbol	Description
$i, k$	Departments, $\{A, B, \dots, L\}$
$j, l$	Areas, $\{1, 2, \dots, 12\}$
$s$	Layout strategies, $\{S_1, S_2, S_3, S_4, S_5\}$
$F_{ik}$	Patient flow from department $i$ to department $k$
$R_{ik}$	Closeness relationship score between departments $i$ and $k$
$D_{jl}$	Rectilinear distance between the centers of areas $j$ and $l$ (m)
$S_i$	Number of patients admitted to department $i$
$E_i$	Entrance accessibility weight of department $i$
$DE_j$	Rectilinear distance between area $j$ and the nearest entrance gate (m)
$A_j$	Area size of location $j$ ( $\text{m}^2$ )
$d_i$	Required area of department $i$ ( $\text{m}^2$ )
$TD_i$	Area satisfaction level of department $i$
$TDO(s)$	Average area satisfaction under strategy $s$
$M_{ijkl}$	Walking distance from department $i$ in area $j$ to department $k$ in area $l$
$M_{ijkl}(s)$	Total inter-department walking distance under strategy $s$
$RV_{ijkl}$	Distance-weighted relationship value between assigned departments
$RV_{ijkl}(s)$	Total relationship value under strategy $s$
$g_1, g_2, g_3$	Normalized objective values
$\alpha$	Weight coefficient of patient flow
$Z$	Final planar adjacency matrix

### 3.2 Input Data

The primary dataset is adopted from [10] and is provided in Appendix A. The entrance accessibility parameters  $E_i$  and the distance matrix  $DE_j$  are generated hypothetically to reflect dual-gate hospital access.

Since the hospital contains two entrance gates ( $X_1$  and  $X_2$ ), the effective distance between area  $j$  and the entrance is defined as the minimum rectilinear distance to either gate:

$$DE_j = \min\{dist_{X_1,j}, dist_{X_2,j}\}.$$

For example, if area 1 is located 10 meters from  $X_2$  and 12 meters from  $X_1$ , then  $DE_1 = 10$ . The generated hypothetical datasets are reported in Appendix B.

The arrangement of the rooms is shown in Figure 1, and the names of the departments along with their abbreviations are also in Table 3. We use the information in Table 4 to transform the relationship matrix.

**Table 3:** Department names and corresponding notations used in the study.

Department	Notation
Internal Medicine	A
Cardiology	B
Pulmonology	C
Dermatology	D
Psychiatry	E
Neurology	F
General Surgery	G
Neurosurgery	H
Plastic Surgery	I
Orthopedics	J
Urology	K
Otolaryngology (ENT)	L

### 3.3 Mathematical Model

The graph-theoretic approach looks at the profit that results from placing each pair of facilities next to one another. It then looks for facility pairings that will maximize the total profit. It locates the maximum planar graph where the weights' total is maximized. The maximum planar graph's arcs indicate which pairs of facilities are to be next to one another. A designer can then use these adjacencies to create a workable plan that satisfies the constraints for shape and

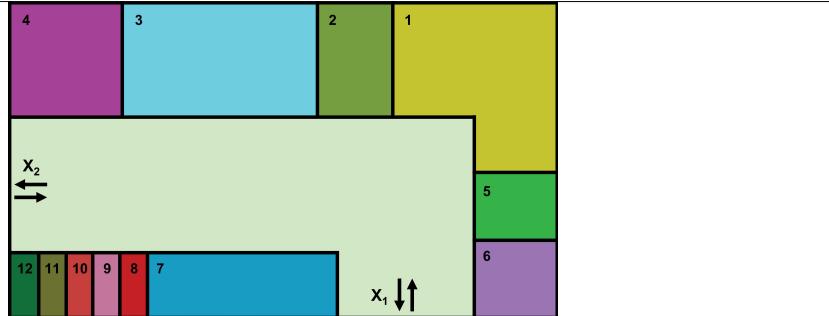


Figure 1: Layout of the healthcare areas.

Table 4: Closeness relationship categories and corresponding quantitative scores.

Closeness Category	Symbol	Score
Too Close	A	10
Closer	E	7
Close	I	5
Far	O	3
Further	U	1
Far-off / Never Close	X	-9

space while placing facility pairs with high interaction next to one another. The graph-theoretic method alone aims to maximize the total of the weights of nearby facilities; it does not take into account the weights of non-neighboring facilities.

The key to solving layout problems with a planar adjacency network efficiently is to represent the graph effectively and utilize algorithms to find the best layout. Adjacency lists and adjacency matrices are two data structures that can be used to represent planar adjacency graphs and help in layout optimization.

### 3.3.1 Conversion

The first step in generating the adjacency matrix is to consolidate the data from the input matrices ( $F_{ik}$  in Table 8,  $R_{ik}$  in Table 9, and  $D_{jl}$  in Table 10). These tables are already presented in an upper triangular format. To ensure a single, comprehensive value for each pair-wise interaction (e.g., combining flow from  $i$  to  $k$  and from  $k$  to  $i$ ), we define the new matrices  $F'_{ik}$ ,  $R'_{ik}$ , and  $D'_{jl}$ . These matrices, defined in Equations (1)-(3), create a new set of upper triangular matrices where each element represents the total combined interaction between two entities.

$$F'_{ik} = \begin{cases} F_{ik} + F_{ki}, & \text{if } i < k, \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

$$R'_{ik} = \begin{cases} R_{ik} + R_{ki}, & \text{if } i < k, \\ 0, & \text{otherwise,} \end{cases} \quad (2)$$

$$D'_{jl} = \begin{cases} D_{jl} + D_{lj}, & \text{if } j < l, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

### 3.3.2 Expansion

The original department set has size  $n$  (here  $n = 12$ ). To incorporate entrance accessibility, we augment the department set with a pseudo-department  $X$ , yielding an expanded set of size  $n + 1$ . Accordingly, the flow and relationship matrices expand from  $\mathbb{R}^{n \times n}$  to  $\mathbb{R}^{(n+1) \times (n+1)}$  by appending one row/column that encodes interactions between each department and the entrance node. Similarly, the area-distance matrix expands from  $\mathbb{R}^{m \times m}$  to  $\mathbb{R}^{(m+1) \times (m+1)}$  by adding one row/column for distances between candidate areas and the entrance.

The patient's entry flow is an important factor. Thus, it is assumed that a new department, named  $X$ , is constituted by the hospital's entrance gate. The inclusion of  $S_i$  (patient admission in Table 11) into  $F'_{ik}$ ,  $E_i$  (closeness to the entrance gate in Table 14) into  $R'_{ik}$ , and  $DE_j$  (distance between area  $j$  and the entrance gate in Table 15) into  $D'_{jl}$  is necessitated. The resulting expanded matrices,  $F''_{ik}$ ,  $R''_{ik}$ , and  $D''_{jl}$ , will all be used in the evaluation section.

$$F''_{ik} = \begin{bmatrix} F'_{ik} & S_i^T \\ 0 & 0 \end{bmatrix},$$

$$R''_{ik} = \begin{bmatrix} R'_{ik} & E_i^T \\ 0 & 0 \end{bmatrix},$$

$$D''_{jl} = \begin{bmatrix} D'_{jl} & DE_j^T \\ 0 & 0 \end{bmatrix}.$$

### 3.3.3 Normalization

Both the  $F''_{ik}$  and  $R''_{ik}$  matrices should be normalized. This will scale both matrices in the range of zero to one.

$$\tilde{F}_{ik} = \frac{F''_{ik}}{\max F''},$$

$$\tilde{R}_{ik} = \frac{R''_{ik} - R''_{min_{ik}}}{R''_{max_{ik}} - R''_{min_{ik}}},$$

where  $\max F''$  is the maximum value in the  $F''_{ik}$  matrix. In Equation (3.3.3),  $R''_{max}$  and  $R''_{min}$  represent the maximum and minimum values in the entire  $R''_{ik}$  matrix, respectively. This min-max normalization scales all relationship values to the range  $[0, 1]$ .

### 3.3.4 Calculate the Adjacency Matrix

The adjacency matrix is calculated by the equation below.

$$Z_{ik} = \alpha \tilde{F}_{ik} + (1 - \alpha) \tilde{R}_{ik}, \quad \alpha \in [0, 1].$$

### 3.3.5 Planar Subgraph Selection (PAG Extraction)

Let  $V$  denote the set of departments augmented with the entrance node  $X$ , and let  $G_c = (V, E_c)$  be the complete undirected graph induced by the weighted adjacency matrix  $Z$ , where each candidate edge  $(i, k) \in E_c$  has weight  $w_{ik} = Z_{ik}$ .

We select a planar subset of edges by introducing a binary decision variable  $y_{ik} \in \{0, 1\}$  indicating whether edge  $(i, k)$  is included in the planar adjacency graph (PAG). The selection problem can be stated as:

$$\max_y \sum_{(i,k) \in E_c} w_{ik} y_{ik},$$

subject to

$$G(y) = (V, E(y)) \text{ is planar, } E(y) = \{(i, k) \in E_c \mid y_{ik} = 1\},$$

$$y_{ik} \in \{0, 1\}, \quad \forall (i, k) \in E_c.$$

In many layout settings, we additionally seek a *maximal planar* solution (i.e., no further edge can be added without violating planarity). For  $|V| \geq 3$ , any maximal planar graph has at most  $3|V| - 6$  edges; thus the procedure stops once  $|E(y)| = 3|V| - 6$  is reached. Because the maximum-weight planar subgraph problem is NP-hard, we use a deterministic greedy edge-addition procedure that guarantees planarity by construction and yields a maximal planar subgraph; global optimality is not guaranteed because maximum-weight planar subgraph selection is NP-hard.

### 3.3.6 Deterministic Greedy Planarization and Complexity

We construct the PAG using greedy edge addition: all candidate edges are sorted in non-increasing order of weight, and an edge is added if and only if the resulting graph remains

planar. Planarity is checked using a linear-time planarity test (as implemented in standard planarity-check routines).

Let  $n = |V|$  and  $|E_c| = \frac{n(n-1)}{2}$ . Sorting edges costs  $O(|E_c| \log |E_c|) = O(n^2 \log n)$ . In the worst case, planarity is tested for many candidate edges. If a planarity test runs in  $T_p(n)$  time, the worst-case complexity of the greedy step is  $O(|E_c| \cdot T_p(n))$ , i.e.,  $O(n^2 \cdot T_p(n))$ ; with linear-time planarity testing, this is  $O(n^3)$  in the worst case. Since a maximal planar graph contains at most  $3n - 6$  edges, the number of accepted edges is  $O(n)$ , and the method is efficient for the problem sizes typical in department-layout design.

### 3.3.7 Problem Formulation

The layout problem is formulated as a Multi-Objective Optimization Problem (MOOP).

Let  $n$  be the number of departments and  $x_{ij}$  be a binary decision variable such that  $x_{ij} = 1$  if department  $i$  is assigned to area  $j$ , and  $x_{ij} = 0$  otherwise.

The objective is to minimize the vector function  $G(x) = [g_1(x), g_2(x), g_3(x)]^T$ , defined as:

$$\text{Maximize } g_1 = \frac{1}{n} \sum_{i=1}^n \min \left( 1, \frac{\sum_{j=1}^m A_j x_{ij}}{d_i} \right),$$

$$\text{Minimize } g_2 = \sum_{i=1}^n \sum_{k=1}^n \sum_{j=1}^m \sum_{l=1}^m F''_{ik} D''_{jl} x_{ij} x_{kl},$$

$$\text{Minimize } g_3 = \sum_{i=1}^n \sum_{k=1}^n \sum_{j=1}^m \sum_{l=1}^m R''_{ik} D''_{jl} x_{ij} x_{kl},$$

subject to assignment constraints:

$$\sum_{j=1}^m x_{ij} = 1, \quad \forall i \in \{1, \dots, n\},$$

$$\sum_{i=1}^n x_{ij} \leq 1, \quad \forall j \in \{1, \dots, m\},$$

$$x_{ij} \in \{0, 1\}.$$

In this study we do not run a continuous multi-objective solver over  $(g_1, g_2, g_3)$ . Instead, we generate a small set of candidate layouts by varying the weighting parameter  $\alpha$  (Strategies S1–S5) in the PAG construction, and we then evaluate and rank these candidate solutions using normalized, unitless scores (Section 3.5).

### 3.4 Strategy Development

In this study, five different strategies are evaluated.

**Strategy S1** : The patient flow as well as the departmental relationship are considered equally significant in this strategy. ( $\alpha = 0.5$ )

**Strategy S2** : In this strategy, all concerns have been paid to arranging the departments based on their patient flow. Hence, the departments with the most patient flow are placed next to each other. The relationship between departments is not significant in this case. ( $\alpha = 1$ )

**Strategy S3** : In contrast with the previous strategy, this strategy organizes departments according to their proximity value. Consequently, departments with the highest relational value are positioned adjacent to each other. The flow of patients between departments is deemed insignificant within this context. ( $\alpha = 0$ )

**Strategy S4** : In this approach, the main focus is on arranging the departments according to their patient flow rather than the departmental relationship. ( $\alpha = 0.75$ )

**Strategy S5** : This method places more emphasis on departmental relationships than patient flow when organizing the departments. ( $\alpha = 0.25$ )

### 3.5 Evaluation Methods

Objectives  $g_2$  (walking-distance) and  $g_3$  (relationship-distance) are costs where smaller values indicate better layouts. For ease of comparison across strategies, we report normalized satisfaction scores  $g_2(s)$  and  $g_3(s)$  in  $[0, 1]$  using monotone transformations so that larger is better. This normalization is used for post hoc comparison and ranking of the candidate strategies and does not alter the underlying cost definitions. To choose the best one among the strategies and evaluate the output layout, three criteria must be calculated for each strategy. To do so, the final patient flow, relationship values, and area distances ( $F''_{ik}$ ,  $R''_{ik}$ , and  $D''_{ik}$  obtained in Section 3.3.2) are used. At last, we compare the attractiveness of each criterion for all strategies.

#### 3.5.1 Area satisfaction level ( $g_1$ )

Higher values of  $g_1$  indicate better satisfaction of departmental area requirements;  $g_1 = 1$  means that all departments meet (or exceed) their required area, while values below 1 reflect area shortfalls for one or more departments.  $A_j$  (Table 13) must satisfy the area satisfaction of  $d_i$  (Table 12).

$$TD_i = \begin{cases} \frac{A_j}{d_i}, & \text{if } \frac{A_j}{d_i} < 1, \\ 1, & \text{otherwise,} \end{cases}$$

$$TDO(s) = \sum_{i=1}^n \frac{TD_i}{n},$$

$$g_1 = \frac{TDO(s)}{\max TDO(s)}.$$

### 3.5.2 Total walking distance of patients ( $g_2$ )

The target is to reach nearly 1 for the value of  $g_2$ . Let  $A_s(i)$  be the area  $j$  assigned to department  $i$  under a given strategy  $s$  ( $s = S1, \dots, S5$ ). The total walking distance for strategy  $s$ ,  $M(s)$ , is a scalar value calculated by summing the patient flow ( $F''_{ik}$ ) multiplied by the corresponding distance ( $D''_{A_s(i), A_s(k)}$ ) for all pairs of departments.

$$M_{ijkl} = F''_{ik} D''_{jl},$$

$$M(s) = \sum_{i=1}^n \sum_{k=1}^n \left( F''_{ik} \times D''_{A_s(i), A_s(k)} \right).$$

The quantity  $M(s)$  is a walking-distance cost; therefore, smaller values indicate better layouts. To report this objective on a common larger-is-better scale in  $[0, 1]$ , we define the normalized score: To normalize this objective, we compare it against the minimum total walking distance found among all strategies,  $M_{\min}$ .

$$g_2(s) = \frac{M_{\min}}{M(s)} \quad \text{where} \quad M_{\min} = \min_{s' \in \{S1 \dots S5\}} M(s').$$

### 3.5.3 Relationship-distance satisfaction level ( $g_3$ )

Along with the previous two objectives,  $g_3$  must be closer to 1 for greater satisfaction.

Similarly, the total distance relationship score for strategy  $s$ ,  $RV(s)$ , is a scalar value calculated by summing the relationship value ( $R''_{ik}$ ) multiplied by the corresponding distance ( $D''_{A_s(i), A_s(k)}$ ) for all pairs.

$$RV_{ijkl} = R''_{ik} D''_{jl},$$

$$RV(s) = \sum_{i=1}^n \sum_{k=1}^n \left( R''_{ik} \times D''_{A_s(i), A_s(k)} \right).$$

This value is then normalized against the minimum score found among all strategies,  $RV_{min}$ .

$$g_3(s) = \frac{RV_{min}}{RV(s)} \quad \text{where} \quad RV_{min} = \min_{s' \in \{S1 \dots S5\}} RV(s')$$

## 4 Implementation

This section presents the complete procedure for implementing the proposed solution approach. The following steps correspond to Strategy S1; the other strategies follow an identical implementation framework.

To ensure computational efficiency and numerical accuracy, the solution procedure was implemented using the Python programming language [26]. Several scientific computing libraries were employed, including NumPy [12] for numerical operations, Pandas [23] for data manipulation, Matplotlib [17] for visualization, and NetworkX [11] for graph-based modeling and analysis.

It is worth emphasizing that the proposed PAG pipeline is fully deterministic. For a fixed input instance, the generated planar subgraph, the corresponding dual graph, and the final objective function values remain exactly reproducible across multiple executions.

### 4.1 Adjacency Matrix

The adjacency matrix ( $Z_{ik}$ ) corresponding to Strategy S1, presented in Table 5, is derived directly from the mathematical formulations introduced in Section 3.3. The matrix entries quantify the normalized interaction strength between department pairs under the defined relationship structure.

### 4.2 Planar Adjacency Graph

The concepts of planar graph and maximum planar graph must first be understood in order to comprehend the graph-theoretic approach. If a network can be drawn in two dimensions without any arc crossing another, it is said to be planar. A planar graph consists of a nonempty, finite collection of nodes and an unordered set of arcs. An adjacency graph is known as a planar adjacency graph (PAG) if it is planar. A PAG is considered maximum if it is maximally planar. The theory of the maximum planar graph and our presentation of the graph-theoretic approach may indicate to the reader that the method is concerned with identifying a maximal PAG in which the sum of the weights of the edges is maximized.

**Table 5:** The calculated adjacency matrix of strategy S1

$Z_{ik}$	A	B	C	D	E	F	G	H	I	J	K	L	X
A	–	0.438	0.378	0.435	0.285	0.360	0.529	0.000	0.266	0.283	0.374	0.363	1.000
B	0.438	–	0.519	0.000	0.296	0.357	0.266	0.266	0.266	0.269	0.266	0.277	0.694
C	0.378	0.519	–	0.320	0.276	0.266	0.289	0.000	0.000	0.355	0.000	0.371	0.801
D	0.435	0.000	0.320	–	0.291	0.266	0.352	0.000	0.535	0.352	0.274	0.277	0.312
E	0.285	0.296	0.276	0.291	–	0.526	0.000	0.271	0.274	0.274	0.266	0.283	0.569
F	0.360	0.357	0.266	0.266	0.526	–	0.269	0.655	0.266	0.294	0.283	0.355	0.870
G	0.529	0.266	0.289	0.352	0.000	0.269	–	0.274	0.288	0.349	0.377	0.283	0.644
H	0.000	0.266	0.000	0.000	0.271	0.655	0.274	–	0.277	0.452	0.269	0.377	0.604
I	0.266	0.266	0.000	0.535	0.274	0.266	0.288	0.277	–	0.529	0.274	0.285	0.576
J	0.283	0.269	0.355	0.352	0.274	0.294	0.349	0.452	0.529	–	0.000	0.280	0.803
K	0.374	0.266	0.000	0.274	0.266	0.283	0.377	0.269	0.274	0.000	–	0.000	0.593
L	0.363	0.277	0.371	0.277	0.283	0.355	0.283	0.377	0.285	0.280	0.000	–	0.807
X	1.000	0.694	0.801	0.312	0.569	0.870	0.644	0.604	0.576	0.803	0.593	0.807	–

The adjacency matrix  $Z_{ik}$  (Table 5) represents a complete weighted graph, where every node is connected to every other node with a specific weight. However, a physical layout cannot have every department adjacent to every other department. The goal of the graph-theoretic approach is to find the *maximal planar subgraph* of this complete graph. This is a standard optimization problem where we select a subset of edges (adjacencies) such that:

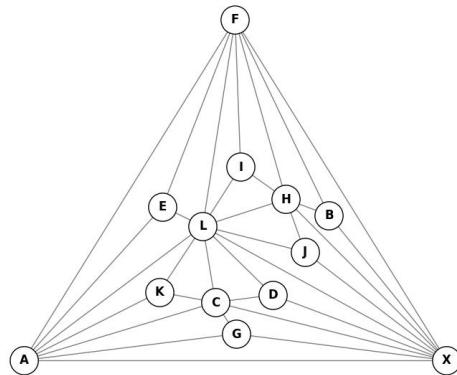
1. The sum of the weights of the selected edges is maximized.
2. The resulting graph is *planar* (it can be drawn on a 2D plane with no edges crossing).

This conversion from the complete weighted matrix ( $Z_{ik}$ ) to the final planar graph (Figure 2) is a key step handled by algorithms within the NetworkX package. The resulting graph is planar by definition, a deterministic greedy edge-addition procedure that guarantees planarity by construction and yields a maximal planar subgraph; global optimality is not guaranteed because maximum-weight planar subgraph selection is NP-hard.

Figure 2 is the output of the Python code that shows the PAG for Strategy S1.

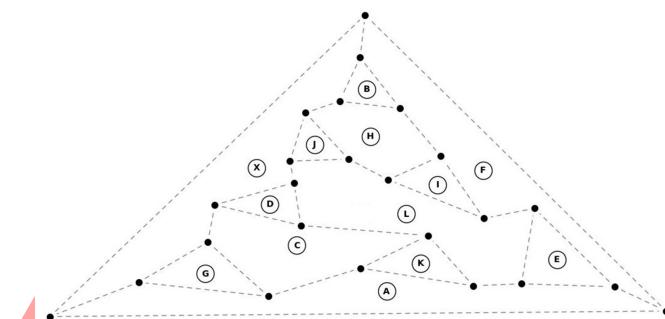
#### 4.3 Dual Graph of PAG

By adding a point inside each face and connecting the points to encompass all nodes, which correlate to the outside boundary, we can create the layout that corresponds to this PAG. From these two, a reasonably basic arrangement can be drawn.



**Figure 2:** The planar adjacency graph for Strategy S1 ( $\alpha = 0.5$ ).

The dual graph of the planar graph for Strategy S1 is shown in Figure 3. For instance, it is apparent that facility  $J$  would be better off being close to facilities  $H$ ,  $L$ , and  $X$  (the entrance gate), or it makes sense for facilities  $I$ ,  $E$ , and  $K$  to be farther away from the entrance gate.



**Figure 3:** The dual graph for Strategy S1 ( $\alpha = 0.5$ ).

#### 4.4 Final Layout

The facility designer must now create a layout with sufficient detail using the data mentioned earlier. This step involves a human designer translating the topological dual graph (Figure 3) into a concrete block layout, which is a common practice in this heuristic approach. The 'trial and error' refers to the manual adjustment process, which is guided by three main criteria:

1. **Adjacency Maintenance:** The final layout must respect the adjacencies specified in the dual graph (e.g., in Figure 3, department J must be adjacent to H, L, and X).
2. **Area Requirements:** Each department block must be scaled to meet its specific area expectation ( $d_i$  from Table 12).
3. **Building Constraints:** The entire layout must fit within the existing building shell and area locations (Figure 1).

The appropriate layout (Figure 4) is the one that best satisfies these three constraints by adjusting the shapes and relative positions of the department blocks.

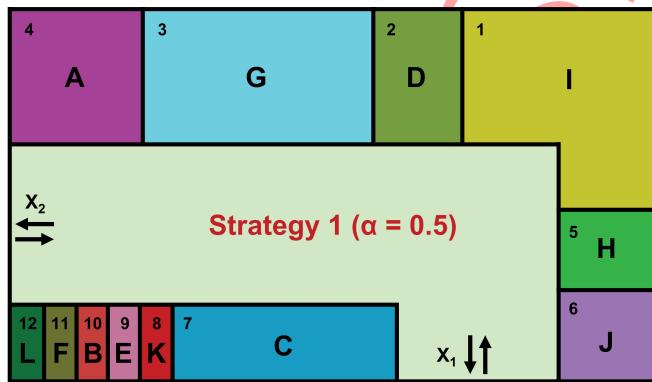


Figure 4: Final layout for Strategy S1 ( $\alpha = 0.5$ ).

## 5 Results

The implementation steps of Section 4 have been completed for all strategies. According to the final layout of Strategy S1 in the previous section and other strategies (Strategies S2–S5) that are included in Appendix C (Figures 7–14), we will evaluate these strategies using the mathematical formulation of Section 3.5.

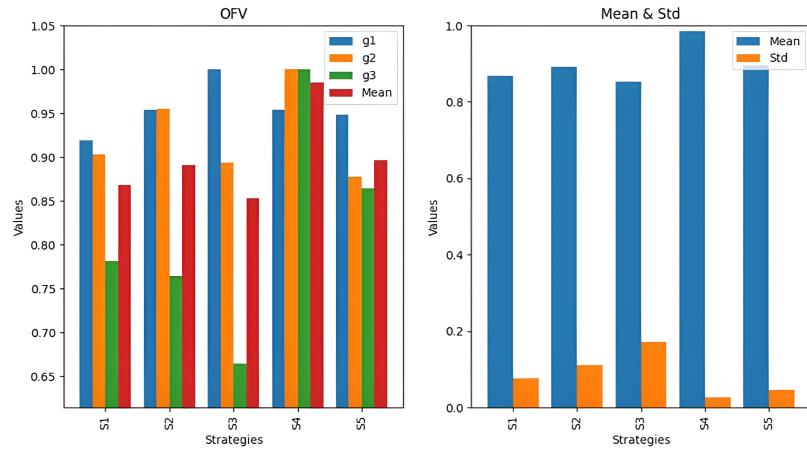
The normalized satisfaction values of the  $g_1$ ,  $g_2$ , and  $g_3$  objectives for all strategies are displayed in Table 6.

Figure 5 demonstrates that Strategy S3 has the highest value of the  $g_1$  objective criteria, while Strategy S4 has the highest values of  $g_2$  and  $g_3$ . Furthermore, it is evident that hybrid strategies S1, S4, and S5 have the lowest standard deviation values.

The best appropriate strategy for the problem is the one that has the highest mean and the lowest standard deviation in the objective. In some cases, a strategy may have the highest average but not the lowest standard deviation or a strategy may have the lowest standard deviation

**Table 6:** Evaluation results for all strategies

	$g_1$	$g_2$	$g_3$	Mean	Std	CV
$S_1$	0.9190	0.9030	0.7817	0.8679	0.0751	0.0865
$S_2$	0.9540	0.9548	0.7643	0.8910	0.1098	0.1232
$S_3$	1.0000	0.8940	0.6646	0.8529	0.1714	0.2010
$S_4$	<b>0.9540</b>	<b>1.0000</b>	<b>1.0000</b>	<b>0.9847</b>	<b>0.0266</b>	<b>0.0270</b>
$S_5$	0.9478	0.8773	0.8646	0.8966	0.0448	0.0500

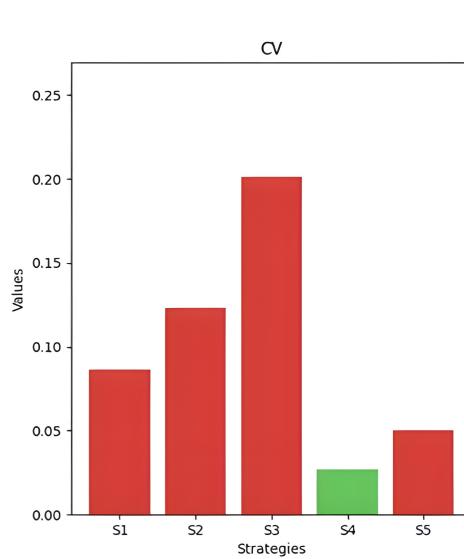
**Figure 5:** The visual results of objectives for all strategies.

but not the highest average. To solve this problem, we need to use the coefficient of variation (CV) ratio.

The reported *Mean*, *Std*, and *CV* in Table 6 are computed across the three normalized objectives ( $g_1, g_2, g_3$ ) for each strategy, and therefore quantify the balance of a solution across objectives (objective-dispersion), not run-to-run variability from repeated algorithm executions.

$$CV_i = \frac{Std(S_i)}{Mean(S_i)}.$$

As can be seen in Figure 6, Strategy S4 has the lowest CV and also has the highest mean and lowest standard deviation. Therefore, the optimal layout arrangement for this problem is the solution of Strategy S4, where  $\alpha = 0.75$ . This suggests that the primary arrangement should concentrate on patient flow while also taking the relationship value between departments into consideration.



**Figure 6:** The final result using CV.

## 6 Discussion

The results demonstrate that Planar Adjacency Graphs (PAGs) offer an effective and practical framework for generating layout topologies that simultaneously satisfy patient-flow efficiency and departmental adjacency requirements under planarity constraints.

The comparative evaluation of the five strategies (S1–S5), introduced in Section 3.4, reveals a clear trade-off between patient flow ( $\alpha = 1$  in S2) and departmental relationships ( $\alpha = 0$  in S3). As reported in Table 6, strategies that exclusively prioritize a single objective tend to underperform with respect to the neglected criterion. In particular, S3—focused solely on relational proximity—achieves the lowest relationship–distance satisfaction score ( $g_3 = 0.6646$ ). This suggests that emphasizing adjacency preferences alone may inadvertently increase travel distances, thereby reducing operational efficiency.

In contrast, Strategy S4 ( $\alpha = 0.75$ ) provides a well-balanced solution. By assigning greater weight to patient flow while still incorporating departmental relationships, S4 achieves the maximum score in both distance-based performance indicators ( $g_2 = 1.00$  and  $g_3 = 1.00$ ). This outcome indicates that the two objectives are not inherently conflicting and, when properly weighted, can be optimized concurrently. The corresponding layout (Figure 12) demonstrates both operational efficiency and adherence to critical clinical adjacencies. Furthermore, the highest mean performance (0.9847) combined with the lowest coefficient of variation ( $CV = 0.0270$ ) confirms the robustness and stability of this strategy.

A key strength of the proposed methodology lies in its capacity to integrate multiple objectives within a unified optimization framework. The approach minimizes travel distances for patients and staff while ensuring that functionally related departments remain in close proximity. This dual consideration reduces inefficiencies, enhances interdepartmental coordination, and supports smoother clinical workflows. The case study results clearly identify Strategy S4 as the most effective compromise solution across all evaluated metrics.

From a practical perspective, the proposed framework provides hospital administrators and facility planners with a systematic decision-support tool for layout design. By balancing operational efficiency with clinical adjacency requirements, the method has the potential to improve workflow performance, patient experience, and overall service quality. The successful implementation in the case study further demonstrates the applicability of the proposed model in real-world healthcare settings.

### 6.1 Comparative Analysis

To assess the effectiveness of the proposed graph-theoretic heuristic (PAG), a comprehensive computational comparison was conducted against the benchmark approaches reported by [10], namely the Genetic Algorithm (GA) and the Quadratic Assignment Problem (QAP) model.

Table 7 summarizes the performance of all strategies across the three methods. Each strategy is evaluated based on the three satisfaction objectives— $g_1$  (area utilization),  $g_2$  (walking distance), and  $g_3$  (relationship-distance)—and ranked according to the Coefficient of Variation (CV). The CV serves as the primary ranking criterion due to its suitability for multi-objective evaluation, as it captures the relative dispersion of performance by normalizing the standard deviation with respect to the mean. Hence, lower CV values indicate more stable and well-balanced solutions.

The comparative results clearly demonstrate the superiority of the proposed approach. PAG Strategy 4 achieves the top overall ranking among all 19 evaluated strategies, with a CV of 0.0270. This value is substantially lower than those of the best-performing benchmark strategies, GA-6 (CV = 0.0331) and QAP-6 (CV = 0.0491), indicating improved robustness and balance.

The superior performance of PAG-S4 can be attributed to two main factors:

1. It attains the highest mean satisfaction score (0.985), reflecting outstanding overall performance across all objectives. Notably, it achieves perfect scores in both the walking-distance ( $g_2$ ) and relationship-distance ( $g_3$ ) criteria.

2. It maintains an exceptionally low standard deviation (0.027), surpassed only marginally by GA-6. This confirms that the solution does not favor a single objective but instead delivers consistently high performance across all evaluation metrics.

Although GA-6 and QAP-6 also produce relatively stable solutions, their mean satisfaction scores (0.847 and 0.855, respectively) remain considerably lower. Therefore, the proposed PAG framework not only ensures stability but also achieves superior overall solution quality. These findings substantiate the effectiveness of the graph-theoretic heuristic in identifying robust, high-quality, and well-balanced layouts for complex multi-objective healthcare facility design problems.

**Table 7:** Full computational comparison of GA, QAP, and PAG strategies (Std and CV indicate dispersion across  $g_1$ ,  $g_2$ , and  $g_3$ ).

Method	Strategy	$g_1$	$g_2$	$g_3$	Mean	Std	CV	Rank
GA	1	0.818	0.743	0.982	0.848	0.122	0.1439	8
	2	1.000	0.640	0.696	0.779	0.194	0.2490	19
	3	0.712	0.995	0.719	0.809	0.161	0.1990	15
	4	0.779	0.695	1.000	0.825	0.158	0.1915	14
	5	0.911	0.681	0.914	0.835	0.133	0.1593	12
	6	<b>0.818</b>	<b>0.874</b>	<b>0.851</b>	<b>0.847</b>	<b>0.028</b>	<b>0.0331</b>	<b>2</b>
	7	0.829	0.623	0.989	0.814	0.184	0.2260	18
QAP	1	0.770	0.704	0.939	0.805	0.121	0.1503	10
	2	1.000	0.682	0.718	0.800	0.174	0.2175	17
	3	0.870	1.000	0.739	0.870	0.130	0.1494	9
	4	0.804	0.635	0.869	0.769	0.121	0.1573	11
	5	0.938	0.734	0.832	0.835	0.102	0.1222	6
	6	<b>0.885</b>	<b>0.807</b>	<b>0.872</b>	<b>0.855</b>	<b>0.042</b>	<b>0.0491</b>	<b>3</b>
	7	0.787	0.681	0.980	0.816	0.152	0.1863	13
PAG	1	0.919	0.903	0.782	0.868	0.075	0.0865	5
	2	0.954	0.955	0.764	0.891	0.110	0.1232	7
	3	1.000	0.894	0.665	0.853	0.171	0.2010	16
	4	<b>0.954</b>	<b>1.000</b>	<b>1.000</b>	<b>0.985</b>	<b>0.027</b>	<b>0.0270</b>	<b>1</b>
	5	0.948	0.877	0.865	0.897	0.045	0.0500	4

## 6.2 Run-to-Run Variability and Robustness

The CV-based ranking reported in Tables 6 and 7 serves as an indicator of *multi-objective balance*. Specifically, the Mean, Standard Deviation (Std), and Coefficient of Variation (CV) are computed across the three objective values ( $g_1, g_2, g_3$ ) for each individual strategy. These statistics therefore quantify how evenly a given strategy performs across objectives. Importantly, they do *not* reflect variability arising from repeated algorithmic executions.

### 6.2.1 Run-to-Run Variance (Proposed Method)

The proposed PAG framework is fully deterministic for a fixed input instance. The procedure consists of: (i) deterministic construction of the weighted adjacency matrix  $Z$ , (ii) greedy edge insertion following a fixed ordering with deterministic tie-breaking rules, and (iii) a planarity verification step that introduces no stochastic components. Consequently, repeated executions on the same instance yield identical planar subgraphs, dual graphs, and objective values. In other words, the PAG method exhibits zero run-to-run variance under identical inputs.

### 6.2.2 Baseline Methods (GA/QAP)

The GA and QAP results presented in Table 7 are reproduced directly from [10]. Readers are referred to that study for detailed descriptions of baseline experimental settings, parameter calibration, and any reported stochastic variability. The primary contribution of the present work lies in the development of the PAG-based methodology and its deterministic evaluation under the same objective definitions used for comparison.

### 6.2.3 Robustness and Sensitivity Analysis

Robustness in this study is examined through sensitivity analysis with respect to the objective-weighting parameter  $\alpha$ . Strategies S1–S5 correspond to  $\alpha \in \{0, 0.25, 0.5, 0.75, 1\}$ , thereby spanning the full spectrum from relationship-dominant to flow-dominant preferences. This design enables assessment of whether the preferred layout remains stable under meaningful shifts in decision priorities.

The findings indicate that the hybrid configuration  $\alpha = 0.75$  (Strategy S4) consistently delivers the most balanced performance across objectives, as evidenced by the lowest objective-dispersion CV.

For further validation, a standard input-perturbation analysis may be conducted by introducing small variations to the flow matrix  $F$ , relationship matrix  $R$ , and entrance-related parameters.

ters, followed by re-execution of the deterministic pipeline. A detailed perturbation protocol is provided in the supplementary material to ensure reproducibility.

## 7 Conclusion

In this study, we applied a graph-theoretic approach to design a hospital healthcare facility layout. This method is highly effective for generating a strong initial solution, particularly when department locations and areas are not yet fixed. By visualizing and optimizing the closeness and adjacency of facilities, this strategy is one of the best options for minimizing flow distances. This study successfully bridges the gap between theory and practice by applying the planar adjacency graph (PAG) approach to a real-world case study, providing validated results and actionable insights for facility planners.

Future studies could extend this research by addressing the following limitations:

- This study focused on a single-floor layout. Future work could introduce constraints for multi-story hospital design.
- A patient reception department, which serves as a central hub for all incoming patients, could be explicitly modeled as a high-flow department with unique relationships to all other units.
- The distances used in the base article were rectilinear. For greater accuracy, Euclidean distances could be implemented.
- The model could be expanded to include more complex entrance gate logic, such as separate entrance and exit gates or specialized access points (e.g., for emergencies).

## Appendix

## A Case Study Data

**Table 8:** Interdepartmental estimated annual patient transfer volumes ( $F_{ik}$ )

**Table 9:** Qualitative interdepartmental relationships ( $R_{ik}$ )

$R_{ik}$	A	B	C	D	E	F	G	H	I	J	K	L
A	-	I	O	I	U	O	E	X	U	U	O	O
B	-	E	X	U	O	U	U	U	U	U	U	U
C	-	O	U	U	U	X	X	O	X	O		
D		-	U	U	O	X	E	O	U	U		
E			-	E	X	U	U	U	U	U		
F				-	U	A	U	U	U	U		
G					-	U	U	O	O	U		
H						-	U	I	U	O		
I							-	E	U	U		
J								-	X	U		
K									-	X		
L										-		

**Table 10:** Rectilinear distances between the centroids of areas  $j$  and  $l$  (in meters), denoted by  $D_{jl}$ 

$D_{jl}$	1	2	3	4	5	6	7	8	9	10	11	12
1	0	20	33.75	47.5	17.5	25	40	50	55	60	65	70
2	20	0	16.25	35	37.5	45	30	40	45	50	55	60
3	33.75	16.25	0	18.75	51.25	58.75	36.25	23.75	28.75	33.75	38.75	43.75
4	47.5	35	18.75	0	65	72.5	50	37.5	32.5	27.5	22.5	25
5	17.5	37.5	51.25	65	0	7.5	32.5	42.5	47.5	52.5	57.5	62.5
6	25	45	58.75	72.5	7.5	0	25	35	40	45	50	55
7	40	30	36.25	50	32.5	25	0	12.5	17.5	22.5	27.5	32.5
8	50	40	23.75	37.5	42.5	35	12.5	0	5	10	15	20
9	55	45	28.75	32.5	47.5	40	17.5	5	0	5	10	15
10	60	50	33.75	27.5	52.5	45	22.5	10	5	0	5	10
11	65	55	38.75	22.5	57.5	50	27.5	15	10	5	0	5
12	70	60	43.75	25	62.5	55	32.5	20	15	10	5	0

**Table 11:** Patient demand for each department, denoted by  $S_i$ 

$S_i$	A	B	C	D	E	F	G	H	I	J	K	L
Demand	722	394	434	450	366	648	474	492	452	552	248	444

**Table 12:** Expected area requirements of departments (in square meters), denoted by  $d_i$ 

$d_i$	A	B	C	D	E	F	G	H	I	J	K	L
Area (m <sup>2</sup> )	336	36	36	72	36	192	72	84	72	180	36	36

**Table 13:** Existing areas and their corresponding sizes (in square meters), denoted by  $A_j$ 

$A_j$	1	2	3	4	5	6	7	8	9	10	11	12
Area (m <sup>2</sup> )	336	72	192	180	72	72	84	36	36	36	36	36

## B Generated Hypothetical Data

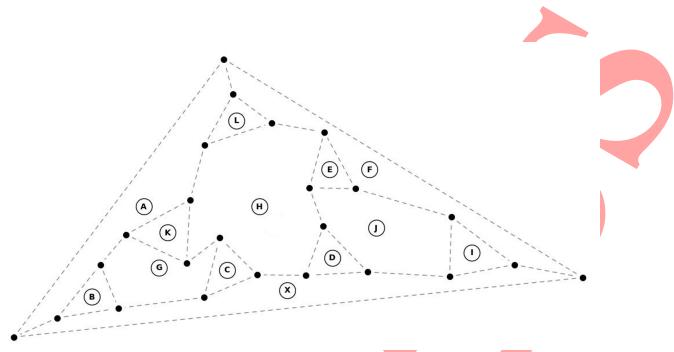
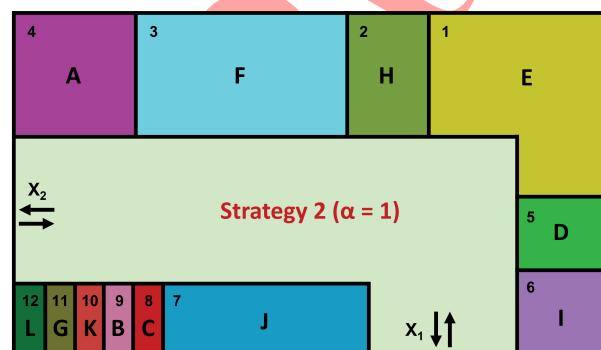
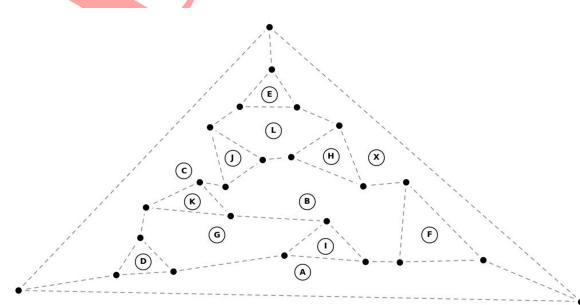
**Table 14:** Qualitative relationships between each department  $i$  and the entrance gate, denoted by  $E_i$ 

$E_i$	A	B	C	D	E	F	G	H	I	J	K	L
Relationship Code	A	E	A	X	O	E	O	U	U	E	E	A

**Table 15:** Rectilinear distance between each area  $j$  and the entrance gate (in meters), denoted by  $DE_j$ 

$DE_j$	1	2	3	4	5	6	7	8	9	10	11	12
Distance (m)	30	35	25	12	10	5	8	30	26	21	16	12

### C Results of Strategies

**Figure 7:** The dual graph for Strategy S2 ( $\alpha = 1$ ).**Figure 8:** Final layout for Strategy S2 ( $\alpha = 1$ ).**Figure 9:** The dual graph for Strategy S3 ( $\alpha = 0$ ).

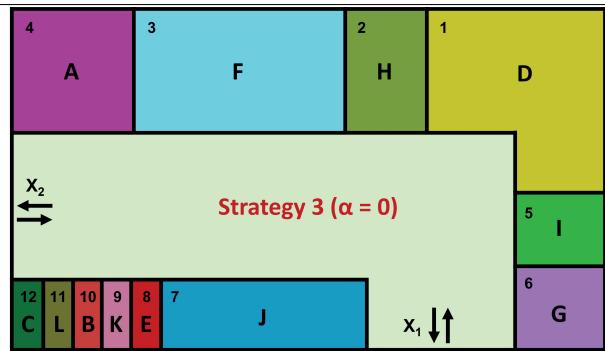


Figure 10: Final layout for Strategy S3 ( $\alpha = 0$ ).

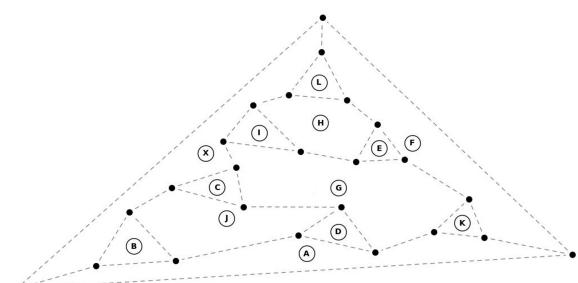


Figure 11: The dual graph for Strategy S4 ( $\alpha = 0.75$ ).

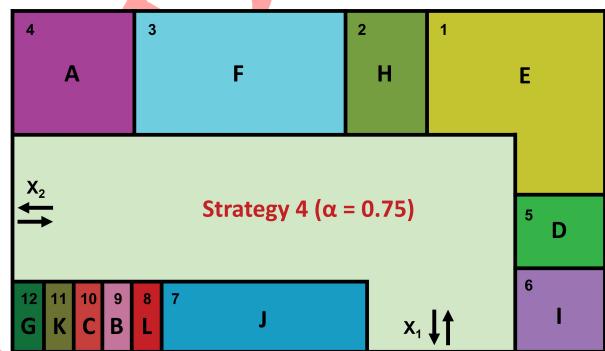


Figure 12: Final layout for Strategy S4 ( $\alpha = 0.75$ ).

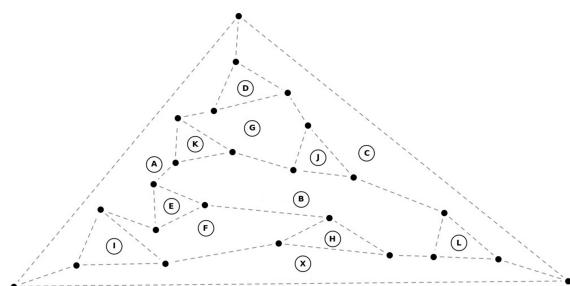
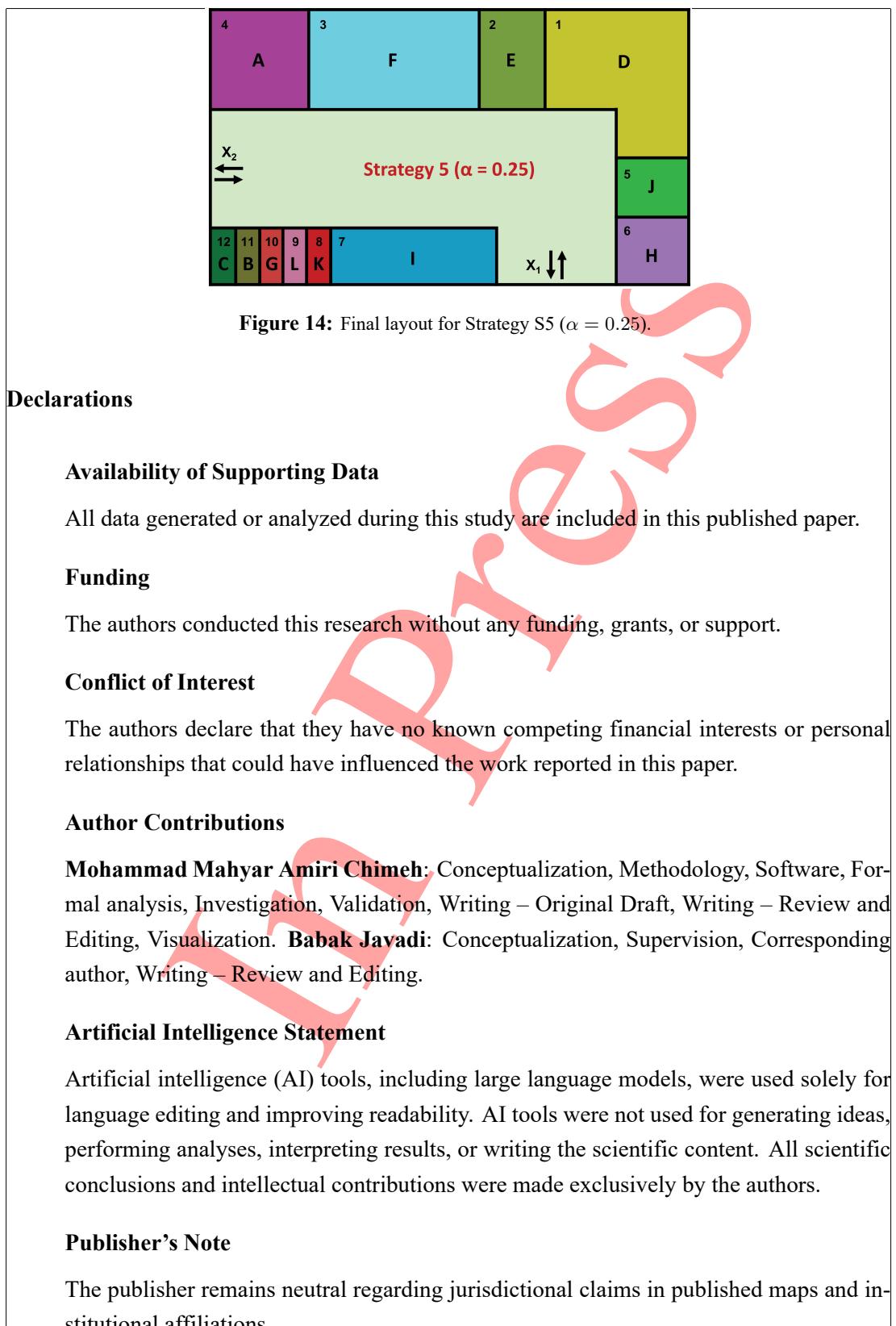


Figure 13: The dual graph for Strategy S5 ( $\alpha = 0.25$ ).



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