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Observer-Based Adaptive Neural Command Filter Control for MIMO Nonlinear Stochastic Systems with Prescribed Partial Tracking Error Constraints

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Abstract. This paper proposes a novel observer-based adaptive neural command filter (CF) output-feedback tracking control scheme for uncertain nonlinear multiple-input multiple-output (MIMO) stochastic systems subject to constrained partial tracking errors (PTEs). The key contributions are threefold. First, a linear Luenberger-type state observer is designed to handle unmeasured states, and radial basis function neural networks (RBFNNs) are employed to approximate unknown nonlinear functions. Second, a dynamic surface control (DSC) strategy augmented with compensating signals simultaneously eliminates the "explosion of complexity" inherent in conventional backstepping and rectifies filter-output errors in the DSC framework. Third, a minimal learning parameter (MLP) technique — based on Young's inequality — reduces the number of online-tunable parameters to a single scalar per subsystem, yielding a computationally efficient adaptive law. The closed-loop system is rigorously proven to be semi-globally uniformly ultimately bounded (SGUUB) in probability via Lyapunov-based stochastic stability analysis, and all PTEs remain within prescribed performance bounds throughout transient and steady-state operation. Comparative simulations on a second-order MIMO stochastic benchmark demonstrate that the proposed approach achieves significantly lower root mean square and integral absolute tracking errors than existing methods, while maintaining bounded and smooth control effort. Current limitations, including the restriction to strict-feedback topologies and the assumption of bounded external disturbances, motivate future extensions to multi-agent and event-triggered control frameworks.

Keywords. Adaptive neural control, Output-feedback control, Nonlinear stochastic systems, Prescribed performance control, Command filtered back-stepping, Dynamic surface control.

MSC. 93C40; 93E03; 93C10.

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1 Introduction

Today, studying the problem of designing a controller for nonlinear stochastic systems (NSSs) is one of the most attractive research areas, due to, stochastic disturbances are a widespread phenomenon in many systems and often cause instability. The development of stochastic control theory has led to numerous advanced control methods, including adaptive, robust, sliding mode, and backstepping control, which are designed to ensure stability in probability. Consequently, numerous results have been procured for NSSs with linearly parametric uncertainties [4, 19, 23]. Moreover, by combining some of the mentioned methods with fuzzy systems or neural networks (NNs) significant results were obtained for NSSs with lower-triangular structures and high uncertainties [1, 5, 7, 33, 34, 35, 36, 37, 43, 44, 45].

The authors of [33] developed adaptive neural backstepping control schemes for SISO NSSs. In [34], an adaptive NN controller scheme was investigated for pure-feedback NSSs. Also, in [7], an adaptive neuro-fuzzy controller for NSSs is presented. Moreover, [35] addressed an adaptive fuzzy control approach for NSSs with non-strict feedback and constant time-delay. However, all of the preceding schemes are obtained under a state feedback framework. To cope with this problem, [5] first introduced an adaptive NN controller with output-feedback (OF) for SISO-NSSs. Afterward, much research was conducted on approximation-based OF adaptive control for NSSs, i.e., NSSs in the presence of dynamic uncertainties [45], and non-strict feedback NSSs [36].

Despite these efforts for stochastic control design, the control schemes obtained in [5, 7, 33, 34, 35, 36, 45] suffer from a "complexity explosion" problem. This refers to the need for repeated differentiation of intermediate control functions (ICFs) within the backstepping design. To eliminate this issue, the study in [29] introduced first-order low-pass filters. Then, in [6] DSC design was combined with an adaptive NN approximator for strict-feedback NSSs. Using the DSC design, the authors of [46] proposed adaptive NN and OF controllers for NSSs. However, [6, 29, 46] neglected the problem of compensating the output errors of filters in the DSC strategy. Hence, it makes stability analysis much more complicated. To remove this drawback, a command filter (CF) backstepping is presented in [12]. In [11], it was developed to an adaptive mode for nonlinear systems (NSs) with strict-feedback. To handle the challenge of high uncertainties and unmeasured states, in [41] an observer-based fuzzy CF-based control scheme was introduced for uncertain NS. Also, [10] studied an observer-based CF control approach for pure-feedback systems. However, papers [10, 11, 12, 41] focus only on the control of deterministic SISO systems. Proving the stability properties of the closed-loop (CL) systems in the above methods is established using the Lyapunov method. Furthermore, the tracking error (TE) is guaranteed to approach a set of residuals. However, in these methods, accessing system states that are constrained by predefined transient performance and steady-state is a very challenging problem. The authors of [2] and [3] offered a prescribed performance control (PPC) method for a specific category of NSs, where performance indicators are adjustable through predefined performance boundaries of TEs. Afterward, some prescribed performance (PP) adaptive methods were introduced for different types of NSs [15, 18, 20, 24]. However, considered systems in [2, 3, 15, 18, 20, 24] are free of stochastic disturbances. In [14] an adaptive PPC using NN for NSSs with time delays is studied. Meanwhile, in [28] an adaptive fuzzy PPC controller is introduced for SISO stochastic systems despite input saturation. However, the PPC schemes in [2, 3, 14, 15, 18, 20, 24, 28] may be violated, and as a result, a singularity problem may occur. To remove this obstacle, [13] proposed a new predetermined operation scheme. In [31], a DSC scheme with partial TE constraints is offered for MIMO strict-feedback NSs, utilizing the approach described above. However, designed controllers in [13] and [31] are valuable for deterministic systems and the authors do not compensate the effects of filters' errors. On the other hand, the quantity of adaptive laws in [13] and [31] is determined by the number of fuzzy rules.

Recent advances have extended adaptive quantized control to various classes of nonlinear systems. For instance, finite-time adaptive quantized control for stochastic nonstrict-feedback constrained systems was investigated in [8], while fuzzy adaptive output feedback for stochastic nonlinear systems under input/output quantization was developed in [38]. Adaptive output feedback strategies for uncertain nonlinear systems with quantized input and output have also been reported in [9, 42]. Furthermore, command filter based approaches for MIMO non-strict feedback systems with input quantization were studied in [47]. More recently, nonsingular predefined-time dynamic surface control [40] and predefined-time adaptive output feedback control [32] for quantized nonlinear systems have been proposed, demonstrating that combining predefined-time stability with quantization can enhance transient performance while reducing communication demand. Beyond the above theoretical developments, intelligent adaptive techniques have also been successfully applied to other complex systems, including energy-efficient clustering in wireless sensor networks using fuzzy neural networks [16] and stabilization of fractional-order chaotic Hopfield neural networks via adaptive model-free control [26]. However, to the best of our knowledge, no existing work simultaneously addresses observer-based adaptive neural command filter control for MIMO stochastic systems with prescribed partial tracking error constraints under input/output quantization, which motivates our future extension.

Given the above-mentioned discussions, this work investigates a CF-based adaptive NN for MIMO-NSSs with OF and PTE constraints. Consequently, the contribution of this study is the development of the controllers from [13] and [31] for MIMO-NSSs using a PPC approach. The design begins by employing a linear state observer for state estimation and RBFNNs for the approximation of uncertain nonlinear functions. Then, a CF and DSC based on PP bounds and a minimal learning parameter method is proposed to scheme an adaptive neural controller.

Finally, the theoretical analysis proves that all CL signals are SGUUB in possibility, and TEs converge to a desirable range.

The proposed scheme offers several distinct advantages compared to the references discussed above, namely:

- For the first time, we propose a CF-based adaptive NN and OF control approach for MIMO-NSSs with OF and constrained partial TEs. However, obtained results in [13] and [31] are suitable for MIMO deterministic systems with state constraints. Consequently, these methods are not applicable to MIMO-NSSs with unmeasured states and the PP method;
- CF overcomes the filters' errors problem in DSC design. In contrast to [10, 11, 12, 41], in this paper, we have addressed an adaptive CF control with a predetermined performance for nonlinear NSs;
- Different from the proposed stochastic control approaches with PPC [14, 28], we no longer encounter the "complexity explosion" problem; and
- The presented approach uses Young's inequality (YI) to significantly reduce the number of tuning parameters. Besides, it is independent of previous knowledge about neural networks. Therefore, the amount of computation is dramatically reduced, and a simplified PPC approach is proposed than [13] and [31].

To clearly highlight the technical contributions and novelty of the proposed method, a structured comparison between this work and several most closely related existing methods is provided in Table 1. Compared to the existing literature, the novelty of this work is reflected in several

Table 1: The qualitative comparative study of the control features of the proposed scheme versus recently related works.

Features	**Proposed**	**Ref. [27]**	**Ref. [30]**	**Ref. [21]**	**Ref. [22]**
MIMO structure	Yes	No (SISO)	No (SISO)	Yes	Yes
Stochastic dynamics	Yes	Yes	No	Yes	No
Output-feedback	Yes	No	Yes	Yes	Yes
PPC	Yes	Yes	Yes	No	Yes
Command Filter (CF)	Yes	No	Yes	No	Yes
MLP technique	Yes	No	No	No	No

aspects. While works like [27] and [21] address prescribed performance for stochastic systems, they are limited to SISO configurations and rely on traditional backstepping, which often leads to the "explosion of complexity" problem. Although [30] introduces command filtering for stochastic systems, it does not account for MIMO dynamics. On the other hand, recent advances such as [22] handle MIMO systems with command filters but are restricted to deterministic environments. In contrast, the proposed scheme simultaneously addresses the challenges of unmeasured states and stochastic disturbances in MIMO non-strict feedback systems, while significantly reducing the computational burden through the integration of the MLP technique.

The rest of this paper is arranged in the following way: An introduction to stochastic control theory is provided in Section 2. Fundamental definitions and preliminary concepts are provided in Section 3. Section 4 addresses controller design and analyzes its CL stability. In Section 5 a numerical simulation example with comparison studies is drawn. The conclusion of the paper is also proposed in Section 6.

2 Stochastic Systems and Stability Analysis

Assume the NSS as defined by:

$$\mathrm{d}x = f(x,t)\,\mathrm{d}t + g(x,t)\,\mathrm{d}w, \tag{1}$$

where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is the drift vector, $g : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^{n\times r}$ is the diffusion vector, satisfying $g(0,t) = 0$. The term w denotes an independent r-dimensional Wiener process with the incremental covariance $E\{\mathrm{d}w\,\mathrm{d}w^T\} = \sigma\sigma^T dt$.

Definition 1 (Itô's Differentiation Rule). Consider the differential operator (Itô's differentiation rule) as follows:

$$\mathcal{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}^T f + \frac{1}{2}\mathrm{Tr}\left\{g^T(x,t)\frac{\partial^2 V}{\partial x^2}\sigma^T g(x,t)\right\}, \tag{2}$$

where $V(x,t)$ represents a Lyapunov function related with the NSS (1), and $\mathrm{Tr}\{A\}$ signifies the trace operator for a matrix A.

Definition 2 (SGUUB in Probability). ([17]) The solution $x(t)$ of NSS (1) is called to be SGUUB in p-moment, if for any $\varepsilon > 0$ and each compact set $\Omega \subset \mathbb{R}^n$ and arbitrary initial state $x(t_0) = x_0 \in \Omega$, there exist a time constant $T = T(x_0,\varepsilon)$ such that $\mathbb{E}[\|x(t)\|^p] < \varepsilon$ for all $t \geq t_0 + T$.

3 Problem Statement and Preliminaries

3.1 System Description and Assumptions

Consider the NSSs with strict-feedback described by

$$
\begin{aligned}
\mathrm{d}x_{j,i_j} &= \big[x_{j,i_j+1} + f_{j,i_j}(\bar{x}_{j,i_j}) + d_{j,i_j}(t)\big]\mathrm{d}t + g_{j,i_j}(\bar{x}_j)\,\mathrm{d}w_j,\\
\mathrm{d}x_{j,n_j} &= \big[u_j + f_{j,n_j}(\bar{x}_{j,n_j}) + d_{j,n_j}(t)\big]\mathrm{d}t + g_{j,n_j}(\bar{x}_j)\,\mathrm{d}w_j,\\
y_j &= x_{j,1}, \quad i_j = 1,\dots,n_j-1, \quad j = 1,\dots,m,
\end{aligned} \tag{3}
$$

where $\bar{x}_{j,n_j} = [x_{j,1},\dots,x_{j,n_j}]^T \in \mathbb{R}^{n_j}$ is the state vector of the j-th subsystem ($\bar{x}_{j,n_j} = \bar{x}_j$), $u_j \in \mathbb{R}$ and $y_j \in \mathbb{R}$ are the control input and the output of the j-th subsystem. $f_{j,i_j}(\cdot)$ and $g_{j,i_j}(\cdot)$ are unknown smooth nonlinear functions and vanish at the origin. $d_{j,i_l}(t)$ is a bounded external disturbance. w_j is an independent r-dimensional standard Wiener process, with the incremental covariance $E\{\mathrm{d}w_j\,\mathrm{d}w_j^T\} = \sigma_j(t)\sigma_j^T(t)\,\mathrm{d}t$. In this study, it is assumed only outputs of the system (3) are available for the measurement.

Control objective: The goal is to develop an adaptive NN-based control approach for the NSS (3), so that while the CL signals remain SGUUB in probability, the partial error constraints are not violated.

The development of the controller relies on the following assumptions and lemmas:

Assumption 1. The trajectory $y_{j,d}(t)$ is bounded and known. In addition, its derivative $\dot{y}_{j,d}(t)$ is smooth and bounded.

Remark 1. Compared with obtained results for stochastic nonlinear NSs [5, 7, 33, 34, 35, 36, 45], only the desired signal information and its derivative $\dot{y}_L$ are needed, while the backstepping design procedure needs the knowledge of y_L^i and DSC design technique needs the knowledge of y_L, $\dot{y}_L$ and $\ddot{y}_L$.

Assumption 2. The disturbance $d_{j,i_j}(t)$ is limited by an unknown positive constant d^*_{j,i_j}, such that $|d_{j,i_j}(t)| \le d^*_{j,i_j}$.

Assumption 3. The disturbance covariance $G_j^T\sigma_j\sigma_j^T G_j$ is bounded, where

$$G_j = [g_{j,1}(\bar{x}_j),\dots,g_{j,n_j}(\bar{x}_j)]^T.$$

Lemma 1 (Young's Inequality)**.** For $(x_1,x_2) \in \mathbb{R}^2$ we have:

$$x_1x_2 \le \frac{\gamma^{p_1}}{p_1}|x_1|^{p_1} + \frac{1}{q_1\gamma^{p_1}}|x_2|^{q_1},$$

where $\gamma > 0, p_1 > 1, q_1 > 1$ and $\frac{1}{p_1} + \frac{1}{q_1} = 1$.

3.2 Neural Network Approximator

Due to their properties of approximation, learning, and fault tolerance, the use of NNs has become popular for the identification and control of NSSs. Also, to handle system uncertainties, RBFNNs are utilized for the approximation of smooth nonlinear functions. With the help of RBFNN, an unknown continuous function $f(Z) : \mathbb{R}^q \to \mathbb{R}$ can be approximated over a compact set $\Omega_Z \subset \mathbb{R}^q$ as:

$$f_{nn}(Z) = \theta^{*t}\Phi(Z),$$

where $Z \in \mathbb{R}^q$ denotes the input vector, and q represents the input-dimension of the NN. Also, $\theta = [\theta_1, \ldots \theta_l]^T \in \mathbb{R}^l$ and l represent the ideal weight vector and the number of nodes in the NN, respectively. The basis function vector is $\Phi(Z) = [\varphi_1(Z), \ldots, \varphi_l(Z)]^T$, where $\varphi_i(Z)$ is defined as:

$$\varphi_i(Z) = \exp\left[-\frac{(Z-\mu_i)^T(Z-\mu_i)}{\eta_i^2}\right], \; i = 1, 2, \ldots l,$$

where μ_i and η_i are the center and width of the Gaussian function, respectively [25]. We can define $f(Z)$ on a closed set $\Omega_Z \subset \mathbb{R}^q$ as follows:

$$f(Z) = \theta^{*t}\Phi(Z) + \delta(Z), \quad \forall Z \in \Omega_Z.$$

Here $\theta^{*t}\Phi(Z)$ is the NN output, and $\delta(Z)$ is the desired accuracy. The ideal constant weight vector θ^* is given by:

$$\theta^* = \arg\min_{\theta\in\mathbb{R}^l}\left\{\sup_{Z\in\Omega_Z}|f(Z) - \theta^T\Phi(Z)|\right\}.$$

Also, $\delta(Z)$ represents the minimum approximation error, and $\delta(Z)| \leq \varepsilon$, where ε is a positive parameter.

3.3 Prescribed Function and Error Transformation

To guarantee the error constraints, we can define $\rho_{j,i_l}(t) : \mathbb{R}^+ \to \mathbb{R}^+/\{0\}$ with $\lim_{t\to\infty}\rho_{j,i_l}(t) = \rho_{\infty j,i_l}$.

$$\rho_{j,i_l}(t) = (\rho_{0j,i_l} - \rho_{\infty j,i_l})e^{-a_{j,i_l}t} + \rho_{\infty j,i_l},$$

where positive constants $\rho_{\infty j,i_l}, \rho_{0j,i_l}, a_{j,i_l}$ are appropriately chosen. The steady-state boundaries and predetermined transient are also ensured applying the following constraints:

$$-\delta_{j,i_l}\rho_{j,i_l}(t) < z_{j,i_l}(t) < \rho_{j,i_l}(t), \quad \text{if } z_{j,i_l}(0) > 0,$$

or

$$-\rho_{j,i_l}(t) < z_{j,i_l}(t) < \delta_{j,i_l}\rho_{j,i_l}(t), \quad \text{if } z_{j,i_l}(0) < 0.$$

Here, $\delta_{j,i_l} \in (0, 1]$ denotes the predetermined parameter, and $\rho_{\infty j,i_l}$ is a constant that limits the TEs in the steady state. The reduction rate a_{j,i_l} adjusts the required convergence speed for the TEs. Selecting parameters $\rho_{j,i_l}(0)$ and δ_{j,i_l} directly adjusts the overshoot [13]. Now, we introduce the transformed constraint error as:

$$\zeta_{j,i_l}(t) = \left(\frac{z_{j,i_l}(t)}{\eta_{j,i_l}(t)}\right), \tag{4}$$

$$\eta_{j,i_l}(t) = q\bar{\eta}_{j,i_l}(t) + (1-q)\underline{\eta}_{j,i_l}(t).$$

Here $q = 1$ if $z_{j,i_l}(t) \geq 0$, otherwise $z_{j,i_l}(t) < 0$. We can also define $\underline{\eta}_{j,i_l}(t)$ and $\bar{\eta}_{j,i_l}(t)$ in the following manner:

$$\begin{cases} \bar{\eta}_{j,i_l}(t) = \rho_{j,i_l}(t), & \text{if } z_{j,i_l}(t) \geq 0, \\ \underline{\eta}_{j,i_l}(t) = -\delta_{j,i_l}\rho_{j,i_l}(t), & \end{cases} \tag{5}$$

$$\begin{cases} \bar{\eta}_{j,i_l}(t) = \delta_{j,i_l}\rho_{j,i_l}(t), & \text{if } z_{j,i_l}(t) < 0, \\ \underline{\eta}_{j,i_l}(t) = -\rho_{j,i_l}(t), & \end{cases} \tag{6}$$

Lemma 2 ([13])**.** The function $\zeta_{j,i_l}(t)$ satisfies the following condition if and only if the parameters $\rho_{0j,i_l}, \rho_{\infty j,i_l}, a_{j,i_l}, \delta_{j,i_l}$ are chosen in accordance with equations (5) and (6):

$$0 < \zeta_{j,i_l}(t) < 1, \quad \forall t > 0 \tag{7}$$

4 Main Results

Now, we propose a CF-based DSC design method to develop an adaptive NN controller. The goal is designing a linear observer for the estimation of the system's unmeasured states. For this purpose, an RBFNN is utilized to identify uncertain nonlinear function. Also, we employ the conventional adaptive method to estimate the upper bounds of the NNs weight norms, culminating in a DSC-based control law derived via Lyapunov stability theory. To simplify the analysis, let the norms of the ideal NN weights be defined as the unknown constant W_j^*:

$$W_j^* = \{N_{j,i_j} = \|\theta^*_{j.i_j}\|^2, j = 1, 2, \ldots, m, i_j = 1, 2, \ldots, n_j\}. \tag{8}$$

Here $N_{j,i_j} \geq \phi^T_{j,i_j}(\cdot)\phi_{j,i_j}(\cdot)$.

4.1 Observer Design

This subsection presents the problem of designing an OF controller, which is constructed using a linear state observer. For this purpose, an observer is presented as:

$$\begin{cases} \dot{\hat{x}}_{j,i_j} = \hat{x}_{j,i_j+1} + k_{j,i_j}(y_j - \hat{x}_{j,1}), \quad j = 1, \ldots m, \quad i_j = 1, \ldots n_j - 1, \\ \dot{\hat{x}}_{j,n_j} = u_j + k_{j,n_j}(y_j - \hat{x}_{j,1}), \end{cases} \tag{9}$$

where $\hat{x}_{j,i_j}$ is the estimation of x_{j,i_j} for $j = 1, \dots m, \quad i_j = 1, \dots n_j - 1$.

We also define the observer error as follows:

$$e_j = [e_{j,1}, e_{j,2} \dots e_{j,n_j}]^T = \bar{x}_j - \hat{x}_j. \tag{10}$$

According to (3), (9) and (10),

$$\mathrm{d}e_j = (A_j e_j + F_j(\bar{x}_j) + D_j)\mathrm{d}t + G_j(\bar{x}_j)\mathrm{d}w, \tag{11}$$

where $F_j(\bar{x}_j) = [f_{j,1}(\bar{x}_{j,1}), \dots f_{j,n_j}(\bar{x}_{j,n_j})]^T$, $D_j = [d_{j,1}, \dots d_{j,n_j}]^T$ and

$$A_j = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_{j,1} & -k_{j,2} & -k_{j,3} & \cdots & -k_{j,n_j} \end{bmatrix}$$

The Hurwitz property of matrix A_j guarantees that for any chosen positive definite matrix (PDM) $Q_j = Q_j^T$, a unique PDM $P_j = P_j^T$, exists such that

$$A_j^T P_j + P_j A_j = -Q_j. \tag{12}$$

Now, we introduce the Lyapunov function as:

$$V_o = \sum_{j=1}^{m} V_{j,0} = \sum_{j=1}^{m} e_j^T P_j e_j. \tag{13}$$

By Itô rule and (12),

$$\mathcal{L}V_o \leq \sum_{j=1}^{N} \big(-\lambda_{\min}(Q_j)\|e_j\|^2 + 2e_j^T P_j F_j + 2e_j^T P_j D_j + \mathrm{Tr}\{\sigma_j G_j^T(\bar{x}_j) P_j G_j(\bar{x}_j)\}\sigma_j^T \big). \tag{14}$$

According to Assumption 3, (8) and YI,

$$2e_j^T P_j F_j \leq 2e_j^T P_j(\Phi_j \theta_j^* + \delta_j) \leq 2\|e_j\|^2 + \|P_j\|^2\|\varepsilon\|^2 + \|P_j\|^2 W_j^*, \tag{15}$$

$$\mathrm{Tr}\{\sigma_j G_j^T(\bar{x}_j) P_j G_j(\bar{x}_j)\} \leq \|P_j\|^2 + |\sigma_j \sigma_j^T|, \tag{16}$$

$$2e_j^T P_j D_j \leq \|e_j\|^2 + \|P_j\|^2 D_j^{*\,2}. \tag{17}$$

Where $\varepsilon_j = [\varepsilon_{j,1}, \varepsilon_{j,2}, \dots \varepsilon_{j,n_j}]^T$, $\delta_j = [\delta_{j,1}, \delta_{j,2}, \dots \delta_{j,n_j}]^T$, $D_j^* = [d_{j,1}^*, d_{j,2}^*, \dots d_{j,n_j}^*]^T$ and $\Phi_j = \mathrm{diag}[\phi_{j,1}^T, \phi_{j,2}^T \dots \phi_{j,n_j}^T]$.

Substituting (15) and (16) into (14) results in

$$\mathcal{L}V_o \leq \big(-(\lambda_{\min}(Q_j) - 3)\|e_j\|^2 + \|P_j\|^2\|\varepsilon_j\|^2 + \|P_j\|^2 D_j^* 2 + \|P_j\|^2 W_j^* + \frac{1}{2}\|P_j\|^2 + \frac{1}{2}|\sigma_j \sigma_j^T|^2\big). \tag{18}$$

4.2 Controller Development and Stability Analysis

This subsection incorporates DSC with CF technique for NSSs in (3) to eliminate the issues of "explosion of complexity" and filters' errors. The proposed design method, like the traditional step-backward design method, contains n_j steps. The i_l-th step of CF-based adaptive DSC design the following coordinate transformation applies for $i_l = 1, \ldots, n_j$:

$$z_{j,1} = y_j - y_{j,d}, \tag{19}$$

$$z_{j,i_j} = \hat{x}_{j,i_j} - \pi_{j,i_j}. \tag{20}$$

where z_{j,i_j} and $z_{i,1}$ are the transformed error level and the TE, respectively. The variable π_{j,i_j} is the state variable generated by filtering an ICF through a first-order filter.

The following definitions are introduced for the partially transformed error surface and the associated virtual errors:

$$z_{j,1} = y_j - y_{j,d}, , \tag{21}$$

$$\zeta_{j,i_l} = \hat{x}_{j,i_l} - \pi_{j,i_l}, \quad i_l = 2, \ldots, m_c, \tag{22}$$

$$\zeta_{j,i_j}(t) = \frac{z_{j,i_j}(t)}{\eta_{j,i_j}(t)}, \quad S_{j,i_j} = \frac{\zeta_{j,i_j}(t)}{1-\zeta_{j,i_j}(t)}, \quad i_j = 1, \ldots, m_c, \tag{23}$$

$$S_{j,i_k} = \hat{x}_{j,i_k} - \pi_{j,i_k}, \quad i_k = m_c + 1, \ldots, n_j, . \tag{24}$$

Step 1: According to (21) and (23), we have

$$\begin{aligned} \mathrm{d}S_{j,1} &= \frac{1}{(1-\zeta_{j,1})^2\eta_{j,1}}\big((x_{j,2} + f_{j,1}(\bar{x}_{j,1}) + d_{j,1} - \dot{\eta}_{j,1}\zeta_{j,1} - \dot{y}_{j,1})\mathrm{d}t + g_{j,1}(\bar{x}_j)\mathrm{d}w_j\big) \\ &= h_{j,1}\big((\zeta_{j,2}\eta_{j,2} + e_{j,2} + \pi_{j,2} + f_{j,1}(\bar{x}_{j,1}) + d_{j,1} - \dot{\eta}_{j,1}\zeta_{j,1} - \dot{y}_{j,d})\mathrm{d}t + g_{j,1}(\bar{x}_j)\mathrm{d}w_j\big), \end{aligned} \tag{25}$$

where $h_{j,1} = \frac{1}{(1-\zeta_{j,1})^2\eta_{j,1}}$.

Variable $\pi_{j,2}$ is proposed to eliminate the need for repeated differentiation of $\alpha_{j,1}$. Suppose $\alpha_{j,1}$ passes via the following first-order filter:

$$\bar{\omega}_{j,2}\dot{\pi}_{j,2} + \pi_{j,2} = \alpha_{j,1}, \quad \pi_{j,2}(0) = \alpha_{j,1}(0). \tag{26}$$

where the parameter $\bar{\omega}_{j,2} > 0$ denotes the filter's time constant. Moreover, a compensation signal $\chi_{j,1}$ is introduced to counteract the filter's output error, with its dynamics given by:

$$\dot{\chi}_{j,1} = h_{j,1}(-c_{j,1}h_{j,1}\chi_{j,1} + \pi_{j,2} - \alpha_{j,1}), \quad \chi_{j,1}(0) = 0. \tag{27}$$

Define the following compensated TE signals as

$$v_{j,1} = S_{j,1} - \chi_{j,1}. \tag{28}$$

Then, from (27) and (28), (25) becomes

$$\mathrm{d}v_{j,1} = h_{j,1}\big((\zeta_{j,2}\eta_{j,2}+e_{j,2}+f_{j,1}(\bar{x}_{j,1})+d_{j,1}-\dot{\eta}_{j,1}\zeta_{j,1}-\dot{y}_{j,d}+c_{j,1}h_{j,1}\chi_{j,1}+\alpha_{j,1})\mathrm{d}t+g_{j,1}(\bar{x}_j)\mathrm{d}w_j\big). \tag{29}$$

Now, we select the Lyapunov function as:

$$V_1 = \sum_{j=1}^{m} V_{j,1} = \sum_{j=1}^{m}\{V_{j,0} + \frac{1}{2}v_{j,1}^2\}. \tag{30}$$

From (29), the Itô differentiation rule of V_1 is given by

$$\begin{aligned}\mathcal{L}V_1 = \sum_{j=1}^{m}\big[&\mathcal{L}V_{j,0} + v_{j,1}h_{j,1}\big(\zeta_{j,2}\eta_{j,2} + e_{j,2} + \bar{f}_{j,1}(\bar{Z}_{j,1}) - \frac{3}{2}v_{j,1}h_{j,1} + d_{j,1}\\ &- \dot{\eta}_{j,1}\zeta_{j,1} - \dot{y}_{j,d} + c_{j,1}h_{j,1}\chi_{j,1} + \alpha_{j,1}\big) + g_{j,1}^T(\bar{x}_j)\sigma_j\sigma_j^T g_{j,1}(\bar{x}_j)h_{j,1}\big],\end{aligned} \tag{31}$$

where

$$\bar{f}_{j,1}(\bar{Z}_{j,1}) = f_{j,1}(\bar{x}_{j,1}) + \frac{3}{2}v_{j,1}h_{j,1} - \dot{y}_{j,d}. \tag{32}$$

Represents an unknown nonlinear function, which is approximated using an RBFNN.

$$\bar{f}_{j,1}(\bar{Z}_{j,1}) = \theta_{j,1}^{*t}\phi_{j,1}(\bar{Z}_{j,1}) + \delta_{j,1}(\bar{Z}_{j,1}), \tag{33}$$

where $\bar{Z}_{j,1} = [\bar{x}_{j,1}, v_{j,1}, h_{j,1}]^T$ and $\delta_{j,1}(\bar{Z}_{j,1})$ is the minimum error for the approximation, and must satisfy $\delta_{j,1}(\bar{Z}_{j,1}) \leq \varepsilon_{j,1}$. Now, YI is used to minimize the number of RBFNN learning parameters, so that a learning parameter should be set as follows:

$$\begin{aligned}v_{j,1}h_{j,1}\bar{f}_{j,1}(\bar{Z}_{j,1}) &= v_{j,1}h_{j,1}\theta_{j,1}^{*t}\phi_{j,1}(\bar{Z}_{j,1}) + v_{j,1}h_{j,1}\delta_{j,1}(\bar{Z}_{j,1}),\\ &\leq \frac{1}{2}v_{j,1}^2h_{j,1}^2 + \frac{1}{2r_{j,1}}v_{j,1}^2h_{j,1}^2W_j^* + \frac{1}{2}r_{j,1} + \frac{1}{2}\varepsilon_{j,1}^2.\end{aligned} \tag{34}$$

In (34), $r_{j,1} > 0$ is a design parameter.

Using Assumptions 2, 3, and YI gives:

$$v_{j,1}h_{j,1}d_{j,1} \leq \frac{1}{2}v_{j,1}^2h_{j,1}^2 + \frac{1}{2}d_{j,1}^{*2}, \tag{35}$$

$$v_{j,1}h_{j,1}e_{j,2} \leq \frac{1}{2}v_{j,1}^2h_{j,1}^2 + \frac{1}{2}\|e_j\|^2, \tag{36}$$

$$h_{j,1}g_{j,1}^T(\bar{x}_j)\sigma_j\sigma_j^T g_{j,1}(\bar{x}_j)h_{j,1} = h_{j,1}^2|\sigma_j\sigma_j^T|. \tag{37}$$

Substituting (34)–(37) into (31), yields

$$\begin{aligned}\mathcal{L}V_1 \leq \sum_{j=1}^{m}\Bigg(&\ell V_{j,0} + v_{j,1}h_{j,1}\Big(\zeta_{j,2}\eta_{j,2} + \frac{1}{2r_{j,1}}v_{j,1}h_{j,1}W_j^* - \dot{\eta}_{j,1}\zeta_{j,1} + c_{j,1}h_{j,1}\chi_{j,1} + \alpha_{j,1}\Big)\\ &+ \frac{1}{2}\|e_j\|^2 + \frac{1}{2}{d_{j,1}^*}^2 + \frac{1}{2}r_{j,1} + \frac{1}{2}\varepsilon_{j,i}^2 + h_{j,i}^2|\sigma_j\sigma_j^T|\Bigg),\end{aligned} \tag{38}$$

Now, take the following ICF:

$$\alpha_{j,1} = -c_{j,1}h_{j,1}S_{j,1} - \eta_{j,2}\zeta_{j,2} - \frac{1}{2r_{j,1}}\hat{W}_j h_{j,1}v_{j,1} + \dot{\eta}_{j,1}\zeta_{j,1}, \tag{39}$$

Replacing (18) and (39), which includes the design parameter $c_{j,1} > 0$, into (38) yields

$$\begin{aligned} \mathcal{L}V_1 \leq \sum_{j=1}^{m}\Big(&-\big(\lambda_{\min}(Q_j) - 3.5\big)\|e_j\|^2 - c_{j,1}v_{j,1}^2h_{j,1}^2 + \|P_j\|^2\|\varepsilon_j\|^2 + \|P_j\|^2 W_j^* \\ &+ \|P_j\|^2\|D_j^*\|^2 + \frac{1}{2}\|\sigma_j\sigma_j^T\|^2 + \frac{1}{2}\|P_j\|^2 + \frac{1}{2}d_{j,1}^{*\,2} \\ &+ \frac{1}{2}r_{j,1} + \frac{1}{2}\varepsilon_{j,1}^2 + h_{j,1}^2\|\sigma_j\sigma_j^T\| + \frac{1}{2r_{j,1}}\tilde{W}_j h_{j,1}^2 v_{j,1}^2\Big). \end{aligned} \tag{40}$$

Step i_l ($2 \leq i_l \leq n_j - 1$): Using (22)–(23), it is obtained

$$\dot{S}_{j,i_l} = h_{j,i_l}\left(\zeta_{j,i_l+1}\eta_{j,i_l+1} + \pi_{j,i_l+1} + k_{j,i_l}e_{j,1} - \dot{\eta}_{j,i_l}\zeta_{j,i_l} - \dot{\pi}_{j,i_l}\right), \tag{41}$$

where $h_{j,i_l} = \frac{1}{(1-\zeta_{j,i_l})^2\eta_{j,i_l}}$.

We present the state variable π_{j,i_l+1}, and pass α_{j,i_l} via the following filter

$$\overline{\sigma}_{j,i_l+1}\dot{\pi}_{j,i_l+1} + \pi_{j,i_l+1} = \alpha_{j,i_l}, \qquad \pi_{j,i_l+1}(0) = \alpha_{j,i}(0). \tag{42}$$

To compensate for the filter's output error, a compensation signal χ_{j,i_l} is described as follows:

$$\dot{\chi}_{j,i_l} = h_{j,i_l}\left(-c_{j,i_l}h_{j,i_l}\chi_{j,i_l} - \chi_{j,i_l-1} + \pi_{j,i_l+1} - \alpha_{j,i_l}\right), \quad \chi_{j,i_l}(0) = 0. \tag{43}$$

v_{j,i_l} is defined by:

$$v_{j,i_l} = S_{j,i_l} - \chi_{j,i_l}. \tag{44}$$

Then, we have

$$\dot{v}_{j,i_l} = h_{j,i_l}\left(\zeta_{j,i_l+1}\eta_{j,i_l+1} + k_{j,i_l}e_{j,1} - \dot{\eta}_{j,i_l}\zeta_{j,i_l} - \dot{\pi}_{j,i_l} + c_{j,i_l}h_{j,i_l}\chi_{j,i_l} + \chi_{j,i_l-1} + \alpha_{j,i_l}\right). \tag{45}$$

Let

$$V_{i_l} = \sum_{j=i_l}^{m}\left(V_{j,i_l-1} + \frac{1}{2}v_{j,i_l}^2\right). \tag{46}$$

The Itô differentiation rule of V_{i_l} along (45) yields

$$\begin{aligned} \mathcal{L}V_{i_l} = \sum_{j=1}^{m}\Big(&\mathcal{L}V_{j,i_l-1} + v_{j,i_l}h_{j,i_l}\Big(\zeta_{j,i_l+1}\eta_{j,i_l+1} + \bar{f}_{j,i_l}\left(\bar{Z}_{j,i_l}\right) \\ &- \frac{1}{2}v_{j,i_l}h_{j,i_l} - \dot{\eta}_{j,i_l}\zeta_{j,i_l} - \dot{\pi}_{j,i_l} + c_{j,i_l}h_{j,i_l}\chi_{j,i_l} + \chi_{j,i_l-1} + \alpha_{j,i_l}\Big)\Big), \end{aligned} \tag{47}$$

where

$$\overline{f}_{j,i_l}\left(\bar{Z}_{j,i_l}\right)=k_{j,i}e_{j,i_l}+\frac{1}{2}v_{j,i_l}h_{j,i_l}. \tag{48}$$

Similar to the previous step, we have

$$v_{j,i_l}h_{j,i_l}f_{j,i_l}\left(\bar{Z}_{j,i_l}\right)\leq\frac{1}{2}v_{j,i_l}^2h_{j,i_l}^2+\frac{1}{2r_{j,i_l}}v_{j,i_l}^2h_{j,i_l}^2W_j^*+\frac{1}{2}r_{j,i_l}+\frac{1}{2}\varepsilon_{j,i_l}^2, \tag{49}$$

where $r_{j,i_l}>0$, and $|\delta_{j,i_l}(\bar{Z}_{j,i_l})|\leq\varepsilon_{j,i_l}$. Substituting (49) into (47) yields

$$\begin{aligned}\mathcal{L}V_{i_j}\leq\sum_{j=1}^{m}\Bigg(&\mathcal{L}V_{j,i_j-1}+v_{j,i_l}h_{j,i_l}\Big(\zeta_{j,i_l+1}\eta_{j,i_l+1}-\dot{\eta}_{j,i_l}\zeta_{j,i_l}-\dot{\pi}_{j,i_l}+\frac{1}{2r_{j,i_l}}v_{j,i_l}h_{j,i_l}W_j^*\\&+c_{j,i_l}h_{j,i_l}\chi_{j,i_l}+\chi_{j,i_l-1}+\alpha_{j,i_l}\Big)+\frac{1}{2}r_{j,i_l}+\frac{1}{2}\varepsilon_{j,i_l}^2\Bigg),\end{aligned} \tag{50}$$

Take the ICF α_{j,i_l} as follows:

$$\alpha_{j,i_l}=-c_{j,i_l}h_{j,i_l}S_{j,i_l}-S_{j,i_l-1}-\zeta_{j,i_l+1}\eta_{j,i_l+1}-\frac{1}{2r_{j,i_l}}v_{j,i_l}h_{j,i_l}\hat{W}_j+\dot{\eta}_{j,i_l}\zeta_{j,i_l}+\dot{\pi}_{j,i_l}, \tag{51}$$

where $c_{j,i_l}>0$ is a positive parameter. Rewriting (50) by (40) and (51) we have

$$\begin{aligned}\mathcal{L}V_{i_l}\leq\sum_{j=1}^{m}\Bigg(&-\left(\lambda_{\min}(Q_j)-3.5\right)\|e_j\|^2-\sum_{k=1}^{i_l}c_{j,k}v_{j,k}^2h_{j,k}^2+\frac{1}{2}\|P_j\|^2+\|P_j\|^2\left\|D_j^*\right\|^2\\&+\|P_j\|^2\|\varepsilon_j\|^2+\|P_j\|^2W_j^*+\frac{1}{2}\left\|\sigma_j\sigma_j^T\right\|^2-\sum_{k=2}^{i_l}h_{j,k}v_{j,k}v_{j,k-1}+\frac{1}{2}\sum_{k=1}^{i_l}\varepsilon_{j,k}^2\\&+\frac{1}{2}\sum_{k=1}^{i_l}r_{j,k}+\frac{1}{2}{d_{j,i}^*}^2+h_{j,i}^2\left\|\sigma_j\sigma_j^T\right\|+\sum_{k=1}^{i_l}\frac{1}{2r_{j,k}}v_{j,k}^2h_{j,k}^2\tilde{W}_j\Bigg).\end{aligned} \tag{52}$$

Step i_k: $m_c+1\leq i_k\leq n_j-1$: using (24), one obtains

$$\dot{S}_{j,i_k}=S_{j,i_k+1}+\pi_{j,i_k+1}+k_{j,i_k}e_{j,1}-\dot{\pi}_{j,i_k}. \tag{53}$$

Now π_{j,i_k+1} is proposed, and suppose α_{j,i_k} passes the following filter:

$$\overline{\sigma}_{j,i_k+1}\dot{\pi}_{j,i_k+1}+\pi_{j,i_k+1}=\alpha_{j,i_k},\qquad\pi_{j,i_k+1}(0)=\alpha_{j,i_k}(0). \tag{54}$$

To mitigate the effect of the output error, a compensation signal χ_{j,i_k} is designed as follows:

$$\dot{\chi}_{j,i_k}=-c_{j,i_k}\chi_{j,i_k}-\chi_{j,i_k-1}+\chi_{j,i_k+1}+\pi_{j,i_k+1}-\alpha_{j,i_k},\quad\chi_{j,i_k}(0)=0. \tag{55}$$

We introduce the compensated signal as follows:

$$v_{j,i_k}=S_{j,i_k}-\chi_{j,i_k}. \tag{56}$$

Then, we have

$$\dot{v}_{j,i_k} = v_{j,i_k+1} + k_{j,i_k}e_{j,1} - \dot{\pi}_{j,i_k} + c_{j,i_k}\chi_{j,i_k} + \chi_{j,i_k-1} + \alpha_{j,i_k}. \tag{57}$$

Now, let

$$V_{i_k} = \sum_{j=1}^{m}\left(V_{j,i_k-1} + \frac{1}{2}v_{j,i_k}^2\right). \tag{58}$$

The Itô differentiation rule of V_{j,n_j} along (57) is given by

$$\begin{aligned}\mathcal{L}V_{i_k} = \sum_{j=1}^{m}\Bigg(&\mathcal{L}V_{j,i_k-1} + v_{j,i_k}\Big(v_{j,i_k+1} + \bar{f}_{j,i_k}\left(\bar{Z}_{j,i_k}\right)\\ &-\frac{1}{2}v_{j,i_k} - \dot{\pi}_{j,i_k} + c_{j,i_k}\chi_{j,i_k} + \chi_{j,i_k-1} + \alpha_{j,i_k}\Big)\Bigg),\end{aligned} \tag{59}$$

where

$$\bar{f}_{j,i_k}\left(\bar{Z}_{j,i_k}\right) = k_{j,i_k}e_{i,1} + \frac{1}{2}v_{j,i_k}. \tag{60}$$

As in the previous steps, we have

$$v_{j,i_k}f_{j,i_k}(\bar{Z}_{j,i_k}) \leq \frac{1}{2}v_{j,i_k}^2 + \frac{1}{2r_{j,i_k}}v_{j,i_k}^2W_j^* + \frac{1}{2}r_{j,i_k} + \frac{1}{2}\varepsilon_{j,i_k}^2, \tag{61}$$

where $r_{j,k} > 0$, and $|\delta_{j,i_k}(\bar{Z}_{j,i_k})| \leq \varepsilon_{j,i_k}$. Substituting (61) into (59) yields

$$\begin{aligned}\mathcal{L}V_{i_k} \leq \sum_{j=1}^{m}\Bigg(&\mathcal{L}V_{j,i_k-1} + v_{j,i_k}\Big(v_{j,i_k+1} - \dot{\pi}_{j,i_k} + \frac{1}{2r_{j,i_k}}v_{j,i_k}W_j^* + c_{j,i_k}\chi_{j,i_k}\\ &+\chi_{j,i_k-1} + \alpha_{j,i_k}\Big) + \frac{1}{2}r_{j,i_k} + \frac{1}{2}\varepsilon_{j,i_k}^2\Bigg),\end{aligned} \tag{62}$$

Take the ICF α_{j,i_k} as follows:

$$\alpha_{j,i_k} = -c_{j,i_k}S_{j,i_k} - S_{j,i_k-1} - \frac{1}{2r_{j,i_k}}v_{j,i_k}\hat{W}_j + \dot{\pi}_{j,i_k}, \tag{63}$$

where $c_{j,i_k} > 0$ represent a design parameter. Now rewriting (62) by (52) and (63) yields:

$$\begin{aligned}\mathcal{L}V_{i_k} \leq \sum_{j=1}^{m}\Bigg(&-(\lambda_{\min}(Q_j) - 3.5)\|e_j\|^2 - \sum_{k=1}^{m_c}c_{j,k}v_{j,k}^2h_{j,k}^2 - \sum_{k=m_c+1}^{i_k}c_{j,k}v_{j,k}^2\\ &+\|P_j\|^2W_j^* + \|P_j\|^2\|D_j\|^{*\,2} + \|P_j\|^2\|\varepsilon_j\|^2 + \frac{1}{2}\|P_j\|^2 + \frac{1}{2}|\sigma_j\sigma_j^T|^2\\ &+\frac{1}{2}d_{j,1}^{*\,2} + \sum_{k=1}^{i_k}\frac{1}{2}\varepsilon_{j,k}^2 + \sum_{k=1}^{i_k}\frac{1}{2}r_{j,k} - \sum_{k=2}^{m_c}h_{j,k}v_{j,k}v_{j,k-1}\\ &+\sum_{k=m_c+1}^{i_k}v_{j,k}\left(v_{j,k+1} - v_{j,i_k-1}\right) + h_{j,1}^2|\sigma_j\sigma_j^T|\\ &+\sum_{k=1}^{m_c}\frac{1}{2r_{j,k}}v_{j,k}^2h_{j,k}^2\tilde{W}_j + \sum_{k=m_c+1}^{i_k}\frac{1}{2r_{j,k}}v_{j,k}^2\tilde{W}_j\Bigg),\end{aligned} \tag{64}$$

Step n_j: From (24), it is obtained

$$\dot{S}_{j,n_j} = u_j + k_{j,n_j} e_{j,1} - \dot{\pi}_{j,n_j}. \tag{65}$$

The compensating signal χ_{j,i_k} is:

$$\dot{\chi}_{j,n_j} = -c_{j,n_j}\chi_{j,n_j} - \chi_{j,n_j-1}, \quad \chi_{j,n_j}(0) = 0. \tag{66}$$

Consider the compensated TE signals be defined as follows:

$$v_{j,n_j} = S_{j,n_j} - \chi_{j,n_j}. \tag{67}$$

Then, we have

$$\dot{v}_{j,n_j} = u_j + k_{j,n_j} e_{j,1} - \dot{\pi}_{j,n_j} + c_{j,n_j}\chi_{j,n_j} + \chi_{j,n_j-1}. \tag{68}$$

An proper Lyapunov function chosen as:

$$V = \sum_{j=1}^{m} V_{j,n_j} = \sum_{j=1}^{m} \left(V_{j,n_j-1} + \frac{1}{2} v_{j,n_j}^2 + \frac{1}{2\gamma_j} \tilde{W}_j^2 \right). \tag{69}$$

Where $\gamma_j > 0$, and the Itô differentiation rule of V along (68) is

$$\mathcal{L}V = \sum_{j=1}^{m} \Bigg(\mathcal{L}V_{j,n_j-1} + v_{j,n_j}\Big(u_j + \bar{f}_{j,n_j}(\bar{Z}_{j,n_j}) - \frac{1}{2} v_{j,n_j} - \dot{\pi}_{j,n_j} + c_{j,n_j}\chi_{j,n_j} + \chi_{j,n_j-1}\Big) - \frac{1}{\gamma_j}\tilde{W}_j \dot{\hat{W}}_j \Bigg), \tag{70}$$

where

$$\bar{f}_{j,n_j}(\bar{Z}_{j,n_j}) = k_{j,n_j} e_{j,1} + \frac{1}{2} v_{j,n_j}. \tag{71}$$

According to the method used in (34), the following inequality is obtained

$$v_{j,n_j} f_{j,n_j}(\bar{Z}_{j,n_j}) \leq \frac{1}{2} v_{j,n_j}^2 + \frac{1}{2r_{j,n_j}} v_{j,n_j}^2 W_j^* + \frac{1}{2} r_{j,n_j} + \frac{1}{2}\varepsilon_{j,n_j}^2, \tag{72}$$

where $r_{j,n_j} > 0$, and $\delta_{j,n_j}(\bar{Z}_{j,n_j}) \leq \varepsilon_{j,n_j}$. Substituting (72) into (70) yields

$$\mathcal{L}V \leq \sum_{j=1}^{m} \Bigg(\mathcal{L}V_{j,n_j-1} + v_{j,n_j}\Big(u_j - \dot{\pi}_{j,n_j} + \frac{1}{2r_{j,n_j}} v_{j,n_j} W_j^* + c_{j,n_j}\chi_{j,n_j} + \chi_{j,n_j-1}\Big) + \frac{1}{2} r_{j,n_j} + \frac{1}{2}\varepsilon_{j,n_j}^2 - \frac{1}{\gamma_j}\tilde{W}_j \dot{\hat{W}}_j \Bigg), \tag{73}$$

Now consider the control signal u_j, and the adaptive rule $\hat{W}_j$ as follows:

$$u_j = -c_{j,n_j} S_{j,n_j} - S_{j,n_j-1} - \frac{1}{2r_{j,n_j}} v_{j,n_j} \hat{W}_j + \dot{\pi}_{j,n_j}, \tag{74}$$

$$\dot{\hat{W}}_j = \sum_{k=1}^{m_c} \frac{\gamma_j}{2r_{j,k}} v_{j,k}^2 h_{j,k}^2 + \sum_{k=m_c+1}^{n_j} \frac{\gamma_j}{2r_{j,k}} v_{j,k}^2 - \gamma_j \sigma_j \hat{W}_j. \tag{75}$$

Where $c_{j,n_j} > 0$ and $\sigma_j > 0$ are positive parameters.

Replacing relations (64), (74) and (75) into the total Lyapunov function derivative (73), we have

$$\begin{aligned} \mathcal{L}V \leq \sum_{j=1}^{m} \Bigg(& - \big(\lambda_{\min}(Q_j) - 3.5\big) \|e_j\|^2 - \sum_{k=1}^{m_c} c_{j,k} v_{j,k}^2 h_{j,k}^2 - \sum_{k=m_c+1}^{n_j} c_{j,k} v_{j,k}^2 - \sigma_j \tilde{W}_j \hat{W}_j \\ & - \sum_{k=2}^{m_c} h_{j,k} v_{j,k} v_{j,k-1} - \sum_{k=m_c+1}^{n_j} v_{j,k} v_{j,k-1} + \sum_{k=m_c+1}^{n_j-1} v_{j,k} v_{j,k+1} + h_{j,1}^2 |\sigma_j \sigma_j^T| \\ & + \frac{1}{2}\|P_j\|^2 + \|P_j\|^2 \|D_j^*\|^2 + \|P_j\|^2 \|\varepsilon_j\|^2 + \|P_j\|^2 W_j^* + \frac{1}{2} |\sigma_j \sigma_j^T|^2 \\ & + \sum_{k=1}^{n_j} \frac{1}{2} \varepsilon_{j,k}^2 + \sum_{k=1}^{n_j} \frac{1}{2} r_{j,k} + \frac{1}{2} d_{j,1}^{*\,2} + h_{j,1}^2 \|\sigma_j \sigma_j^T\| \Bigg), \end{aligned} \tag{76}$$

Applying YI, yields

$$\tilde{W}_j \hat{W}_j \leq -\frac{1}{2} \tilde{W}_j^2 + \frac{1}{2} W_j^{*2}, \tag{77}$$

$$-v_{j,k} v_{j,k-1} \leq \frac{1}{2} v_{j,k}^2 + \frac{1}{2} v_{j,k-1}^2, \quad k = m_c + 1, \ldots, n_j, \tag{78}$$

$$v_{j,k} v_{j,k+1} \leq \frac{1}{2} v_{j,k}^2 + \frac{1}{2} v_{j,k+1}^2, \quad k = m_c + 1, \ldots, n_j - 1, \tag{79}$$

$$-h_{j,k} v_{j,k} v_{j,k-1} \leq \frac{1}{2} h_{j,k}^2 v_{j,k}^2 + \frac{1}{2} v_{j,k-1}^2, \quad k = 2, \ldots, m_c. \tag{80}$$

Now, we choose

$$\begin{cases} c_j + \frac{1}{2} < c_{j,1} h_{j,1}^2, \\ c_j + \frac{1}{2} h_{j,k}^2 + \frac{1}{2} < c_{j,k} h_{j,k}^2, \quad (k = 2, \ldots, m_c), \\ c_j + 1.5 < c_{j,k}, \quad (k = m_c + 1, \ldots, n_j - 1), \\ c_j + 1 < c_{j,n_j}. \end{cases} \tag{81}$$

Here, we provide further clarification on the physical intuition and practical selection of parameters related to Equation (81). The inequalities in Equation (81) are crucial constraints derived from the stability analysis using the Lyapunov function approach (see Equation (76)). Specifically, they dictate the relationship between the design parameters c_j and $c_{j,k}$ that are necessary to ensure the derivative of the Lyapunov function, $\mathcal{L}V$, can be shown to be negative definite (or negative semi-definite), which guarantees stability, as formally stated in Theorem 1.

Intuitive Interpretation: These parameters, c_j and $c_{j,k}$, can be intuitively understood as representing design choices that reflect the desired *penalty* or *cost* associated with the overall system's state (c_j) and the individual tracking errors of the subsystems ($c_{j,k}$), respectively. The inequalities in Equation (81) enforce a specific hierarchy and balance among these costs. For instance:

- The first inequality in (81) suggests that the overall system cost (c_j) must be sufficiently lower than a term scaled by the error cost and dynamics of the first subsystem ($c_{j,1}h_{j,1}^2$).
- Similarly, the subsequent inequalities establish relative bounds for the costs associated with other subsystems ($k = 2, \ldots, n_j$).

Essentially, these conditions guide the controller design to prioritize the management of dominant error sources while ensuring the stability of the entire system. A larger value for $c_{j,k}$ might indicate that the error in subsystem k is more critical to manage, thereby requiring a stricter bound on the overall cost c_j relative to it.

Practical Selection for Controller Implementation: From a practical standpoint, these conditions provide a valuable framework for tuning the controller parameters. Engineers aiming to implement this controller can select values for c_j and $c_{j,k}$ that satisfy these inequalities. This selection process should be guided by the specific performance requirements of the application and the relative importance of mitigating errors in each subsystem. By adhering to these constraints, one can ensure that the controller is not only theoretically stable but also robust in practice, effectively stabilizing the system despite its inherent dynamics and uncertainties. Further tuning based on simulation or experimental results may be performed around these conditions to optimize performance.

By choosing the parameters given in (81) and in view of (77) and (81), (76) becomes

$$\begin{aligned}\mathcal{L}V \leq \sum_{j=1}^{m}\Bigg(& -\big(\lambda_{\min}(Q_j) - 3.5\big)\|e_j\|^2 - c_j\sum_{k=1}^{n_j} v_{j,k}^2 - \frac{1}{2}\sigma_j\tilde{W}_j^2 + \sum_{k=1}^{n_j}\frac{1}{2}r_{j,k} \\ & + \sum_{k=1}^{n_j}\frac{1}{2}\varepsilon_{j,k}^2 + \frac{1}{2}d_{j,1}^{*\,2} + \|P_j\|^2\|\varepsilon_j\|^2 + \|P_j\|^2 W_j^* + \|P_j\|^2\|D_j^*\|^2 \\ & + \frac{1}{2}\|P_j\|^2 + \frac{1}{2}|\sigma_j\sigma_j^T|^2 + h_{j,1}^2|\sigma_j\sigma_j^T| + \frac{1}{2}\sigma_j W_j^{*\,2}\Bigg),\end{aligned} \tag{82}$$

Now, we define the following constants

$$C_1 = \min_{1\leq j\leq m}\Big\{\big(\lambda_{\min}(Q_j) - 3.5\big)/\lambda_{\max}(P_j),\ 2c_j,\ \gamma_j\sigma_j\Big\}, \tag{83}$$

$$C_2 = \sum_{j=1}^{m}\Bigg(\|P_j\|^2 W_j^* + \|P_j\|^2\|D_j^*\|^2 + \|P_j\|^2\varepsilon_j^2 + \frac{1}{2}\sum_{k=1}^{n_j} r_{j,k} + \sum_{k=1}^{n_j}\frac{1}{2}\varepsilon_{j,k}^2 + \frac{1}{2}d_{j,1}^{*\,2} + \frac{1}{2}\sigma_j W_j^* + \frac{1}{2}\|P_j\|^2 + \frac{1}{2}|\sigma_j\sigma_j^T|^2 + h_{j,1}^2|\sigma_j\sigma_j^T|\Bigg), \tag{84}$$

As a result, (82) becomes

$$\mathcal{L}V \le -C_1 V + C_2. \tag{85}$$

Integrating (85) over $[0, T]$ gives

$$0 \le E[V(t)] \le e^{-C_1 t}V(0) + \frac{C_2}{C_1}(1 - e^{-C_1 t}) \le V(0) + \frac{C_2}{C_1}, \tag{86}$$

From (46) and (86), we have

$$E\left[\sum_{j=1}^{m}\sum_{i=1}^{m_c}\frac{1}{2}v_{j,i_l}^2\right] \le V(0) + \frac{C_2}{C_1} \le V' + \frac{C_2}{C_1}, \le \sum_{j=1}^{m}\sum_{i=1}^{m_c} v_{j,i_l}'^{\,2} + V_W' + \frac{C_2}{C_1}, \tag{87}$$

where $V' = \frac{1}{2}\sum_{j=1}^{m}\left(\sum_{i_l=1}^{m_c} v_{j,i_l}'^{\,2} + \sum_{i_k=m_c+1}^{n_j} v_{j,i_k}'^{\,2} + \frac{1}{\gamma_j}\tilde{W}_j^2\right)$, $\sup_{\hat{x}_{j,i_l}} |v_{i,j}(\hat{x}_{j,i_l}, W_j(0))| \le v_{j,i_l}'$ and $V_W' = \frac{1}{2}\sum_{j=1}^{m}\sum_{i_k=m_c+1}^{n_i}\left(v_{j,i_k}'^{\,2} + \frac{1}{\gamma_j}|W_{j,\max}^* - W_j(0)|^2\right)$, $j = 1, \ldots, m$. Hence,

$$E\left[\sum_{j=1}^{m} v_{j,i_l}^2\right] \le \sum_{j=1}^{m} v_{j,i_l}'^{\,2} + 2\left(V_W' + \frac{C_2}{C_1}\right), \quad \forall i_l = 1, \ldots, m_c, \tag{88}$$

which provides

$$E\left[\sum_{j=1}^{m}|v_{j,i_l}|\right] \le \sqrt{\sum_{j=1}^{N} v_{j,i_l}'^{\,2} + 2\left(V_W' + \frac{C_2}{C_1}\right)}, \quad \forall i_l = 1, \ldots, m_c, \tag{89}$$

According to [10, Lemma 3], it can be inferred that compensating signals χ_{j,i_l} are bounded as

$$E[|\chi_{j,i_l}|] \le \frac{\mu_j}{2c_{j,0}}(1 - e^{-2c_{j,0}t}), \tag{90}$$

where $c_{j,0} = \frac{1}{2}\min_{j,i_j}(c_{j,i_j})$ and $|\pi_{j,i_j+1} - \alpha_{j,i_l}| \le \mu_j$. On the other hand, from (88) we can observe v_{j,i_l} is bounded in probability. Thus, (44) implies that S_{j,i_l} is bounded in probability as follows:

$$E[|S_{j,i_l}|] \le \frac{\mu_j}{2c_{j,0}}(1 - e^{-2c_{j,0}t}) + \sqrt{\sum_{j=1}^{N} v_{j,i_l}'^{\,2} + 2\left(V_W' + \frac{C_2}{C_1}\right)}, \tag{91}$$

Then, from (39) and (67), we have

$$E\left[\sum_{j=1}^{m}\sum_{i_k=m_c+1}^{n_j}\frac{1}{2}v_{j,i_k}^2\right]\leq V(0)+\frac{C_2}{C_1}, \tag{92}$$

and

$$E\left[\sum_{j=1}^{m}|v_{j,i_k}|\right]\leq\sqrt{2\left(V(0)+\frac{C_2}{C_1}\right)},\quad \forall i_k=m_c+1,\ldots,n_j, \tag{93}$$

Similar to (90), we know that ξ_{j,i_k} is bounded in probability. Furthermore, equation (94) demonstrates that $v_{j,k}$ is bounded in probability. Therefore, from (56) the boundedness of S_{j,i_k} in probability can be concluded as

$$E[|S_{j,i_k}|]\leq\frac{\mu_j}{2c_{j,0}}(1-e^{-2c_{j,0}t})+\sqrt{2\left(V(0)+\frac{C_2}{C_1}\right)}. \tag{94}$$

Therefore, $E[|S_{j,i_j}|]$ is confined in the set $\Omega_{|S_{j,i_j}}$, such that

$$\begin{cases}\Omega_{S_{j,i_l}}=\left\{S_{j,i_l}\in\mathbb{R}^{n_j}:E[|S_{j,i_l}|]\leq\frac{\mu_j}{2c_{j,0}}(1-e^{-2c_{j,0}t})+\sqrt{\sum_{j=1}^{N}v'^{\,2}_{j,i_l}+2\left(V'_W+\frac{C_2}{C_1}\right)}\right\}, \\ i_l=1,\ldots,m_c, \\ \\ \Omega_{S_{j,i_k}}:=\left\{S_{j,i_k}\in\mathbb{R}:E[|S_{j,i_k}|]\leq\frac{\mu_j}{2c_{j,0}}(1-e^{-2c_{j,0}t})+\sqrt{2\left(V(0)+\frac{C_2}{C_1}\right)}\right\}, \\ i_k=m_c+1,\ldots,n_j.\end{cases} \tag{95}$$

Due to the fact that $0<E(V(t))\leq V(0)+\frac{C_2}{C_1}\leq V'+\frac{C_2}{C_1}$, we have

$$\sum_{j=1}^{m}\frac{1}{2\gamma_j}\left|W_j^*-\hat{W}_j\right|^2\leq V'+\frac{C_2}{C_1},\qquad i_j=1,\ldots,n_j. \tag{96}$$

As a consequence of (96), $\hat{W}_j$ is guaranteed to remain in the compact set

$$\Omega_{W_j}=\left\{W_j\in\mathbb{R}:\left|\tilde{W}_j\right|\leq\sqrt{2\gamma_j\left(V'+\frac{C_2}{C_1}\right)},\ j=1,\ldots,m\right\}. \tag{97}$$

ii) Lemma 2 shows that $0<\zeta_{j,i_l}(0)<1$ and $\zeta_{j,i_l}(t)$ are continuous. Based on the definition of $0<\zeta_{j,i_l}(0)<1$, we can infer that $E\left[z_{j,i_l}(t)\right]<|\eta_{j,i_l}(t)|$, and conversely. Therefore, the PTEs $E\left[z_{j,i_l}\right]$ are confined to the set $\Omega_{S_{j,i_l}}$. This ensures that the PTE performance (both transient and steady-state) adheres to the prescribed constraints.

iii) Given results (i) and (ii), it follows that $E\left[v_{j,i_l}\right], E\left[v_{j,i_k}\right], E\left[S_{j,i_l}\right], E\left[S_{j,i_k}\right], E\left[z_{j,i_l}\right]$ and $|\hat{W}_j|, j = 1,\dots,m, i_l = 1,\dots,m_c, i_k = m_c+1,\dots,n_j$, are bounded in probability. We conclude that the control input signal u_j is bounded in probability.

iv) From (95), we have

$$\frac{1}{2}E\left[S_{j,i_k}^2\right] \le \frac{\mu_j}{2c_{j,0}}(1-e^{-2c_{j,0}t}) + (1-e^{-ct})\frac{C_2}{C_1} + V(0)e^{-c_1 t}, \qquad i_l = 1,\dots,m_c. \tag{98}$$

A slight modification of (98) lead to the following inequality:

$$\begin{aligned} E\left[\sum_{j=1}^{n_j}|z_{j,i_l}|\right] \le& \sqrt{2}\sum_{j=1}^{n_j}|\eta_{j,i_l}|(1-\zeta_{j,i_l}) \\ &\times\left(\sqrt{\frac{\mu_j}{2c_{j,0}}(1-e^{-2c_{j,0}t}) + (1-e^{-C_1 t})\frac{C_2}{C_1} + V(0)e^{-C_1 t}}\right), \end{aligned} \tag{99}$$

for $i_j = 1,\dots,m_j$.

As $t \to \infty$,

$$E\left[\sum_{j=1}^{n_j}|z_{j,i_l}|\right] \le \sqrt{2}\sum_{j=1}^{n_j}|\eta_{j,i_l}|(1-\zeta_{j,i_l})\left(\sqrt{\frac{\mu_j}{2c_{j,0}} + \frac{C_2}{C_1}}\right).$$

Consequently, the magnitude of the PTEs does not exceed the specified error bounds

Theorem 1 (Main Result)**.** Consider the stochastic CL system described by (3), the state observer (9), the intermediate control laws (39), (51) and (63), the control signal (74) and adaptive law (75) under Assumptions 1-3. Then, for initial conditions subject to $-\delta_{j,i_l}\rho_{j,i_l}(0) < z_{j,i_l}(0) < \rho_{j,i_l}(0)$ or $-\rho_{j,i_l}(0) < z_{j,i_l}(0) < \delta_{j,i_l}\rho_{j,i_l}(0)$, there exist suitable design parameters ensuring that all the signals in the controlled CL systems are SGUUB in probability. Also, the transient and steady-state performance of TEs within a predetermined range are guaranteed.

Proof. To establish the stability of the closed-loop stochastic system, we define the composite Lyapunov function candidate V exactly as formulated in equation (69):

$$V = \sum_{j=1}^{m} V_{j,n_j} = \sum_{j=1}^{m}\left(V_{j,n_j-1} + \frac{1}{2}v_{j,n_j}^2 + \frac{1}{2\gamma_j}\tilde{W}_j^2\right), \tag{100}$$

where V_{j,n_j-1} is the cumulative Lyapunov function from the preceding steps of the j-th subsystem. This term inherently includes the observer error energy $V_{j,0} = e_j^t P_j e_j$ and the compensated tracking errors $v_{j,k}$ for $k = 1,\dots,n_j-1$. The term $\tilde{W}_j = W_j^* - \hat{W}_j$ represents the neural network weight estimation error. Applying the infinitesimal generator $\mathcal{L}$ to (100) along the trajectories of the system yields:

$$\mathcal{L}V = \sum_{j=1}^{m}\left(\mathcal{L}V_{j,n_j-1} + v_{j,n_j}\mathcal{L}v_{j,n_j} + \frac{1}{2}\mathrm{Tr}\left\{\mathbf{G}_j^t\frac{\partial^2 V}{\partial\hat{x}_j^2}\mathbf{G}_j\right\} - \frac{1}{\gamma_j}\tilde{W}_j\dot{\hat{W}}_j\right). \tag{101}$$

By substituting the adaptive law (75), into the derivative of the weight error term, we obtain:

$$-\frac{1}{\gamma_j}\tilde{W}_j\dot{\hat{W}}_j = -\tilde{W}_j\left(\sum_{k=1}^{m_c}\frac{1}{2r_{j,k}}v_{j,k}^2 h_{j,k}^2 + \sum_{k=m_c+1}^{n_j}\frac{1}{2r_{j,k}}v_{j,k}^2\right) + \sigma_j\tilde{W}_j\hat{W}_j. \tag{102}$$

This expression is specifically designed to cancel the terms arising from the approximation of unknown nonlinearities using RBFNNs and to handle the quadratic error terms associated with the stochastic disturbances. By applying the property $\sigma_j\tilde{W}_j\hat{W}_j \leq -\frac{\sigma_j}{2}\tilde{W}_j^2 + \frac{\sigma_j}{2}W_j^{*2}$ and using the recursive design inequalities for each step k, the generator $\mathcal{L}V$ can be bounded as:

$$\mathcal{L}V \leq \sum_{j=1}^{m}\left(-\lambda_{\text{min}}(Q_j)\|e_j\|^2 - \sum_{k=1}^{n_j}c_{j,k}v_{j,k}^2 - \frac{\sigma_j}{2}\tilde{W}_j^2\right) + \beta, \tag{103}$$

where $c_{j,k}$ are positive design constants and $\beta = \sum_{j=1}^{m}(\frac{\sigma_j}{2}W_j^{*2} + \Delta_j)$, with Δ_j being a constant representing approximation errors and external disturbances. This leads to:

$$\mathcal{L}V \leq -\alpha V + \beta, \tag{104}$$

where $\alpha = \min_{j,k}\{\frac{\lambda_{\text{min}}(Q_j)}{\lambda_{\text{max}}(P_j)}, 2c_{j,k}, \gamma_j\sigma_j\}$. According to the stochastic stability theory, the inequality $\mathcal{L}V \leq -\alpha V + \beta$ implies that the expectation of the Lyapunov function $E[V(t)]$ remains bounded for all $t \geq 0$, which ensures that all signals in the closed-loop system, including e_j, $v_{j,k}$, and $\tilde{W}_j$, are SGUUB in probability. Furthermore, since $v_{j,k}$ is bounded and the compensation signals are bounded by design, the transformed errors $z_{j,k}$ are also bounded. Given the error transformation properties in (4), this guarantees that the original tracking errors $e_{j,k}$ satisfy $-\delta_{j,k}\rho_{j,k}(t) < e_{j,k}(t) < \rho_{j,k}(t)$, thus maintaining the prescribed performance for all $t \geq 0$. This completes the proof. □

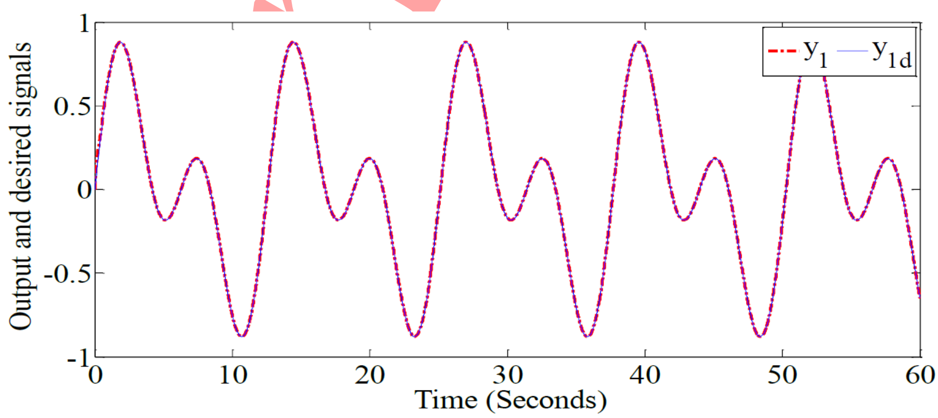

Figure 1: Tracking performance of the first output.The actual output $y_1(t)$ (solid line) closely follows the reference trajectory $y_{1,r}(t)$ (dash-dotted line), demonstrating accurate closed-loop tracking despite stochastic uncertainties.

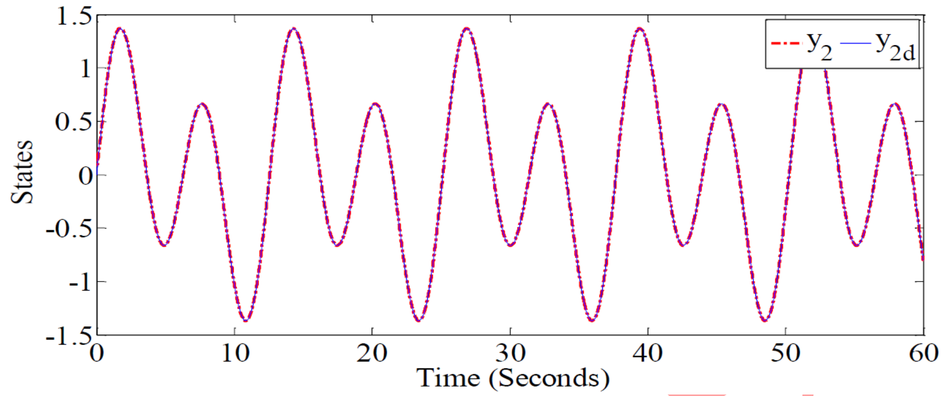

Figure 2: Tracking performance of the second output. The actual output $y_2(t)$ (solid) accurately follows the desired reference trajectory $y_{2,r}(t)$ (dash-dot), confirming the overall effectiveness of the adaptive controller in a stochastic MIMO setting.

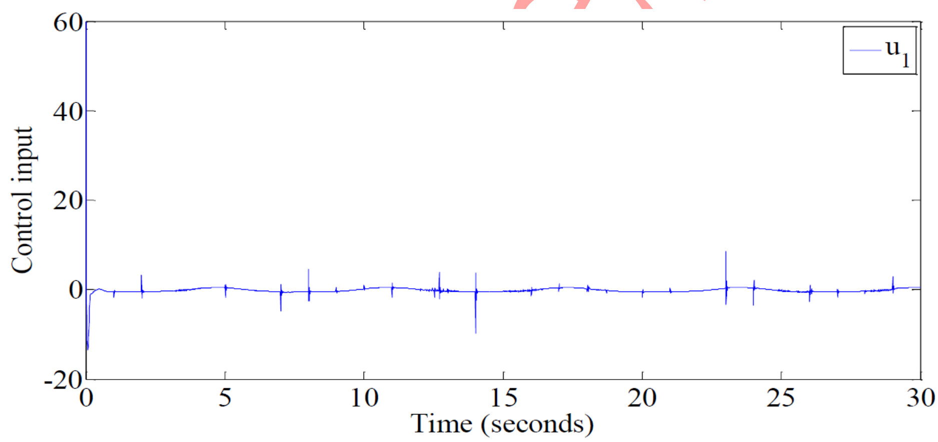

Figure 3: Time evolution of the first control input $u_1(t)$. The control signal remains smooth and bounded, consistent with the stability properties guaranteed by Theorem 1.

5 Simulation Results

The effectiveness of the proposed adaptive NN controller with OF for NSSs is demonstrated via simulation on a stochastic MIMO NSs (105). Moreover, comparisons with the proposed controllers in [41] and [13] are presented.

5.1 Simulation Example

Consider a MIMO-NSS with the following dynamics:

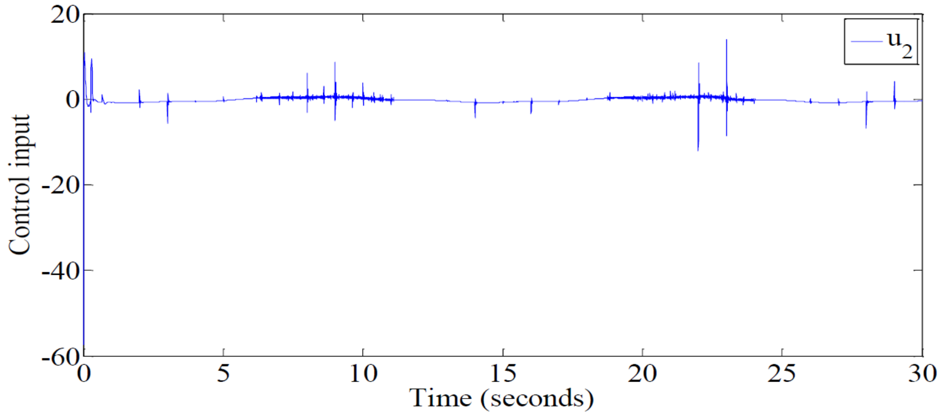

Figure 4: Time evolution of the second control input $u_2(t)$. The control effort remains bounded and well-regulated, showing that the adaptive law avoids excessive control activity.

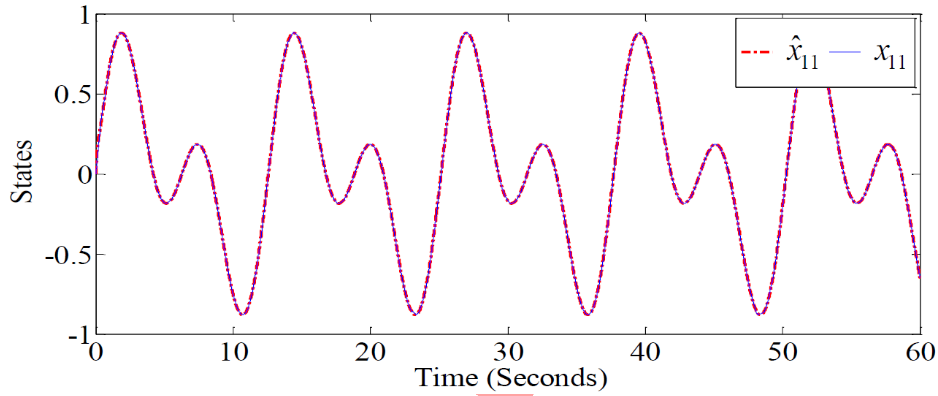

Figure 5: Comparison of the actual ($x_{1,1}(t)$ (dash-dot)) and estimated ($\hat{x}_{1,1}(t)$ (solid)) values of the first state. The close overlap of the curves verifies the accuracy of the adaptive neural estimator.

$$\begin{cases} dx_{j,1} = (x_{j,2} + f_{j,1}(\bar{x}_{j,1}) + d_{j,1})dt + g_{j,1}(\bar{x}_j)dw_j, \\ dx_{j,2} = (u_j + f_{j,2}(\bar{x}_{j,2}) + d_{j,2})dt + g_{j,2}(\bar{x}_j)dw_j, \\ y_j = x_{j,1}, \end{cases} \tag{105}$$

Here, $f_{1,1}(\bar{x}_{1,1}) = 0$, $f_{1,2}(\bar{x}_{1,2}) = x_{12}^2$, $f_{2,1}(\bar{x}_{2,1}) = 0.1x_{21}$, $f_{2,2}(\bar{x}_{2,2}) = \sin(x_{22})$, the stochastic disturbance functions are

$g_{1,1}(\bar{x}_1) = 0.1x_{11}\sin(x_{11}), g_{1,2}(\bar{x}_1) = 0.1x_{12}^2, g_{2,1}(\bar{x}_2) = 0.1\sin(x_{21}), g_{2,2}(\bar{x}_2) = 0.1\sin(x_{22}),$

external disturbance are $d_{11} = d_{12} = 0.1\sin(t), d_{21} = d_{22} = 0.1\cos(0.5t)$, and w_j is considered a Gaussian noise with mean zero, and variance 1.0.

The reference trajectories for the system are given as $y_{1,d} = 0.5\sin(t) + 0.5\sin(0.5t)$ and $y_{2,d} = 0.5\sin(0.5t) + \sin(t)$.

Our parameter selections for the observer (9), adaptation law (75), intermediate control (39), and control signal (74) are designed to ensure boundedness of all CL signals and accurate

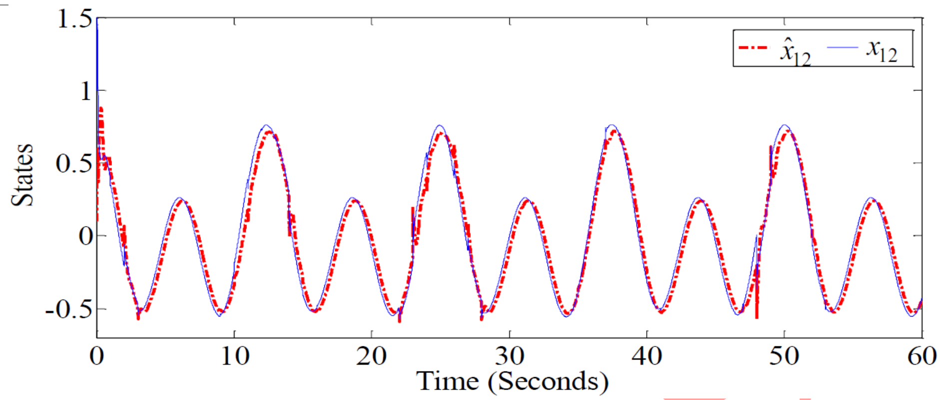

Figure 6: State variables (the curves of $x_{1,2}(t)$ (dash-dot) and $\hat{x}_{1,2}(t)$ (solid)).

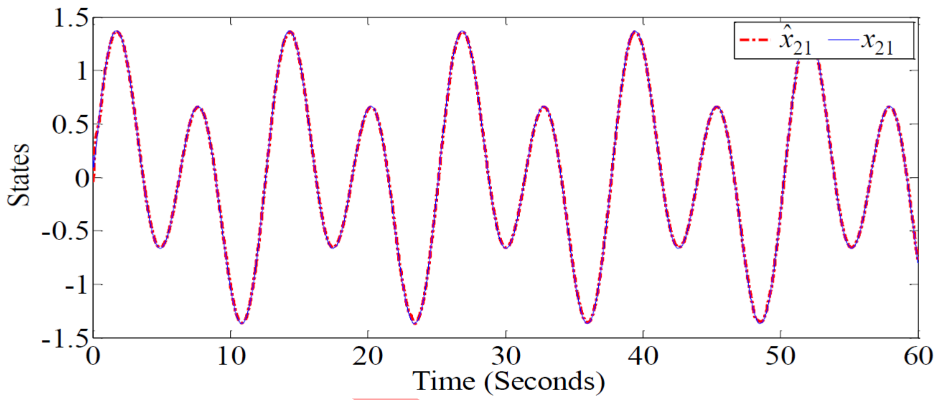

Figure 7: State variables (the curves of $x_{2,1}(t)$ (dash-dot) and $\hat{x}_{2,1}(t)$ (solid)). The estimator successfully captures the underlying stochastic dynamics and adapts to uncertainties in real time.

output trajectory tracking:

$$c_{11} = c_{12} = c_{21} = c_{22} = 1,$$

$$k_{1,1} = k_{2,1} = 7, k_{1,2} = 120, k_{2,2} = 100, \ell_{1,1} = \ell_{1,2} = 0.01, \gamma_1 = \gamma_2 = 1,$$

$$r_{11} = r_{12} = r_{21} = r_{22} = 1, \sigma_1 = 0.6, \sigma_2 = 0.7.$$

To achieve the desired performance, the following functions and parameters are chosen for controller (74):

$$\rho_{1,1} = 0.35e^{-2t} + 0.05,$$

$$\rho_{2,1} = 0.35e^{-2t} + 0.05,$$

$\delta_{1,1} = 0.5$ and $\delta_{2,1} = 0.5$.

We set the initial conditions

$$x_{11}(0) = 0, x_{12}(0) = 1, x_{21}(0) = 0.1, x_{22}(0) = -0.2, \hat{x}_{12}(0) = 0.1, \hat{x}_{22}(0) = 0.5,$$

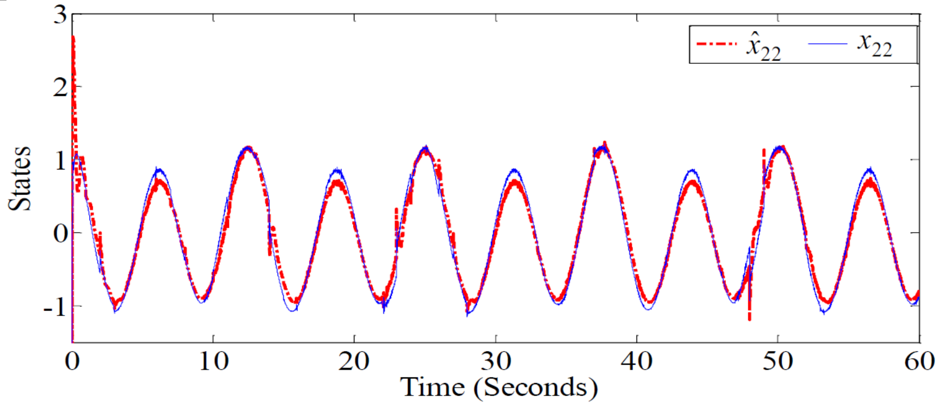

Figure 8: State variables (the curves of $x_{2,2}(t)$ (dash-dot) and $\hat{x}_{2,2}(t)$ (solid)). The close matching indicates reliable adaptive state approximation under stochastic disturbances.

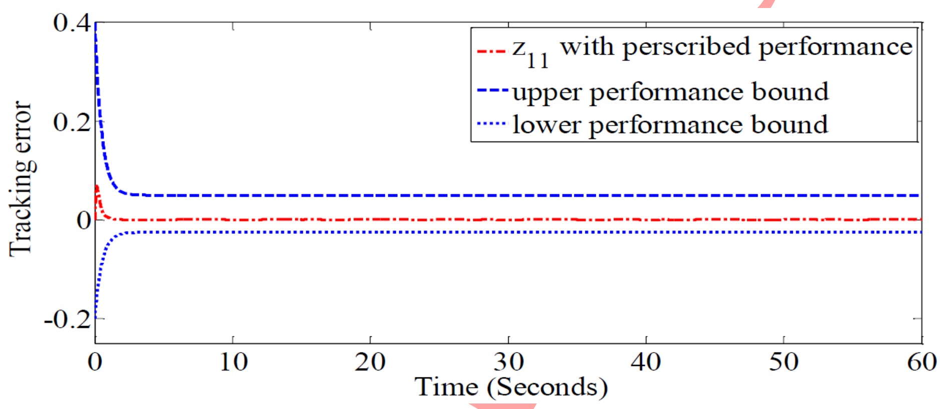

Figure 9: Tracking error $e_{1,1}(t)$ and performance bounds $\pm\varrho_{1,1}(t)$.

with all remaining states initialized to zero.

We apply the suggested adaptive NN control approach with OF to the MIMO-NSS (105). Figures 1 to 11 show the simulation results. Figures 1 and 2 compare the output trajectories with the desired signals. The corresponding control efforts are plotted by Figures 3 and 4, which show that they are bounded and reasonable. The convergence of the system states to their estimates in Figures 5–8 confirms the satisfactory performance of the state observer. From Figures 9 and 10 it can be inferred that the proposed approach satisfies the PP tracking and despite the different unknown functions present in (105), Figure 11 shows that only one parameter per subsystem needs to be tuned online.

As shown in the figures, the proposed adaptive NN control guarantees two key properties: (1) all CL signals are SGUUB, and (2) the TEs are restricted within predetermined performance ranges. Also, the above results show good control performance and TEs for the CL stochastic MIMO systems regardless of the structured uncertainties, unmeasured states and the stochastic and external disturbances.

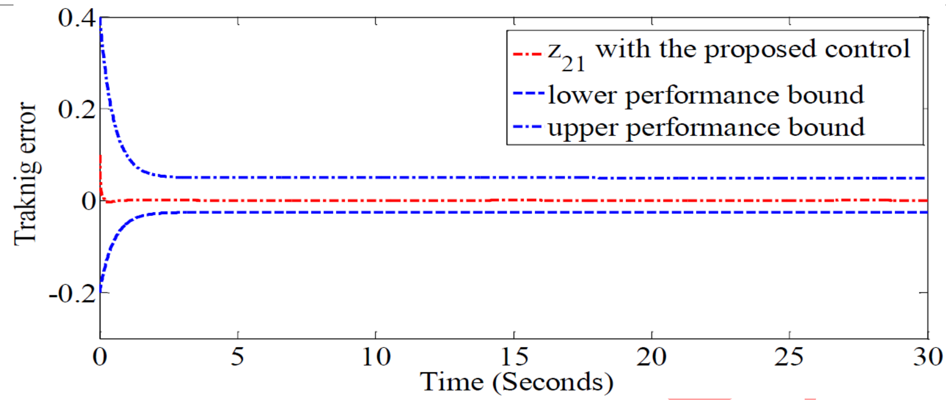

Figure 10: Tracking error $e_{2,1}(t)$ and performance bounds $\pm\varrho_{2,1}(t)$ for subsystem 2.

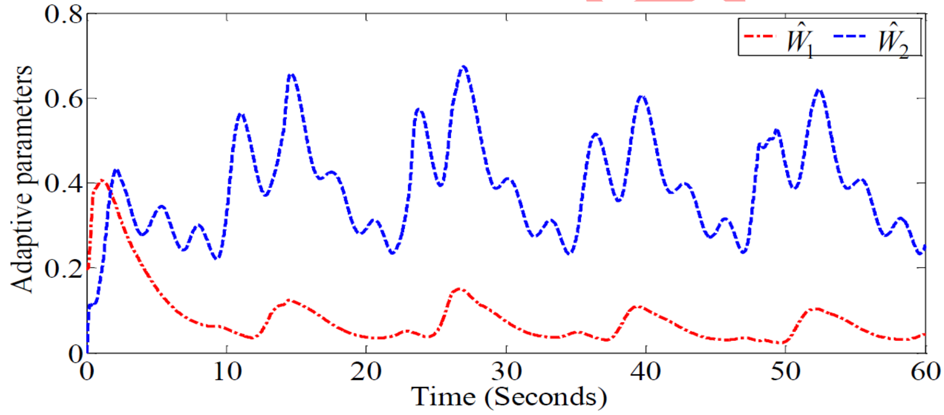

Figure 11: Evolution of the adaptive parameters $\hat{\theta}_1(t)$ and $\hat{\theta}_2(t)$ corresponding to the unknown lumped nonlinear functions. Their bounded adaptation behavior agrees with the Lyapunov-based theoretical analysis and illustrates how the controller compensates for stochastic uncertainties during operation.

Remark 2. It should be stated that the problem studied in this paper has also been investigated in [13] and [31]. But, the controlled systems in [13] and [31] are in the form of deterministic systems, not stochastic systems. Hence, these methods cannot be applied to control MIMO stochastic systems. On the other hand, in contrast to our proposed method, the control approaches in [14] and [28] suffer from “explosion of complexity”. Besides, they are not valuable for stochastic MIMO NSs.

5.2 Simulation Comparison

This subsection provides a comparison of the proposed control scheme against the existing algorithms found in [41] and [14]. We employ identical initial conditions and parameters for all approaches. The controller’s performance is governed by the following parameters and functions:

$$\rho_{1,1} = \rho_{2,1} = 0.35e^{-2t} + 0.05,$$

$\delta_{1,1} = 0.5$ and $\delta_{2,1} = 0.5$. Now, we employ the control scheme [41] and [14] to control MIMO-NSS (105). The simulation results are plotted by Figures 12 and 13.

Moreover, for a quantitative comparison, the performance indexes of the three controllers are summarized in Table I. For a fair comparison of the mentioned control schemes in quantitative terms, the following performance indicators are defined in [39]:

- $RMS(z_{j,1})$ represents the *Root Mean Square* of the TE. It is applied to compare the average tracking performance as follows:

$$RMS(z_{j,1}) = \sqrt{(1/T_f)\int_0^{T_f} |z_{j,1}|^2 dt}, \quad j = 1, 2,$$

  where T_f and z_{j1} are the total execution time and TE for the j-th subsystem, respectively.

- $RMS(u_j)$ represents the RMS of the control effort. It is applied to compare the amount of control efforts performance as follows:

$$RMS(u_j) = \sqrt{(1/T_f)\int_0^{T_f} |u_j|^2 dt}, \quad j = 1, 2.$$

- $IAE(z_{j,1})$ represents the *Integral of Absolute Error (IAE)* as follows:

$$IAE(z_{j,1}) = \int_0^{T_f} |z_{j,1}|, \quad j = 1, 2.$$

- $MAE(z_{j1})$, or maximum absolute error, is used to evaluate controllers' transient performance.

$$MAE(z_{j,1}) = \max_t\{|z_{j1}|\}, \quad j = 1, 2.$$

It is observed from Figures 12 and 13 that the control approach [41] does not yield satisfactory performance, and TEs violate the desired error range. Besides, a comparison in Table 2 further shows the proposed approach's superiority in both steady-state and transient performance over the controller from [41]. This is because the control approach is not equipped with the PP approach. Furthermore, the results in Figures 12 and 13 and Table 2 indicate that the control approach in [14] successfully restricts TEs to the predefined bounds, similar to the method proposed herein. However, the performance indexes for TEs and control efforts in this paper are smaller than in [14].

Remark 3. According to the simulation results, Figures 1 and 2 illustrate that the system outputs closely follow the reference trajectories, confirming the closed-loop tracking performance.

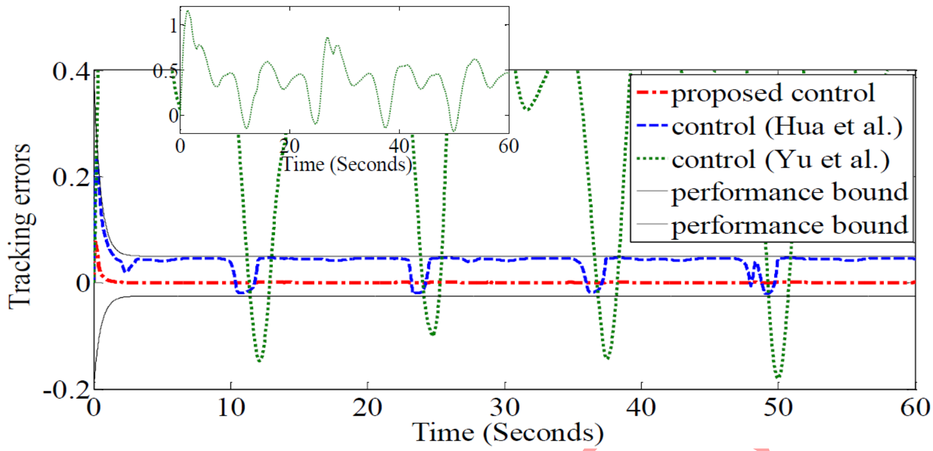

Figure 12: Comparison of tracking errors for the first subsystem using different controllers.

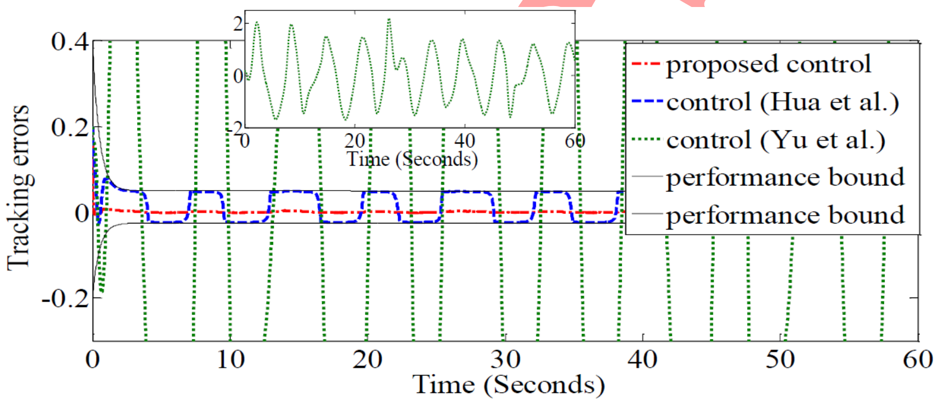

Figure 13: Comparison of tracking errors for the second subsystem using different controllers.

Figures 3 and 4 show that the control inputs remain smooth and bounded, as guaranteed by Theorem 1. Figures 5 and 8 compare the actual and estimated states, where the close matching of the curves verifies the accuracy of the adaptive NN estimation.

Also, Figures 9 and 10 demonstrate that the tracking errors $(e_{1,1}(t), e_{2,1}(t))$ are confined within the derived performance bounds $(\pm\varrho_{1,1}(t), \pm\varrho_{2,1}(t))$, providing empirical validation for the closed-loop performance guarantees established in Theorem 1.

Moreover, Figures 12 and 13 present a comparative analysis of the proposed controller against existing methods [41] and [14]. The results clearly indicate that while both the proposed method and [14] achieve successful convergence, our approach exhibits superior performance in terms of control effort and convergence rate. Notably, method [41] fails to achieve convergence under the given conditions. Figure 11 displays the adaptive parameters associated with the unknown nonlinear functions. These parameters remain bounded and their evolution reflects the real-time compensation of uncertainties. Their convergence behavior is fully consistent with the Lyapunov-based analysis presented in Theorem 1.

Table 2: The quantitative comparative study of the control schemes in [41], [14] and the proposed control scheme.

Performance Index	Proposed control	Control [41]	Control [14]
RMS(z_{11})	0.006	0.66	0.06
RMS(z_{21})	0.005	1.41	0.05
IAE(z_{11})	0.068	25.03	2.55
IAE(z_{21})	0.079	51.29	2.07
MAE(z_{11})	0.07	0.98	0.24
MAE(z_{21})	0.008	2.75	0.08
RMS(u_1)	2.03	1.43	2.155
RMS(u_2)	0.83	4.88	4.96

5.2.1 Minimal Learning Parameter Technique

Our proposed adaptive control framework incorporates a *minimal learning parameter technique*. This approach is designed to regulate the adaptation of the learning parameters, preventing overly aggressive or rapid updates that can destabilize the system or lead to undesirable control actions. Specifically, the adaptation law is formulated such that parameter convergence is smooth and robust, guided by a carefully designed learning rate that considers system dynamics and uncertainty bounds. By limiting the magnitude and rate of parameter adjustments, this technique effectively reduces control signal oscillations (chattering) and consequently minimizes the overall control effort. This contrasts with conventional adaptive methods that might employ simpler, less constrained adaptation laws, potentially leading to higher control signals and increased tracking errors when faced with significant uncertainties or disturbances.

5.2.2 Compensation Signals for Filter Errors

Furthermore, the robustness and accuracy of our controller are enhanced by explicit *compensation signals for filter errors*. In practical implementations, adaptive filters (used for state estimation, noise reduction, or uncertainty bounding) may introduce residual errors. These errors, if unaddressed, can propagate and degrade the overall tracking performance. Our control law integrates terms that actively estimate and compensate for these filter-induced errors. By feeding back a signal that counteracts the inaccuracies stemming from the filter's operation, we ensure that the control system maintains its trajectory-tracking precision. This mechanism is crucial for mitigating the impact of filter imperfections and unmodeled dynamics, allowing for more precise control compared to methods that do not explicitly account for such error sources.

5.2.3 Analysis of Control Effort and Tracking Errors

Table 2 presents a quantitative comparison of our proposed method against existing approaches [14] and [41]. As evidenced by the data, our method achieves a significantly lower control effort and reduced tracking errors. These performance gains are directly attributable to the specialized components of our control strategy. The *minimal learning parameter technique* plays a pivotal role in reducing control effort by ensuring smoother and more regulated adaptation of control parameters, thereby preventing excessive control signal amplitudes and oscillations. Concurrently, the implementation of *compensation signals for filter errors* actively mitigates performance degradation caused by inaccuracies in state estimation or filtering stages. This dual approach allows our controller to maintain high tracking precision while demanding less control action, outperforming methods [14] and [41] which, lacking these specific mechanisms, exhibit higher control effort and larger tracking deviations, particularly under challenging operating conditions.

6 Conclusion

This paper has addressed the challenging problem of adaptive OF tracking control with PP for uncertain MIMO NSSs. The proposed control framework integrates four key elements: (i) a linear Luenberger-type state observer for handling unmeasured states; (ii) RBFNNs for approximating unknown nonlinear dynamics; (iii) a CF-augmented DSC strategy with compensation signals to simultaneously resolve the "explosion of complexity" and filter-output error problems inherent in conventional backstepping; and (iv) an MLP technique based on Young's inequality, which reduces the number of online-adjustable parameters to a single scalar per subsystem. Rigorous Lyapunov-based stochastic stability analysis establishes that all closed-loop signals are SGUUB in probability, and that the PTEs remain confined within user-defined PP bounds at all times. Comparative simulation studies on a second-order MIMO stochastic benchmark confirm significant improvements in tracking precision and control efficiency relative to the methods of Hua et al. [14] and Yu et al. [41].

Research Limitations: Several limitations of the current work should be acknowledged. First, the proposed architecture is restricted to systems in strict-feedback form; pure-feedback and non-strict-feedback topologies are not covered. Second, the PP framework requires the initial tracking errors to lie strictly within the prescribed bounds, which depends on accurate initialization. Third, the linear observer design assumes that the system nonlinearities satisfy a growth condition compatible with the observer gain selection; rapidly growing nonlinearities may compromise observer performance. Fourth, the analysis assumes bounded external disturbances; the case of unbounded or structurally unknown disturbances remains open.

Future Research Directions: Based on the above limitations, the following extensions are recommended for future investigation: (1) Extension to MASs with distributed observers and event-triggered communication, addressing scalability and communication delay constraints. (2) Generalization to pure-feedback and non-triangular stochastic systems using implicit function theorem-based approaches. (3) Integration of finite-time or fixed-time convergence guarantees within the PP framework. (4) Application to physical engineering systems such as robotic manipulators, flexible joint mechanisms, or networked control systems subject to packet dropouts. (5) Incorporating input constraints such as saturation and quantization within the current design framework.

Declarations

Availability of Supporting Data

All data generated or analyzed during this study are included in this published article.

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Conflict of Interest

The author declares no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Author Contributions

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Javad Zanganeh: Conceptualization, Software and Validation, Data Curation, Writing – Review & Editing, Visualization, Supervision

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