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Comparison of Some MCDM Techniques in a Hesitant Fuzzy Environment

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Abstract. Multi-criteria decision-making (MCDM) often involves situations characterized by uncertainty, ambiguity, and vagueness. To address such complexities, MCDM techniques play a crucial role. This paper presents a comparative analysis of two widely used methods—Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR)—within a hesitant fuzzy environment. Hesitant fuzzy sets allow decision-makers to express hesitation by assigning multiple possible membership values to an element rather than a single value. In this framework, the TOPSIS ranks alternatives based on their closeness to the positive and negative ideal solutions, while the VIKOR identifies a compromise solution by balancing individual and collective regret measures. The effectiveness of the comparison is demonstrated through illustrative numerical examples. Moreover, some real life applications of these methods are discussed.

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1 Introduction

In daily life, individuals frequently face decision-making situations ranging from simple choices to complex problems that require systematic evaluation methods. Multi-Criteria Decision-Making (MCDM) provides structured approaches for addressing such problems and plays an important role in situations involving multiple, often conflicting, criteria. These methods have been widely applied in fields such as economics, engineering, environmental studies, and management science to support decision-makers in selecting the most appropriate alternative among competing options. Over time, several MCDM techniques have been developed, including the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) proposed by Hwang and Yoon [14], the VIKOR method introduced by Opricovic and Tzeng [19], the Analytic Hierarchy Process (AHP) developed by Saaty [21], and the ELimination and Choice Expressing REality (ELECTRE) method proposed by Benayoun et al. [9].

Fuzzy set theory was introduced to address uncertainty, imprecision, and vagueness inherent in real-world data and has found extensive applications in control systems, decision-making, artificial intelligence, and medical diagnosis. It is particularly effective for modeling linguistic variables and situations where precise numerical boundaries are difficult to define. To handle more complex forms of uncertainty, several generalizations of fuzzy sets have been proposed. Atanassov [3] introduced intuitionistic fuzzy sets (IFS) in 1985 by incorporating both membership and non-membership degrees. Later, Torra [24] proposed hesitant fuzzy sets (HFS) in 2009, allowing decision-makers to express hesitation by assigning multiple possible membership values rather than a single one.

Among classical MCDM techniques, VIKOR and TOPSIS are well-known methods for identifying ideal solutions. VIKOR focuses on obtaining a compromise solution in the presence of conflicting criteria, whereas TOPSIS ranks alternatives based on their relative closeness to the ideal solution and distance from the negative-ideal solution. Opricovic and Tzeng [19] compared VIKOR with TOPSIS and ELECTRE, highlighting the advantages and limitations of each approach and emphasizing the effectiveness of VIKOR in resolving compromise-based decision problems.

Subsequent research extended these methods to more complex fuzzy environments. Interval-valued intuitionistic fuzzy and hesitant fuzzy variants of MCDM methods were developed to address higher levels of uncertainty. Hesitant fuzzy sets, in particular, provide an effective mechanism for representing vagueness and hesitation in human judgment, although their application in MCDM remains comparatively limited. Liao and Xu [15, 16, 17] proposed several hesitant fuzzy decision-making frameworks, including VIKOR-based approaches, highlighting their effectiveness in handling uncertainty and hesitation. Verma and Sharma [25] introduced new operational laws for HFSs to enhance decision-making capabilities, while Torra and Narukawa [18] contributed further conceptual developments. Zhang and Wu [29] investigated weighted fuzzy sets in MCDM, emphasizing the role of criterion weights in hesitant fuzzy environments. Hansen and Devlin [13] explored applications of multi-criteria decision analysis in healthcare, and Garg et al. [12] proposed a hesitant fuzzy decision framework integrating TOPSIS with the Choquet integral. Zhang and Wei [28] further extended the VIKOR method for decision-making problems involving HFSs.

Recent theoretical developments have also focused on hesitant fuzzy equations and algebraic structures that support advanced modeling of decision uncertainty. Babakordi and Taghi-Nezhad [5] introduced hesitant fuzzy equations and analyzed market equilibrium prices under hesitant fuzzy environments. Babakordi and Allahviranloo [4] proposed solution techniques for hesitant fuzzy systems of linear equations, while Taghi-Nezhad and Babakordi [23] developed fully hesitant parametric fuzzy equations extending classical fuzzy models. Further analytical properties of hesitant fuzzy equation systems were studied by Babakordi et al. [6]. Additionally, Babakordi [7, 8] proposed new arithmetic operations on generalized trapezoidal hesitant fuzzy numbers and introduced new classes of hesitant fuzzy soft sets, contributing to the mathematical foundations of hesitant fuzzy theory and enabling more flexible multi-criteria decision modeling.

In recent years, significant progress has been made in hybrid fuzzy MCDM techniques that incorporate hesitant fuzzy information to better capture decision uncertainty. Akram and Ali [2] proposed a hybrid hesitant fuzzy MCDM framework for system selection problems, while Ferrara, et al. [11] developed a hesitant fuzzy expert-based decision model for analyzing complex financial environments. Abdel-Basset et al. [1] introduced a hybrid fuzzy MCDM scheme combining MEREC-G and RATMI methods, and Saha et al. [22] proposed a consensus-based MULTIMOORA framework under a probabilistic hesitant fuzzy environment for manufacturing vendor selection.

The present study compares the TOPSIS and VIKOR methods within a hesitant fuzzy framework, aiming to systematically analyze their relative methodological behavior under identical hesitant fuzzy settings and to provide practical guidance for selecting appropriate techniques in real-world MCDM applications, rather than proposing a new theoretical model.

The remainder of this paper is organized as follows. Section 2 introduces the fundamental concepts of hesitant fuzzy elements. Section 3 describes the HF-VIKOR method and the TOPSIS technique for multi-criteria decision-making. Section 4 presents numerical illustrations for both MCDM techniques. Section 5 provides a comparative analysis of the two MCDM methods, and Section 6 concludes the paper.

2 Preliminaries

Before presenting the comparative analysis, it is necessary to outline the fundamental concepts that form the basis of this study. This section reviews essential definitions and mathematical formulations related to hesitant fuzzy elements (HFEs), which constitute the core component of the hesitant fuzzy environment employed throughout the subsequent methodology and analysis.

2.1 Some Basic Definitions

Definition 1. [27] A fuzzy set $\tilde{A}$ in a universal set X is defined as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\},$$

where $\mu_{\tilde{A}}(x) : X \to [0,1]$ denotes the membership degree of x in $\tilde{A}$.

Definition 2. [10] A fuzzy set $\tilde{A}$ on $\mathbb{R}$ is called a fuzzy number if

1. $\tilde{A}$ is normal, i.e., $h(\tilde{A}) = 1$,
2. for every $\alpha \in (0,1]$, the α-cut of $\tilde{A}$ is a closed interval,
3. the support of $\tilde{A}$ is bounded.

Definition 3. [3] An intuitionistic fuzzy set (IFS) $\tilde{A}^I$ on a universal set X is defined as

$$\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)\rangle \mid x \in X\},$$

where $\mu_{\tilde{A}^I}(x)$ and $\nu_{\tilde{A}^I}(x)$ denote the membership and non-membership degrees, satisfying

$$0 \leq \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1.$$

The hesitation degree is given by

$$\pi_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x).$$

Definition 4. [18, 24] A hesitant fuzzy set (HFS) $\tilde{H}$ on a universal set X is defined as

$$\tilde{H} = \{\langle x, h_H(x)\rangle \mid x \in X\},$$

where each hesitant fuzzy element (HFE) is

$$h_H(x) = \{\gamma_1, \gamma_2, \ldots, \gamma_n\}, \quad \gamma_i \in [0,1].$$

Definition 5. [15] For an HFE h, the score function is defined as

$$s(h) = \frac{1}{l_h} \sum_{\gamma \in h} \gamma,$$

where l_h denotes the cardinality of h.

Example 1. Let $a = (0.1, 0.4, 0.7)$ and $b = (0.7, 0.2)$. Then

$$s(a) = \frac{0.1 + 0.4 + 0.7}{3} = 0.4, \quad s(b) = \frac{0.7 + 0.2}{2} = 0.45.$$

Since $s(a) < s(b)$, it follows that $b > a$.

Definition 6. [15] The variance function of an HFE h is defined as

$$v(h) = \frac{1}{l_h} \sqrt{\sum_{\gamma_i, \gamma_j \in h} (\gamma_i - \gamma_j)^2}.$$

Example 2. Let $a = (0.2, 0.3, 0.7)$ and $b = (0.7, 0.2)$. Then

$$v(a) = \frac{1}{3} \sqrt{(0.2 - 0.3)^2 + (0.2 - 0.7)^2 + (0.3 - 0.7)^2} = 0.14,$$

$$v(b) = \frac{1}{2} \sqrt{(0.7 - 0.2)^2} = 0.25.$$

Since $v(b) > v(a)$, it follows that $a > b$.

Definition 7. [15] The Manhattan distance between two HFEs a and b is defined as

$$d(a, b) = \frac{1}{l} \sum_{i=1}^{l} \left| a^{\sigma(i)} - b^{\sigma(i)} \right|,$$

where $a^{\sigma(i)}$ and $b^{\sigma(i)}$ denote the i-th smallest elements and l is the maximum cardinality.

Example 3. Let $a = (0.2, 0.3, 0.7)$ and $b = (0.7, 0.8, 0.5)$. Then

$$d(a, b) = \frac{1}{3} \left(|0.2 - 0.7| + |0.3 - 0.8| + |0.7 - 0.5| \right) = 0.4.$$

Definition 8. [26] For two adjusted HFEs a and b, let $h_{j(i)}$ denote the i-th smallest element of h_j, and let l be the maximum cardinality. The hesitant normalized Hamming and Euclidean distances are defined as

$$d_{hnh}(a, b) = \frac{1}{l} \sum_{i=1}^{l} \left| h_{1(i)} - h_{2(i)} \right|,$$

$$d_{hne}(a, b) = \sqrt{\frac{1}{l} \sum_{i=1}^{l} \left(h_{1(i)} - h_{2(i)} \right)^2}.$$

3 Algorithms

This section presents the algorithmic procedures of the HF-VIKOR and TOPSIS methods in the hesitant fuzzy environment.

3.1 Algorithm to Tackle the Problem Using the Hesitant Fuzzy VIKOR Approach

Building upon the conceptual framework described earlier, this subsection presents the algorithmic procedure of the hesitant fuzzy VIKOR approach for solving multi-criteria decision-making (MCDM) problems. The stepwise procedure is primarily adapted from the hesitant fuzzy VIKOR models proposed by Liao and Xu [15] and Zhang and Wei [28].

3.2 Algorithm to Tackle the Problem Using the Hesitant Fuzzy TOPSIS Approach

4 Numerical Examples

This section presents numerical examples to demonstrate the applicability and effectiveness of the proposed methods

4.1 HF-VIKOR

To further illustrate the practical applicability of the hesitant fuzzy VIKOR methodology described in the preceding subsection, this part presents numerical examples that demonstrate its step-by-step implementation. These examples show how the HF-VIKOR approach is applied to a multi-criteria decision-making problem and highlight the interpretation of results within a hesitant fuzzy environment.

Example 4. Consider an MCDM problem with hesitant fuzzy sets, where the set of alternatives is A = $\{A_1, A_2, A_3\}$ and the criteria are defined as C = $\{C_1, C_2, C_3\}$. The criterion weights are specified as $w = (0.4, 0.3, 0.3)$, as detailed in the table below.

Step 1: Construct the hesitant fuzzy decision matrix $H = [h_{ij}]$:

The hesitant fuzzy decision matrix is summarized in Table 1.

Step 2: Compute the average value for every HFE.

The mean value for HFE is given by

Algorithm 1 Hesitant Fuzzy VIKOR Approach

Input: Alternatives A_i $(i = 1, \dots, m)$, criteria C_j $(j = 1, \dots, n)$, weights w_j $(\sum w_j = 1)$, hesitant fuzzy evaluations H_{ij}.

Output: Compromise ranking and best alternative(s).

1. **Construct the hesitant fuzzy decision matrix [15, 28].** Define $H_{ij} = \{h_{ij}^1, \dots, h_{ij}^{k_{ij}}\} \subseteq [0, 1]$ and form $H = (H_{ij})$.
2. **Compute mean values [15].**
$$h_{ij} = \frac{1}{k_{ij}} \sum_{l=1}^{k_{ij}} h_{ij}^l.$$
3. **Normalize the matrix [28].** Let $h_j^{\text{min}} = \min_i h_{ij}$ and $h_j^{\text{max}} = \max_i h_{ij}$. Then
$$h'_{ij} = \begin{cases} \dfrac{h_{ij} - h_j^{\text{min}}}{h_j^{\text{max}} - h_j^{\text{min}}}, & \text{benefit,} \\ \dfrac{h_j^{\text{max}} - h_{ij}}{h_j^{\text{max}} - h_j^{\text{min}}}, & \text{cost.} \end{cases}$$
4. **Weighted normalized values [15, 28].**
$$v_{ij} = w_j h'_{ij}.$$
5. **Utility and regret measures [15].** Let
$$f_j = \begin{cases} \max_i v_{ij}, & \text{benefit,} \\ \min_i v_{ij}, & \text{cost.} \end{cases}$$
$$S_i = \sum_{j=1}^{n} (f_j - v_{ij}), \qquad R_i = \max_j |f_j - v_{ij}|.$$
6. **Compromise index [15, 28].** Let $S_{\text{min}}, S_{\text{max}}, R_{\text{min}}, R_{\text{max}}$ be the extreme values. Then
$$Q_i = v \frac{S_i - S_{\text{min}}}{S_{\text{max}} - S_{\text{min}}} + (1 - v) \frac{R_i - R_{\text{min}}}{R_{\text{max}} - R_{\text{min}}},$$
where $v \in [0, 1]$ (commonly $v = 0.5$). Clearly, $Q_i \in [0, 1]$ and smaller values indicate better performance.
7. **Ranking and compromise solution [28].** Rank alternatives increasingly by Q_i. The best alternative A_1 is accepted if (i) $Q(A_2) - Q(A_1) \geq \frac{1}{m-1}$, (ii) A_1 is also top-ranked by S_i or R_i.
If one of the above conditions is not satisfied, a set of compromise solutions may be proposed. Sensitivity analysis with respect to v can also be conducted to examine the robustness of the ranking.

Algorithm 2 Hesitant Fuzzy TOPSIS Method

Input: Set of alternatives $A = \{A_1, \ldots, A_m\}$, set of criteria $C = \{C_1, \ldots, C_n\}$, criteria weights w_j ($j = 1, \ldots, n$) with $w_j \geq 0$ and $\sum_{j=1}^{n} w_j = 1$, and hesitant fuzzy evaluations H_{ij}.
Output: Ranking of alternatives.

1. **Construct the hesitant fuzzy decision matrix [12].** For each alternative A_i under criterion C_j, express the evaluation as a hesitant fuzzy element

$$H_{ij} = \{h_{ij}^1, h_{ij}^2, \ldots, h_{ij}^{k_{ij}}\},$$

where $h_{ij}^l \in [0, 1]$. Form the matrix $H = (H_{ij})_{m \times n}$.

2. **Compute the mean score of each hesitant fuzzy element [12].** Transform each H_{ij} into a crisp value by

$$d_{ij} = \frac{1}{k_{ij}} \sum_{l=1}^{k_{ij}} h_{ij}^l.$$

Obtain the aggregated decision matrix $D = (d_{ij})$.

3. **Normalize the decision matrix [12].** Let

$$d_j^{\text{min}} = \min_i d_{ij}, \qquad d_j^{\text{max}} = \max_i d_{ij}.$$

For benefit criteria,

$$d'_{ij} = \frac{d_{ij} - d_j^{\text{min}}}{d_j^{\text{max}} - d_j^{\text{min}}}.$$

For cost criteria,

$$d'_{ij} = \frac{d_j^{\text{max}} - d_{ij}}{d_j^{\text{max}} - d_j^{\text{min}}}.$$

Denote the normalized matrix by $D' = (d'_{ij})$.

4. **Construct the weighted normalized matrix [12].**

$$v_{ij} = w_j \, d'_{ij}.$$

Form $V = (v_{ij})$.

5. **Determine the positive and negative ideal solutions [12].**

$$v_j^+ = \max_i v_{ij}, \qquad v_j^- = \min_i v_{ij}.$$

Define

$$A^+ = \{v_1^+, \ldots, v_n^+\}, \quad A^- = \{v_1^-, \ldots, v_n^-\}.$$

6. **Compute the separation measures [12].**

$$S_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}, \qquad S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}.$$

7. **Calculate the closeness coefficient [12].**

$$r_i = \frac{S_i^-}{S_i^+ + S_i^-}, \qquad i = 1, \ldots, m.$$

8. **Rank the alternatives [12].** Rank alternatives in descending order of r_i. The larger r_i, the better the alternative.

Table 1: Hesitant fuzzy decision matrix.

Alternatives / Criteria	C_1	C_2	C_3
A_1	$\{0.5, 0.4, 0.3\}$	$\{0.6, 0.8\}$	$\{0.3, 0.6, 0.9\}$
A_2	$\{0.4, 0.5\}$	$\{0.7, 0.2, 0.8\}$	$\{0.5, 0.8, 0.9\}$
A_3	$\{0.4, 0.8, 0.6\}$	$\{0.5, 0.7, 0.4\}$	$\{0.3, 0.6, 0.7\}$

$$h_{ij} = \frac{\sum_{k=1}^{n} h_k}{n},$$

where h_k are the elements of the hesitant fuzzy set.

For Criteria C_1 :

$$h_{11} = \frac{0.5 + 0.4 + 0.3}{3} = 0.4, \quad h_{21} = \frac{0.4 + 0.5}{2} = 0.45, h_{31} = \frac{0.4 + 0.8 + 0.6}{3} = 0.6.$$

For Criteria C_2 :

$$h_{12} = 0.7, \quad h_{22} = 0.5667, \quad h_{32} = 0.5333.$$

For Criteria C_3 :

$$h_{13} = 0.6, \quad h_{23} = 0.7333, \quad h_{33} = 0.5333.$$

The mean decision matrix is reported in Table 2.

Table 2: Mean decision matrix.

Alternatives	C_1	C_2	C_3
A_1	0.40	0.70	0.60
A_2	0.45	0.5667	0.7333
A_3	0.60	0.5333	0.5333

Step 3: Normalize the hesitant decision matrix.

To find the normalized decision matrix, first compute $h_j^{\min}$ and $h_j^{\max}$ for each criterion C_j, $j = 1, 2, 3$:

$$h_1^{\max} = 0.6, \; h_1^{\min} = 0.4, \qquad h_2^{\max} = 0.7, \; h_2^{\min} = 0.5333, \qquad h_3^{\max} = 0.7333, \; h_3^{\min} = 0.5333.$$

Assuming all criteria are benefit type, the normalized decision matrix is obtained as follows.

For Criteria C_1 :

$$h'_{11} = \frac{0.4 - 0.4}{0.6 - 0.4} = 0.00, \quad h'_{21} = \frac{0.45 - 0.4}{0.6 - 0.4} = 0.25, h'_{31} = \frac{0.6 - 0.4}{0.6 - 0.4} = 1.00.$$

For Criteria C_2 :

$$h'_{12} = 1.00, \quad h'_{22} = 0.20, \quad h'_{32} = 0.00.$$

For Criteria C_3 :

$$h'_{13} = 0.33, \quad h'_{23} = 1.00, \quad h'_{33} = 0.00.$$

Now, all the HFEs are normalized. The normalized hesitant decision matrix $H' = [h'_{ij}]$ is shown in Table 3.

Table 3: Normalized decision matrix.

Alternatives	C_1	C_2	C_3
A_1	0.00	1.00	0.33
A_2	0.25	0.20	1.00
A_3	1.00	0.00	0.00

Step 4: Find the weighted normalized decision matrix

To obtain the weighted normalized decision matrix, each normalized value is multiplied by its corresponding criterion weight

$$w_j = (0.4,\, 0.3,\, 0.3),$$

such that

$$v_{i1} = 0.4\, h'_{i1}, \quad v_{i2} = 0.3\, h'_{i2}, \quad v_{i3} = 0.3\, h'_{i3}, \quad i = 1, 2, 3.$$

Table 4 displays the weighted normalized decision matrix.

Table 4: Weighted normalized decision matrix.

Alternatives	C_1	C_2	C_3
A_1	0.00	0.30	0.10
A_2	0.10	0.06	0.30
A_3	0.40	0.00	0.00

Step 5: Compute the utility and the regret measure values.

The values of S_i and R_i are determined using the following formulas:

$$S_i = \sum_{j=1}^{m} (f_j - v_{ij}), \quad R_i = \max_j \{|f_j - v_{ij}|\},$$

where f_j represents the ideal value of criterion j. Since the all criteria are of benefit type, so the ideal values for the criteria are $C_1 = 0.40$, $C_2 = 0.30$, $C_3 = 0.30$ i.e., $f_j = \max_i v_{ij}$ for each criterion C_j.

For alternative A_1 :

$$S_1 = (0.4 - 0.0) + (0.3 - 0.3) + (0.3 - 0.1) = 0.60$$

$$R_1 = \max\{|0.4 - 0.0|\,,\ |0.3 - 0.3|\,,\ |0.3 - 0.1|\} = 0.4$$

For alternative A_2 :

$$S_2 = (0.4 - 0.1) + (0.3 - 0.06) + (0.3 - 0.3) = 0.54$$

$$R_2 = \max\{|0.4 - 0.1|\,,\ |0.3 - 0.06|\,,\ |0.3 - 0.3|\} = 0.3$$

For alternative A_3 :

$$S_3 = (0.4 - 0.4) + (0.3 - 0.0) + (0.3 - 0.0) = 0.60$$

$$R_3 = \max\{|0.0 - 0.4|\,,\ |0.3 - 0.0|\,,\ |0.3 - 0.0|\} = 0.3$$

The S_i and R_i for each alternative are reported in Table 5.

Table 5: Utility and regret measures.

Alternative	S_i	R_i
A_1	0.60	0.40
A_2	0.54	0.30
A_3	0.60	0.30

Step 6: Calculate the compromise index Q_i.

The compromise index Q_i is calculated as

$$Q_i = v \cdot \frac{S_i - S_{\text{min}}}{S_{\text{max}} - S_{\text{min}}} + (1 - v) \cdot \frac{R_i - R_{\text{min}}}{R_{\text{max}} - R_{\text{min}}},$$

where v is the weight assigned to maximum group utility. For this example, take $v = 0.5$:

$$S_{\text{min}} = 0.54, \quad S_{\text{max}} = 0.60, \quad R_{\text{min}} = 0.3, \quad R_{\text{max}} = 0.4$$

For alternative A_1 :

$$Q_1 = 0.5\left(\frac{0.60 - 0.54}{0.60 - 0.54}\right) + 0.5\left(\frac{0.4 - 0.3}{0.4 - 0.3}\right) = 1.00$$

For alternative A_2 :

$$Q_2 = 0.5\left(\frac{0.54 - 0.54}{0.60 - 0.54}\right) + 0.5\left(\frac{0.3 - 0.3}{0.4 - 0.3}\right) = 0.00$$

For alternative A_3 :

$$Q_3 = 0.5\left(\frac{0.60 - 0.54}{0.60 - 0.54}\right) + 0.5\left(\frac{0.3 - 0.3}{0.4 - 0.3}\right) = 0.50$$

The compromise index Q_i is summarized in Table 6.

Table 6: Compromise index values.

Alternative	Q_i
A_1	1.00
A_2	0.00
A_3	0.50

Step 7: Find the rank of alternatives and the compromise solution. From the calculations

$$Q_2 < Q_3 < Q_1, \quad S_2 < S_3 < S_1, \quad R_2 < R_3 < R_1.$$

The alternative A_2 with the smallest Q_2 is considered as the best choice among alternatives. To declare A_2 as a compromise solution, first check the condition of acceptability:

$$Q(A_3) - Q(A_2) \geq D_Q, \quad D_Q = \frac{1}{m-1}.$$

where $m = 3$ and $Q(A_2)$, $Q(A_3)$ represent the alternatives in the first and second positions in the ranking list, respectively.

$$Q(A_3) - Q(A_2) = 0.5 \geq 0.5, \quad D_Q = 0.5.$$

The condition is satisfied. A_2 is considered as compromise solution.

Table 7: Sensitivity analysis of the compromise index Q for selected values of the parameter v in Example 4.

v	$Q(A_1)$	$Q(A_2)$	$Q(A_3)$	Ranking (best $\rightarrow$ worst)
0.00	1.00	0.00	0.00	$A_2 \prec (A_3 \sim A_1)$
0.25	1.00	0.00	0.25	$A_2 \prec A_3 \prec A_1$
0.50	1.00	0.00	0.50	$A_2 \prec A_3 \prec A_1$
0.75	1.00	0.00	0.75	$A_2 \prec A_3 \prec A_1$
1.00	1.00	0.00	1.00	$A_2 \prec (A_3 \sim A_1)$

From Table 7, it is evident that the ranking of alternatives remains unchanged despite the change in parameter v. The variation of Q_i values for different values of v is illustrated in Figure 1 and reported in Table 7. The value of A_2 remains consistently the lowest, indicating that it is the best compromise choice under all decision-making settings. When v increases, the weight given to the group utility rises; therefore, $Q(A_3)$ gradually increases while $Q(A_1)$ remains constant. However, no crossover occurs, and A_2 continues to outperform both A_1 and A_3, demonstrating the robustness of the VIKOR decision in this hesitant fuzzy environment.

Example 5. Consider an MCDM problem with a set of four alternatives as $A = \{A_1, A_2, A_3, A_4\}$ and the criteria are defined as $C = \{C_1, C_2, C_3\}$. The criterion weights are specified as $w = (0.1, 0.4, 0.5)$, as detailed in below.

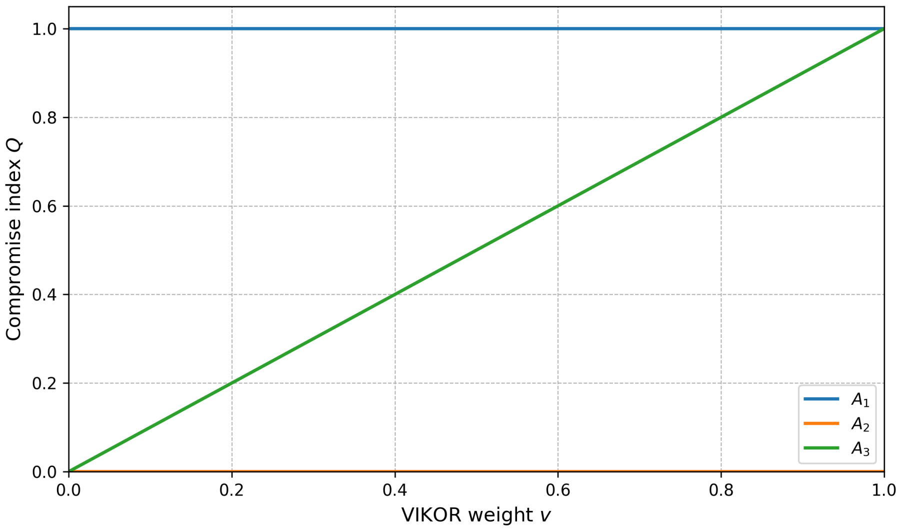

Figure 1: Sensitivity analysis of the compromise index (Q) for selected values of the VIKOR parameter (v) in Example 4.

Step 1: Construct the hesitant fuzzy decision matrix $H = [h_{ij}]$

The hesitant fuzzy decision matrix is summarized in Table 8.

Table 8: Hesitant fuzzy decision matrix showing the performance of each alternative under criteria C_1, C_2, and C_3.

Alternatives / Criteria	C_1	C_2	C_3
A_1	$\{0.6, 0.7, 0.2\}$	$\{0.7, 0.5\}$	$\{0.6, 0.7\}$
A_2	$\{0.1, 0.5\}$	$\{0.6, 0.7, 0.8\}$	$\{0.3, 0.5\}$
A_3	$\{0.4, 0.8, 0.7\}$	$\{0.3, 0.6\}$	$\{0.5, 0.8\}$
A_4	$\{0.7, 0.4\}$	$\{0.4, 0.5\}$	$\{0.5, 0.8, 0.9\}$

Step 2: Compute the average value for every HFE.

The mean value for HFE is given by

$$h_{ij} = \frac{\sum_{k=1}^{n} h_k}{n},$$

where h_k are the elements of the hesitant fuzzy set.

For Criteria C_1 :

$$h_{11} = \frac{0.6 + 0.7 + 0.2}{3} = 0.5, \quad h_{21} = \frac{0.1 + 0.5}{2} = 0.3,$$

$$h_{31} = \frac{0.4 + 0.8 + 0.7}{3} = 0.63, \quad h_{41} = \frac{0.7 + 0.4}{2} = 0.55.$$

For Criteria C_2 :

$$h_{12} = 0.6, \quad h_{22} = 0.7, \quad h_{32} = 0.45, \quad h_{42} = 0.45.$$

For Criteria C_3 :

$$h_{13} = 0.65, \quad h_{23} = 0.40, \quad h_{33} = 0.65, \quad h_{43} = 0.73.$$

The mean decision matrix is shown in Table 9.

Table 9: Mean values computed for the hesitant fuzzy evaluations of all alternatives.

Alternative	C_1	C_2	C_3
A_1	0.50	0.60	0.65
A_2	0.30	0.70	0.40
A_3	0.63	0.45	0.65
A_4	0.55	0.45	0.73

Step 3: Normalize the hesitant decision matrix.

To find the normalized decision matrix, first determine h_j^{min} and h_j^{max} for each criterion $C_j \ (j = 1, 2, 3)$:

$$h_1^{\text{max}} = 0.63, \ h_1^{\text{min}} = 0.30, \quad h_2^{\text{max}} = 0.70, \ h_2^{\text{min}} = 0.45, \quad h_3^{\text{max}} = 0.73, \ h_3^{\text{min}} = 0.40.$$

Assuming C_1 and C_2 are benefit criteria and C_3 is a cost criterion, the normalized decision matrix is obtained as follows.

For Criteria C_1 :

$$h'_{11} = \frac{0.50 - 0.30}{0.63 - 0.30} = 0.61, \quad h'_{21} = \frac{0.30 - 0.30}{0.63 - 0.30} = 0.00,$$

$$h'_{31} = \frac{0.63 - 0.30}{0.63 - 0.30} = 1.00, \quad h'_{41} = \frac{0.55 - 0.30}{0.63 - 0.30} = 0.76.$$

For Criteria C_2 :

$$h'_{12} = 0.60, \quad h'_{22} = 1.00, \quad h'_{32} = 0.00, \quad h'_{42} = 0.00.$$

For Criteria C_3 :

$$h'_{13} = 0.24, \quad h'_{23} = 1.00, \quad h'_{33} = 0.24, \quad h'_{43} = 0.00.$$

Now, all the HFEs are normalized. The normalized hesitant decision matrix $H' = [h'_{ij}]$ is summarized in Table 10.

Table 10: Normalized decision matrix for Example 5.

Alternative	C_1	C_2	C_3
A_1	0.61	0.60	0.24
A_2	0.00	1.00	1.00
A_3	1.00	0.00	0.24
A_4	0.76	0.00	0.00

Step 4: Find the weighted normalized decision matrix.

To obtain the weighted normalized decision matrix, each normalized value is multiplied by its corresponding criterion weight

$$w_j = (0.1,\ 0.4,\ 0.5),$$

such that

$$v_{i1} = 0.1\,h'_{i1}, \quad v_{i2} = 0.4\,h'_{i2}, \quad v_{i3} = 0.5\,h'_{i3}, \quad i = 1, 2, 3, 4.$$

Table 11 displays the weighted normalized decision matrix.

Table 11: Weighted normalized decision matrix obtained using the given criteria weights.

Alternative	C_1	C_2	C_3
A_1	0.061	0.24	0.12
A_2	0.000	0.40	0.50
A_3	0.100	0.00	0.12
A_4	0.076	0.00	0.00

Step 5: Compute the utility and the regret measure values.

The values of S_i and R_i are determined using the following formulas:

$$S_i = \sum_{j=1}^{m} (f_j - v_{ij}), \qquad R_i = \max_j \{|f_j - v_{ij}|\},$$

where f_j represents the ideal value of criterion j. Since the criterion C_1 and C_2 are of benefit type and C_3 is of cost type. So the ideal values for the criteria are: $C_1 = 0.10$, $C_2 = 0.40$, $C_3 = 0.00$. That is, $f_j = \max_i v_{ij}$ for benefit criteria $(j = 1, 2)$ and $f_j = \min_i v_{ij}$ for the cost criterion $(j = 3)$.

For alternative A_1 :

$$S_1 = (0.100 - 0.061) + (0.40 - 0.24) + (0.00 - 0.12) = 0.076.$$

$$R_1 = \max\{|0.100 - 0.061|\,,\ |0.40 - 0.24|\,,\ |0.00 - 0.12|\} = 0.16.$$

For alternative A_2 :

$$S_2 = (0.10 - 0.00) + (0.40 - 0.40) + (0.00 - 0.50) = -0.40.$$

$$R_2 = \max\{|0.10 - 0.00|\,,\ |0.40 - 0.40|\,,\ |0.00 - 0.50|\} = 0.50.$$

For alternative A_3 :

$$S_3 = (0.10 - 0.10) + (0.40 - 0.00) + (0.00 - 0.12) = 0.28.$$

$$R_3 = \max\{|0.10 - 0.10|\,,\ |0.40 - 0.00|\,,\ |0.00 - 0.12|\} = 0.40.$$

For alternative A_4 :

$$S_4 = (0.100 - 0.076) + (0.40 - 0.00) + (0.00 - 0.00) = 0.424.$$

$$R_4 = \max\{|0.100 - 0.076|\,,\ |0.40 - 0.00|\,,\ |0.00 - 0.00|\} = 0.40$$

It is noted that the utility measure S_i may assume negative values in the presence of cost criteria, since S_i is computed relative to the ideal solution. The S_i and R_i for each alternative are reported in Table 12.

Table 12: Utility (S_i) and regret (R_i) measures for each alternative.

Alternative	S_i	R_i
A_1	0.076	0.16
A_2	-0.40	0.50
A_3	0.28	0.40
A_4	0.424	0.40

Step 6: Calculate the compromise index Q_i.

The compromise index Q_i is calculated as

$$Q_i = v \cdot \frac{S_i - S_{\text{min}}}{S_{\text{max}} - S_{\text{min}}} + (1 - v) \cdot \frac{R_i - R_{\text{min}}}{R_{\text{max}} - R_{\text{min}}},$$

where v is the weight assigned to maximum group utility. For this example, take $v = 0.5$.

$$S_{\text{min}} = -0.400, \quad S_{\text{max}} = 0.424, \quad R_{\text{min}} = 0.16, \quad R_{\text{max}} = 0.50.$$

For alternative A_1 :

$$Q_1 = 0.5\left(\frac{0.076 + 0.400}{0.424 + 0.400}\right) + 0.5\left(\frac{0.16 - 0.16}{0.50 - 0.16}\right) = 0.289.$$

For alternative A_2 :

$$Q_2 = 0.5\left(\frac{-0.400 + 0.400}{0.424 + 0.400}\right) + 0.5\left(\frac{0.50 - 0.16}{0.50 - 0.16}\right) = 0.5.$$

For alternative A_3 :

$$Q_3 = 0.5\left(\frac{0.280 + 0.400}{0.424 + 0.400}\right) + 0.5\left(\frac{0.40 - 0.16}{0.50 - 0.16}\right) = 0.765.$$

For alternative A_4 :

$$Q_4 = 0.5\left(\frac{0.424 + 0.400}{0.424 + 0.400}\right) + 0.5\left(\frac{0.40 - 0.16}{0.50 - 0.16}\right) = 0.853.$$

The compromise index Q_i is summarized in Table 13.

Table 13: Compromise index Q_i values for all alternatives.

Alternative	Q_i
A_1	0.289
A_2	0.5
A_3	0.765
A_4	0.853

Step 7: Find the rank of alternatives and the compromise solution.

From the above calculations, the ranking is obtained as

$$Q_1 < Q_2 < Q_3 < Q_4, \quad S_2 < S_1 < S_3 < S_4, \quad R_1 < R_3 = R_4 < R_2.$$

The alternative A_1 with the smallest Q_1 is considered the best choice among A_1, A_2, A_3, A_4. To declare A_1 as a compromise solution, we check the acceptable condition:

$$Q(A_2) - Q(A_1) \geq D_Q, \quad D_Q = \frac{1}{m-1},$$

where $m = 4$ and $Q(A_1)$, $Q(A_2)$ represent the alternatives in the first and second positions in the ranking list, respectively. Substituting the values:

$$Q(A_2) - Q(A_1) = 0.211 < 0.33, \quad D_Q = 0.33.$$

Since the condition is not satisfied, the alternatives A_1 and A_2 are considered the compromise solutions.

The variation of Q_i values for different values of v is illustrated in Figure 2 and reported in Table 14. The results in Table 14 clearly show that the compromise index Q is sensitive

Table 14: Sensitivity analysis of the compromise index Q for selected values of the parameter v in Example 5.

v	$Q(A_1)$	$Q(A_2)$	$Q(A_3)$	$Q(A_4)$	Ranking (best $\to$ worst)
0.00	0.000	1.00	0.706	0.706	$A_1 \prec (A_3 \sim A_4) \prec A_2$
0.25	0.145	0.875	0.736	0.804	$A_1 \prec A_3 \prec A_4 \prec A_2$
0.50	0.289	0.5	0.765	0.853	$A_1 \prec A_2 \prec A_3 \prec A_4$
0.75	0.434	0.125	0.794	0.902	$A_2 \prec A_1 \prec A_3 \prec A_4$
1.00	0.579	0.000	0.824	1.00	$A_2 \prec A_1 \prec A_3 \prec A_4$

to the choice of the parameter v. When v is low, the decision is driven mainly by regret (R_i) and alternative A_2 becomes the preferred option. As v increases, the weight of group utility (S_i) dominates, causing A_1 to gradually emerge as the optimal solution. A crossover shift is observed near $v \approx 0.40$, indicating that the final decision strongly depends on the decision maker's attitude toward utility versus regret.

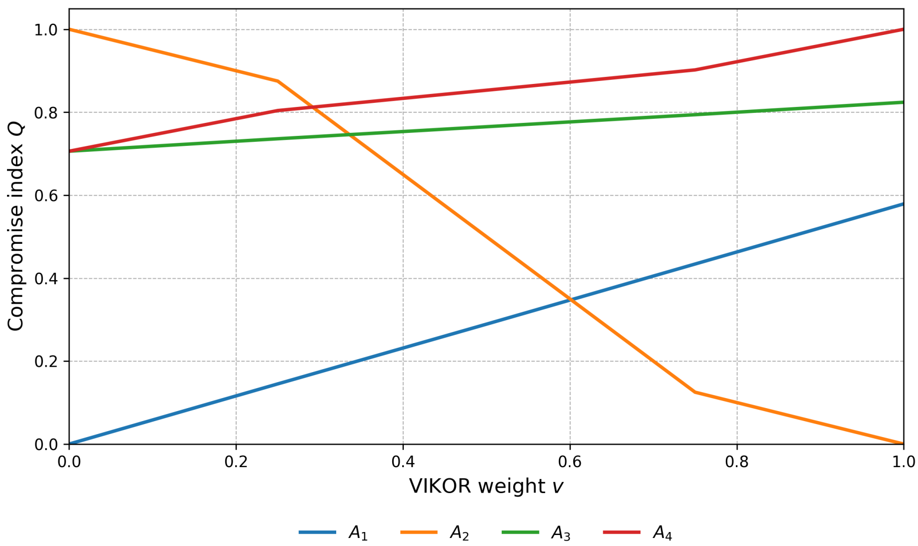

Figure 2: Sensitivity analysis of the compromise index (Q) for selected values of the VIKOR parameter (v) in Example 5.

Example 6. Consider an MCDM problem with set of 3 alternatives as $A = \{A_1, A_2, A_3\}$ and the criteria are defined as $C = \{C_1, C_2, C_3\}$. The criterion weights are specified as $w = (0.4, 0.35, 0.25)$, as detailed in below. The hesitant decision matrix is given in Table 15.

Following the same computational steps described in Examples 4 and 5—including aggregation of hesitant fuzzy elements, normalization of benefit and cost criteria, weighting of criteria, and evaluation of the utility measure S_i, regret measure R_i, and compromise index Q_i—the final decision results are obtained.

Table 15: Hesitant fuzzy decision matrix for Example 6.

Alternatives / Criteria	C_1	C_2	C_3
A_1	$\{0.7, 0.8, 0.9\}$	$\{0.5, 0.6, 0.7\}$	$\{0.4, 0.5\}$
A_2	$\{0.6, 0.7\}$	$\{0.7, 0.8, 0.9\}$	$\{0.5, 0.6, 0.7\}$
A_3	$\{0.8, 0.9\}$	$\{0.6, 0.7\}$	$\{0.3, 0.4, 0.5\}$

The computed values of the weighted normalized decision matrix, utility measure S_i, regret measure R_i, and compromise index Q_i are summarized in Tables 16–18.

Table 16: Weighted normalized decision matrix produced by combining normalized values with the assigned criteria weights.

Alternatives / Criteria	C_1	C_2	C_3
A_1	0.30	0.00	0.188
A_2	0.00	0.35	0.00
A_3	0.4	0.087	0.25

Table 17: Utility and regret measures for each alternative.

Alternatives	S_i	R_i
A_1	0.262	0.35
A_2	0.40	0.40
A_3	0.013	0.263

Based on the compromise index values, the ranking of alternatives is obtained as

$$Q_3 < Q_1 < Q_2,$$

indicating that alternative A_3 is the most preferred option.

The acceptable advantage condition is satisfied, and therefore A_3 is identified as the compromise solution for this decision problem.

To examine the robustness of the ranking results, a sensitivity analysis of the compromise index Q_i with respect to the VIKOR parameter v is conducted. The variation of Q_i values for different values of v is illustrated in Figure 3 and reported in Table 19. The results demonstrate that although the numerical values of Q_i vary slightly with changes in v, the ranking of alternatives remains unchanged. This confirms the stability and robustness of the proposed hesitant fuzzy VIKOR method.

Example 7. Let us consider a real-life MCDM problem under hesitant fuzzy sets, where a farmer aims to select the most suitable fertilizer for rice cultivation. Let $A = \{A_1, A_2, A_3\}$

Table 18: Compromise index Q_i values for all alternatives.

Alternatives	Q_i
A_1	0.6392
A_2	1.00
A_3	0.00

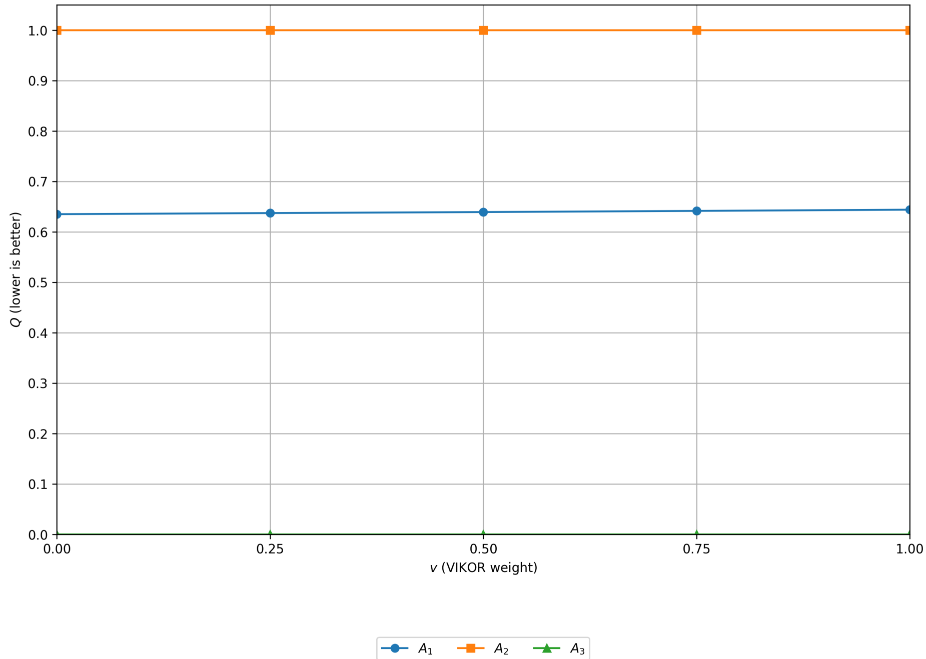

Figure 3: Sensitivity analysis of the compromise index (Q) for selected values of the VIKOR parameter (v) in Example 6.

denote the set of alternatives, where A_1 represents a urea-based fertilizer, A_2 denotes organic compost, and A_3 corresponds to an NPK mixed fertilizer. The criteria set is defined as $C = \{C_1, C_2, C_3\}$, where C_1 represents the expected increase in crop yield (benefit criterion), C_2 denotes soil health improvement (benefit criterion), and C_3 corresponds to application cost (cost criterion). The criterion weights are assigned by domain experts as $w = (0.45, 0.35, 0.20)$. The hesitant fuzzy decision matrix reflecting expert evaluations is presented below.

The hesitant fuzzy decision matrix constructed based on expert evaluations is reported in Table 20.

Following the same HF–VIKOR computational procedure described in Example 4 - including mean aggregation of hesitant fuzzy elements, normalization, weighting, and computation of the utility measure S_i, regret measure R_i, and compromise index Q_i—the final results are obtained.

Table 19: Sensitivity analysis of the compromise index Q for selected values of the parameter v Example 6.

v	$Q(A_1)$	$Q(A_2)$	$Q(A_3)$	Ranking (best $\rightarrow$ worst)
0.00	0.6350	1.00	0.00	$A_3 \prec A_1 \prec A_2$
0.25	0.6372	1.00	0.00	$A_3 \prec A_1 \prec A_2$
0.50	0.6392	1.00	0.00	$A_3 \prec A_1 \prec A_2$
0.75	0.6415	1.00	0.00	$A_3 \prec A_1 \prec A_2$
1.00	0.6439	1.00	0.00	$A_3 \prec A_1 \prec A_2$

Table 20: Hesitant fuzzy decision matrix showing the interval-valued performance of each alternative under criteria C_1, C_2, C_3.

Alternatives / Criteria	C_1	C_2	C_3
A_1	$\{0.6, 0.7, 0.8\}$	$\{0.4, 0.5, 0.6\}$	$\{0.6, 0.7\}$
A_2	$\{0.7, 0.8, 0.9\}$	$\{0.7, 0.8\}$	$\{0.3, 0.4\}$
A_3	$\{0.5, 0.6\}$	$\{0.6, 0.7, 0.8\}$	$\{0.4, 0.5, 0.6\}$

Table 21 reports the compromise index values for all alternatives. The ranking obtained is

$$Q_2 < Q_1 < Q_3,$$

indicating that alternative A_2 (organic compost) is the most preferred option.

The acceptable advantage condition is satisfied, and therefore A_2 is identified as the compromise solution.

To examine the robustness of the decision, a sensitivity analysis of the compromise index Q_i with respect to the VIKOR parameter v is performed. The variation of Q_i values is illustrated in Figure 4 and summarized in Table 22. The results show that A_2 consistently achieves the minimum Q_i value for all values of $v \in [0, 1]$, confirming the stability and robustness of the proposed hesitant fuzzy VIKOR decision outcome.

4.2 HF-TOPSIS

To demonstrate the practical implementation of the hesitant fuzzy TOPSIS methodology discussed in the previous subsection, this part provides illustrative numerical examples. These examples show how the TOPSIS approach is applied to an MCDM problem under hesitant fuzzy information and help interpret and validate the obtained rankings.

Example 8. Let us consider the MCDM problem hesitant fuzzy sets, let $A = \{A_1, A_2, A_3\}$ denotes the set of alternatives and $C = \{C_1, C_2, C_3\}$ represents the criteria and consider the criterion weights $w = (0.4, 0.3, 0.3)$ with the information given below.

Table 21: Compromise index Q_i values for all alternatives.

Alternatives	Q_i
A_1	0.80
A_2	0.00
A_3	0.925

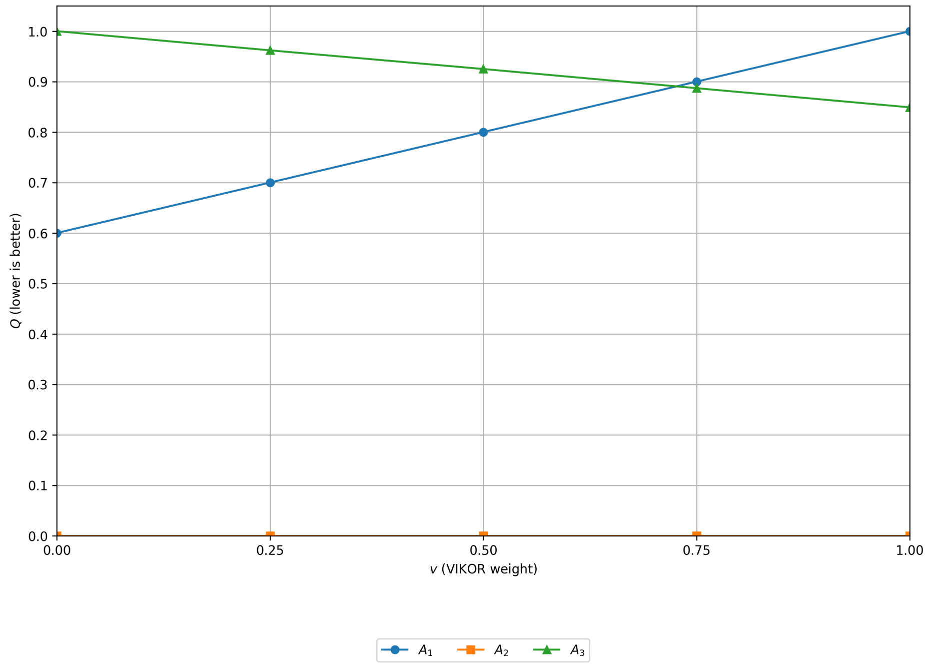

Figure 4: Sensitivity analysis of the compromise index (Q) for selected values of the VIKOR parameter (v) in Example 7.

Step 1: Hesitant fuzzy decision matrix.

The hesitant fuzzy decision matrix is denoted by $H = [h_{ij}]$ and is given in Table 23.

Step 2: Mean value of hesitant fuzzy elements

The mean value of each hesitant fuzzy element (HFE) is computed as

$$d_{ij} = \frac{1}{n} \sum_{k=1}^{n} h_k,$$

where h_k are the elements of the corresponding hesitant fuzzy set.

For C_1 :

$$d_{11} = 0.40,\ d_{21} = 0.45,\ d_{31} = 0.60,$$

For C_2 :

Table 22: Sensitivity analysis of the compromise index Q_i with respect to parameter v Example 7.

v	$Q(A_1)$	$Q(A_2)$	$Q(A_3)$	Ranking (best $\rightarrow$ worst)
0.00	0.60	0.00	1.00	$A_2 \prec A_1 \prec A_3$
0.25	0.70	0.00	0.962	$A_2 \prec A_1 \prec A_3$
0.50	0.80	0.00	0.925	$A_2 \prec A_1 \prec A_3$
0.75	0.90	0.00	0.887	$A_2 \prec A_1 \prec A_3$
1.00	1.00	0.00	0.849	$A_2 \prec A_3 \prec A_1$

Table 23: Hesitant fuzzy decision matrix for the set of alternatives and criteria.

Alternatives / Criteria	C_1	C_2	C_3
A_1	$\{0.5, 0.4, 0.3\}$	$\{0.6, 0.8\}$	$\{0.3, 0.6, 0.9\}$
A_2	$\{0.4, 0.5\}$	$\{0.7, 0.2, 0.8\}$	$\{0.5, 0.8, 0.9\}$
A_3	$\{0.4, 0.8, 0.6\}$	$\{0.5, 0.7, 0.4\}$	$\{0.3, 0.6, 0.7\}$

$$d_{12} = 0.70,\ d_{22} = 0.5667,\ d_{32} = 0.5333,$$

For C_3 :

$$d_{13} = 0.60,\ d_{23} = 0.7333,\ d_{33} = 0.5333.$$

The mean value decision matrix is presented in Table 24.

Table 24: Mean values of hesitant fuzzy elements for each criterion.

Alternatives / Criteria	C_1	C_2	C_3
A_1	0.4	0.7	0.6
A_2	0.45	0.5667	0.7333
A_3	0.6	0.5333	0.5333

Step 3: Normalization of the decision matrix

The minimum and maximum values for each criterion are

For C_1 :

$$d_1^{\min} = 0.40,\ d_1^{\max} = 0.60,$$

For C_2 :

$$d_2^{\min} = 0.5333,\ d_2^{\max} = 0.70,$$

For C_3 :

$$d_3^{\min} = 0.5333,\ d_3^{\max} = 0.7333.$$

Since all criteria are benefit criteria, min–max normalization is applied:

$$d'_{ij} = \frac{d_j^{\max} - d_{ij}}{d_j^{\max} - d_j^{\min}}.$$

For C_1 :

$$d'_{11} = 1.00,\ d'_{21} = 0.75,\ d'_{31} = 0.00,$$

For C_2 :

$$d'_{12} = 0.00,\ d'_{22} = 0.81,\ d'_{32} = 1.00,$$

For C_3 :

$$d'_{13} = 0.33,\ d'_{23} = 1.00,\ d'_{33} = 0.00.$$

The normalized decision matrix is reported in Table 25.

Table 25: Normalized hesitant fuzzy decision matrix.

Alternatives / Criteria	C_1	C_2	C_3
A_1	1.00	0.00	0.33
A_2	0.75	0.81	1.00
A_3	0.00	1.00	0.00

Step 4: Weighted normalized decision matrix

The weighted normalized values are obtained as

$$v_{ij} = w_j\, d'_{ij}.$$

$$v_{i1} = 0.4\, d'_{i1}, \quad v_{i2} = 0.3\, d'_{i2}, \quad v_{i3} = 0.3\, d'_{i3}, \quad i = 1, 2, 3.$$

The weighted normalized decision matrix is presented in Table 26.

Table 26: Weighted normalized decision matrix.

Alternatives / Criteria	C_1	C_2	C_3
A_1	0.40	0.00	0.099
A_2	0.30	0.243	0.30
A_3	0.00	0.30	0.00

Step 5: Ideal solutions

$$A^+ = \{0.40, 0.30, 0.30\}, A^- = \{0.00, 0.00, 0.00\}.$$

Step 6: Separation measures

$$S_i^+ = \sqrt{\sum_{j=1}^{3}(v_{ij} - d_j^+)^2}, \qquad S_i^- = \sqrt{\sum_{j=1}^{3}(v_{ij} - d_j^-)^2}.$$

For A_1 :

$$S_1^+ = 0.3611,\ S_1^- = 0.4121,$$

For A_2 :

$$S_2^+ = 0.1149,\ S_2^- = 0.4889,$$

For A_3 :

$$S_3^+ = 0.50,\ S_3^- = 0.30.$$

The separation measure is reported in Table 27.

Table 27: Separation measures for the alternatives.

Alternatives	S_i^+	S_i^-
A_1	0.3611	0.4121
A_2	0.1149	0.4889
A_3	0.50	0.30

Step 7: Relative closeness.

$$r_i = \frac{S_i^-}{S_i^+ + S_i^-}.$$

$$r_1 = 0.533, \quad r_2 = 0.810, \quad r_3 = 0.375.$$

The closeness degree is shown in Table 28.

Table 28: Relative closeness values of the alternatives.

Alternatives	r_i (Relative closeness)
A_1	0.533
A_2	0.81
A_3	0.375

Thus, the ranking is

$$A_3 < A_1 < A_2,$$

and A_2 is the best alternative.

Example 9. Let us consider the MCDM problem hesitant fuzzy sets, let $A = \{A_1, A_2, A_3, A_4\}$ denotes the set of alternatives and $C = \{C_1, C_2, C_3\}$ represents the criteria and consider the criterion weights $w = (0.1, 0.4, 0.5)$ with the information given below.

Step 1: Hesitant fuzzy decision matrix.

The hesitant fuzzy decision matrix is denoted by $H = [h_{ij}]$, and is presented in Table 29.

Table 29: Hesitant fuzzy decision matrix for the set of alternatives and criteria.

Alternatives / Criteria	C_1	C_2	C_3
A_1	$\{0.6, 0.7, 0.2\}$	$\{0.7, 0.5\}$	$\{0.6, 0.7\}$
A_2	$\{0.1, 0.5\}$	$\{0.6, 0.7, 0.8\}$	$\{0.3, 0.5\}$
A_3	$\{0.4, 0.8, 0.7\}$	$\{0.3, 0.6\}$	$\{0.5, 0.8\}$
A_4	$\{0.7, 0.4\}$	$\{0.4, 0.5\}$	$\{0.5, 0.8, 0.9\}$

Step 2: Mean value of hesitant fuzzy elements.

The mean value of each hesitant fuzzy element (HFE) is computed as

$$d_{ij} = \frac{1}{n} \sum_{k=1}^{n} h_k,$$

where h_k are the elements of the corresponding hesitant fuzzy set.

For criterion C_1 :

$$d_{11} = 0.50, \; d_{21} = 0.30, \; d_{31} = 0.63, \; d_{41} = 0.55,$$

For criterion C_2 :

$$d_{12} = 0.60, \; d_{22} = 0.70, \; d_{32} = 0.45, \; d_{42} = 0.45,$$

For criterion C_3 :

$$d_{13} = 0.65, \; d_{23} = 0.40, \; d_{33} = 0.65, \; d_{43} = 0.73.$$

The resulting mean value decision matrix is shown in Table 30.

Table 30: Mean values of hesitant fuzzy elements for each criterion.

Alternatives / Criteria	C_1	C_2	C_3
A_1	0.50	0.60	0.65
A_2	0.30	0.70	0.40
A_3	0.63	0.45	0.65
A_4	0.55	0.45	0.73

Step 3: Normalization of the decision matrix.

The minimum and maximum values for each criterion are computed as follows:

For C_1:

$$d_1^{\min} = 0.30,\ d_1^{\max} = 0.63,$$

For C_2:

$$d_2^{\min} = 0.45,\ d_2^{\max} = 0.70,$$

For C_3:

$$d_3^{\min} = 0.40,\ d_3^{\max} = 0.73.$$

Assume that C_1 and C_2 are benefit criteria, while C_3 is a cost criterion. Using min–max normalization, the normalized decision matrix is obtained as:

For criterion C_1 :

$$d'_{11} = 0.39,\ d'_{21} = 1.00,\ d'_{31} = 0.00,\ d'_{41} = 0.24,$$

For criterion C_2 :

$$d'_{12} = 0.40,\ d'_{22} = 0.00,\ d'_{32} = 1.00,\ d'_{42} = 1.00,$$

For criterion C_3 :

$$d'_{13} = 0.76,\ d'_{23} = 0.00,\ d'_{33} = 0.76,\ d'_{43} = 1.00.$$

The normalized hesitant fuzzy decision matrix $H' = [d'_{ij}]$ is provided in Table 31.

Table 31: Normalized hesitant fuzzy decision matrix.

Alternatives / Criteria	C_1	C_2	C_3
A_1	0.39	0.40	0.76
A_2	1.00	0.00	0.00
A_3	0.00	1.00	0.76
A_4	0.24	1.00	1.00

Step 4: Weighted normalized decision matrix.

The weighted normalized values are computed by $v_{ij} = w_j\, d'_{ij}$ where $w = (0.1, 0.4, 0.5)$.

$$v_{i1} = 0.1\, d'_{i1}, \quad v_{i2} = 0.4\, d'_{i2}, \quad v_{i3} = 0.5\, d'_{i3}, \quad i = 1, 2, 3, 4.$$

The weighted normalized decision matrix is reported in Table 32.

Table 32: Weighted normalized decision matrix.

Alternatives / Criteria	C_1	C_2	C_3
A_1	0.039	0.16	0.38
A_2	0.10	0.00	0.00
A_3	0.00	0.40	0.38
A_4	0.024	0.40	0.5

Step 5: Positive and negative ideal solutions.

The positive ideal solution (PIS) and negative ideal solution (NIS) are defined as

$$A^+ = \{d_1^+, d_2^+, d_3^+\}, A^- = \{d_1^-, d_2^-, d_3^-\}.$$

Accordingly,

$$A^+ = \{0.10,\ 0.40,\ 0.00\}, A^- = \{0.00,\ 0.00,\ 0.50\}.$$

Step 6: Separation measures.

The separation distances from the ideal and negative ideal solutions are computed as

$$S_i^+ = \sqrt{\sum_{j=1}^{3}(v_{ij} - d_j^+)^2}, \qquad S_i^- = \sqrt{\sum_{j=1}^{3}(v_{ij} - d_j^-)^2}.$$

For A_1 :

$$S_1^+ = 0.4536,\ S_1^- = 0.2037,$$

For A_2 :

$$S_2^+ = 0.4536,\ S_2^- = 0.5099,$$

For A_3 :

$$S_3^+ = 0.3929,\ S_3^- = 0.4176,$$

For A_4 :

$$S_4^+ = 0.5057,\ S_4^- = 0.4007.$$

The separation measures are summarized in Table 33.

Step 7: Relative closeness coefficient.

The relative closeness degree of each alternative is calculated as

$$r_i = \frac{S_i^-}{S_i^+ + S_i^-}, \quad i = 1, 2, 3, 4.$$

Table 33: Separation measures for the alternatives.

Alternatives	S_i^+	S_i^-
A_1	0.4536	0.2037
A_2	0.40	0.5099
A_3	0.3929	0.4176
A_4	0.5057	0.400

Table 34: Relative closeness values of the alternatives.

Alternatives	r_i (Relative closeness)
A_1	0.3099
A_2	0.5603
A_3	0.5152
A_4	0.4420

$$r_1 = 0.3099, \quad r_2 = 0.5603, \quad r_3 = 0.5152, \quad r_4 = 0.4420.$$

The closeness coefficients are reported in Table 34.

Based on the descending order of r_i, the ranking of alternatives is

$$A_2 > A_3 > A_4 > A_1.$$

Hence, A_2 is identified as the best alternative.

Example 10. Let us consider the MCDM problem hesitant fuzzy sets, let $A = \{A_1, A_2, A_3\}$ denotes the set of alternatives and $C = \{C_1, C_2, C_3\}$ represents the criteria and consider the criterion weights $w = (0.4, 0.35, 0.25)$ with the information given below.

The hesitant decision matrix is given in Table 35 Following the same computational proce-

Table 35: Hesitant fuzzy decision matrix for the set of alternatives and criteria.

Alternatives / Criteria	C_1	C_2	C_3
A_1	$\{0.7, 0.8, 0.9\}$	$\{0.5, 0.6, 0.7\}$	$\{0.4, 0.5\}$
A_2	$\{0.6, 0.7\}$	$\{0.7, 0.8, 0.9\}$	$\{0.5, 0.6, 0.7\}$
A_3	$\{0.8, 0.9\}$	$\{0.6, 0.7\}$	$\{0.3, 0.4, 0.5\}$

dure described in Examples 8 and 9, the hesitant fuzzy elements are first transformed into their mean values, normalized using the min–max approach, and subsequently weighted according to the criterion weights. For brevity, intermediate computational steps are omitted.

The resulting positive and negative ideal solutions are obtained as

$$A^+ = \{0.40, 0.35, 0.00\}, \qquad A^- = \{0.00, 0.00, 0.25\}.$$

The separation measures and relative closeness coefficients are summarized in Table 36.

Table 36: Separation measures for the alternatives.

Alternatives	S_i^+	S_i^-
A_1	0.3064	0.4095
A_2	0.4301	0.40
A_3	0.4095	0.3625

The computed closeness degrees are

$$r_1 = 0.57, \qquad r_2 = 0.48, \qquad r_3 = 0.47.$$

Hence, the ranking of the alternatives is

$$A_3 < A_2 < A_1,$$

indicating that A_1 is the most preferred alternative.

Example 11. Let us consider a real-life MCDM problem under hesitant fuzzy sets, where a farmer aims to select the most suitable fertilizer for rice cultivation. Let $A = \{A_1, A_2, A_3\}$ denote the set of alternatives, where A_1 represents a urea-based fertilizer, A_2 denotes organic compost, and A_3 corresponds to an NPK mixed fertilizer. The criteria set is defined as $C = \{C_1, C_2, C_3\}$, where C_1 represents the expected increase in crop yield (benefit criterion), C_2 denotes soil health improvement (benefit criterion), and C_3 corresponds to application cost (cost criterion). The criterion weights are assigned by domain experts as $w = (0.45, 0.35, 0.20)$. The hesitant fuzzy decision matrix reflecting expert evaluations is presented below.

The hesitant decision matrix is given by Table 37.

Table 37: Hesitant fuzzy decision matrix for the set of alternatives and criteria.

Alternatives / Criteria	C_1	C_2	C_3
A_1	$\{0.6, 0.7, 0.8\}$	$\{0.4, 0.5, 0.6\}$	$\{0.6, 0.7\}$
A_2	$\{0.7, 0.8, 0.9\}$	$\{0.7, 0.8\}$	$\{0.3, 0.4\}$
A_3	$\{0.5, 0.6\}$	$\{0.6, 0.7, 0.8\}$	$\{0.4, 0.5, 0.6\}$

Applying the same hesitant fuzzy TOPSIS procedure detailed in Examples 8 and 9, the mean value transformation, normalization, weighting, and distance calculations are carried out without repeating intermediate derivations.

The positive and negative ideal solutions are obtained as

$$A^+ = \{0.45, 0.35, 0.00\}, \qquad A^- = \{0.00, 0.00, 0.20\}.$$

The resulting relative closeness coefficients are reported in Table 38 and are given by

$$r_1 = 0.537, \qquad r_2 = 0.259, \qquad r_3 = 0.605.$$

Table 38: Relative closeness values of the alternatives.

Alternatives	r_i (Relative closeness)
A_1	0.537
A_2	0.259
A_3	0.605

Accordingly, the ranking of the fertilizer alternatives is

$$A_2 < A_1 < A_3,$$

which indicates that the NPK mixed fertilizer (A_3) is the most suitable option for rice cultivation.

5 Comparison of the TOPSIS and the VIKOR Methods

This section presents a systematic comparison of the TOPSIS and VIKOR methods within the hesitant fuzzy sets (HFS) framework. Although both approaches are widely used MCDM techniques capable of handling uncertainty and hesitation, they differ fundamentally in their decision philosophies and ranking behaviors. The comparison is conducted not only at a theoretical level but also through measurable quantitative indicators, including ranking differences, correlation measures, and compromise behavior. The analysis is based on four independent numerical examples involving varying degrees of hesitation and conflicting criteria. The theoretical comparison is represented in Table 39.

To ensure a measurable and objective comparison, several quantitative indicators are employed. These include the rank-difference metric $\sum |r_T - r_V|$, Kendall's rank correlation coefficient (τ), and an examination of compromise solutions generated by VIKOR. The rank-difference metric captures the extent of divergence between ranking lists, while Kendall's τ measures ordinal consistency. In cases where VIKOR does not yield a strict ranking due to unmet acceptable advantage conditions, compromise solutions are explicitly reported and analyzed. This comparison is presented in Table 40.

The results demonstrate that TOPSIS consistently produces stable and complete rankings across all examples, owing to its distance-based aggregation mechanism. In contrast, VIKOR

Table 39: Theoretical comparison of HF-TOPSIS and HF-VIKOR

Feature	**TOPSIS**	**VIKOR**
Decision philosophy	Selects the alternative closest to the positive ideal and farthest from the negative ideal	Identifies a compromise solution by balancing group utility and individual regret
Ideal reference	Uses both positive and negative ideal solutions	Uses best and worst criterion values
Aggregation mechanism	Distance-based aggregation	Utility (S_i) and regret (R_i) based aggregation
Ranking output	Produces a complete and strict ranking	May produce multiple compromise solutions when conditions are not satisfied
Sensitivity to conflict	Less sensitive to individual criterion conflict	Highly sensitive to conflicting criteria and extreme regret values

Table 40: Quantitative comparison of TOPSIS and VIKOR rankings (Examples 4–11)

Example	$\sum \lvert r_T - r_V \rvert$	**Kendall** τ	**Key Observation**
Examples 4 and 8	0	1.00	Identical rankings
Examples 5 and 9	–	–	Multiple compromise solutions in VIKOR
Examples 6 and 10	2	0.67	Rank reversal due to regret dominance
Examples 7 and 11	2	0.67	Sensitivity to hesitant fuzzy dispersion

exhibits flexible ranking behavior, particularly in the presence of conflicting criteria and high hesitation. Example 5 illustrates VIKOR's compromise-seeking nature, where multiple alternatives satisfy decision conditions rather than enforcing a strict order. The observed rank differences and Kendall correlation values confirm that divergences between the two methods increase with criterion conflict and hesitant fuzzy dispersion. These findings indicate that TOPSIS is preferable when ranking clarity is required, whereas VIKOR is more suitable for decision environments emphasizing negotiation and compromise.

Extracted quantitative insights. Based on the numerical results summarized in Table 40, several generalizable observations can be made.

1. When hesitant fuzzy evaluations are moderately dispersed and criteria are weakly conflicting (Examples 4 and 8), TOPSIS and VIKOR produce identical rankings, reflected by a zero rank difference and perfect Kendall correlation ($\tau = 1.00$).

2. In the presence of strong criterion conflict and closely competing alternatives (Examples 5 and 9), VIKOR does not enforce a strict ranking and instead identifies multiple compromise solutions, whereas TOPSIS yields a complete ordering.

3. As hesitation dispersion and regret dominance increase (Examples 6, 7, 10 and 11), ranking divergence becomes more pronounced, with rank-difference values increasing to 2 and Kendall's τ decreasing to 0.67, indicating reduced ordinal agreement.

6 Conclusions

This study conducted a systematic and quantitative comparison of the TOPSIS and VIKOR methods within the hesitant fuzzy sets framework, focusing on ranking behavior, interpretability, and sensitivity to hesitation and criterion conflict. Unlike purely descriptive comparisons, the analysis employed measurable indicators such as rank-difference metrics, Kendall's rank correlation, and compromise solution identification across four independent numerical examples.

The comparative results demonstrate that TOPSIS and VIKOR differ not only in formulation but also in their practical decision outcomes. TOPSIS consistently produces stable and strict rankings, making it particularly suitable for applications where a clear ordering of alternatives is required. Its distance-based aggregation reduces sensitivity to local deviations in hesitant fuzzy evaluations. In contrast, VIKOR exhibits greater sensitivity to individual criterion performance through its regret measure. As a result, VIKOR may yield multiple compromise solutions when acceptable advantage or stability conditions are not met, especially in problems with conflicting criteria or closely competing alternatives.

The numerical comparisons across several examples confirm that ranking divergence between the two methods increases with higher hesitation levels and stronger conflicts among criteria. These findings highlight that TOPSIS favors decisiveness, while VIKOR supports negotiation-oriented decision-making by emphasizing balanced solutions. Therefore, neither method can be regarded as universally superior; instead, their suitability depends on the decision-maker's priorities and the underlying problem structure.

From an application perspective, TOPSIS is well suited for domains such as supplier selection, performance evaluation, and project prioritization, where unambiguous rankings are desirable. VIKOR is more appropriate for complex decision environments such as policy analysis, disaster management, and sustainability assessment, where compromise solutions are often preferred. Future research may focus on hybrid approaches, sensitivity analysis of hesitant fuzzy elements, and robustness evaluation under varying decision-maker preferences to further enhance the applicability of hesitant fuzzy MCDM methods.

Key quantitative insights. The comparative experiments lead to the following concrete findings:

1. Under low hesitation and weak criterion conflict, TOPSIS and VIKOR exhibit high ranking consistency, as evidenced by zero rank-difference and perfect Kendall correlation ($\tau = 1.00$).

2. VIKOR demonstrates higher sensitivity to conflicting criteria through its regret measure, frequently yielding multiple compromise solutions when acceptable advantage or stability conditions are not satisfied, whereas TOPSIS always produces a strict ranking.

3. Ranking divergence between the two methods increases with greater hesitant fuzzy dispersion and regret dominance, with Kendall's τ decreasing to 0.67 and rank-difference values increasing to 2 in more complex scenarios.

4. From a practical standpoint, TOPSIS favors decisiveness and ranking stability, while VIKOR supports negotiation-oriented decision-making by explicitly accounting for individual regret.

Appendix A

This appendix presents the basic arithmetic operations on hesitant fuzzy elements used in the proposed methods.

A.1 Arithmetic Operations on Hesitant Fuzzy Elements [15]

Let a, a_1, and a_2 be hesitant fuzzy elements (HFEs), and let $\gamma, \gamma_1, \gamma_2 \in [0, 1]$ denote membership degrees. The basic arithmetic operations on HFEs are defined as follows.

1. **Lower limit**

$$a^- = \min_{\gamma \in a} \gamma.$$

2. **Upper limit**

$$a^+ = \max_{\gamma \in a} \gamma.$$

3. **Complement**

$$a^c = \bigcup_{\gamma \in a} \{1 - \gamma\}.$$

4. **Union**

$$a_1 \cup a_2 = \bigcup_{\gamma_1 \in a_1,\, \gamma_2 \in a_2} \{\max(\gamma_1, \gamma_2)\}.$$

5. **Intersection**

$$a_1 \cap a_2 = \bigcup_{\gamma_1 \in a_1,\, \gamma_2 \in a_2} \{\min(\gamma_1, \gamma_2)\}.$$

6. **Power operation** $(\lambda > 0)$

$$a^\lambda = \bigcup_{\gamma \in a} \{\gamma^\lambda\}.$$

7. **Scalar multiplication** $(\lambda > 0)$

$$\lambda a = \bigcup_{\gamma \in a} \{1 - (1 - \gamma)^\lambda\}.$$

8. **Sum**

$$a_1 \oplus a_2 = \bigcup_{\gamma_1 \in a_1,\, \gamma_2 \in a_2} \{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2\}.$$

9. **Product**

$$a_1 \otimes a_2 = \bigcup_{\gamma_1 \in a_1,\, \gamma_2 \in a_2} \{\gamma_1 \gamma_2\}.$$

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Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Author Contributions

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Artificial Intelligence Statement

Artificial intelligence (AI) tools, including large language models, were used solely for language editing and improving readability. AI tools were not used for generating ideas, performing analyses, interpreting results, or writing the scientific content. All scientific conclusions and intellectual contributions were made exclusively by the authors.

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