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A Model-Free Mamdani Fuzzy Controller for Moving Target Tracking with Safety Distance Enforcement in Non-Holonomic Mobile Robots

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Abstract. Non-holonomic mobile robots are widely deployed in industrial and service environments, yet designing controllers for moving-target tracking under nonholonomic constraints remains a challenging open problem. In this paper, we present a Mamdani-type fuzzy logic tracking controller specifically designed for non-holonomic mobile robots pursuing dynamic targets with arbitrary movement patterns. Although tailored to differential-drive platforms, the proposed architecture can be extended to other types of mobile robots and autonomous systems. A key feature of the controller is the explicit enforcement of a predefined safe distance between the robot and the target, preventing collision while simultaneously supporting covert or low-detection tracking applications. A significant advantage of this model-free architecture is its computational efficiency: the system operates on minimal sensor inputs, requires no dynamic model of the robot, and can be seamlessly deployed on low-cost sensing hardware, making it well-suited for energy-constrained platforms. Lyapunov-based stability analysis is provided for the closed-loop system, and the methodology is validated through simulation on a differential-drive robot model across multiple complex scenarios in a virtual environment, including cases with high measurement noise. The comprehensive simulation results confirm that the controller achieves robust stability and high tracking precision, demonstrating its practical acceptability for real-time target tracking applications.

Keywords. Fuzzy logic, Moving target tracking, Non-holonomic mobile robot, Mamdani fuzzy controller.

MSC. 93C42; 93C85; 93D05.

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1 Introduction

Autonomous mobile robots have become an integral part of modern life and are widely deployed in industrial, service, and reconnaissance applications [12]. Among the most active research directions in mobile robotics is the control of systems subject to nonholonomic constraints, which remains a challenging problem owing to the inherently nonlinear structure of such systems [2, 18, 25, 37]. Nonholonomic mobile robots—particularly differential-drive platforms—occupy a prominent position in this landscape due to their mechanical simplicity and broad industrial relevance [1, 32]. Consequently, a substantial body of work has addressed their modelling, control, and practical deployment [2, 18, 25, 37].

Tracking a moving target introduces an additional layer of complexity beyond conventional point-to-point navigation. Unlike fixed-destination routing, where the robot can converge to a static goal via a pre-planned path or a stable control law, in the moving-target problem the destination evolves continuously in time, causing both position error and heading error to vary without bound unless actively regulated [8, 15]. The target's motion is typically more diverse and dense than in structured trajectory-following tasks, with higher clutter density and unpredictable heading changes. Tracking a moving ground target using non-holonomic robots has therefore long been an important topic of interest, with applications spanning cooperative robot identification [21], military and reconnaissance operations, multi-robot system alignment [6, 19], companion robots for the elderly and mobility-impaired, trajectory tracking [28, 36], and autonomous target docking [32]. Unmanned ground robots (UGRs) are particularly well suited for this task: since their speed is comparable to that of typical ground targets—unlike unmanned aerial robots (UARs)—they can maintain a consistent tracking distance during pursuit, enabling stealth and covert operation. Human factors studies have further highlighted the importance of proper safety-distance management in robot–target interactions [23], reinforcing the need for explicit distance constraints in controller design.

The problem of mobile robot navigation and tracking has been studied extensively from multiple perspectives [8, 10, 15, 22, 24]. Path-planning approaches typically separate the navigation problem into global planning and local execution phases [10, 15, 17, 38], while reactive methods attempt to generate control actions directly from sensor measurements without an explicit map [14, 22, 24]. Some methods rely on full dynamic models and no-slip assumptions, which lead to increased computational complexity and longer controller runtime [7, 26]. Others require high-bandwidth sensing modalities such as cameras or laser rangefinders, imposing additional economic and computational costs on the platform [4, 9, 13, 30].

Among the approaches that avoid explicit dynamic modelling, fuzzy logic controllers have attracted considerable attention due to their ability to encode human driving intuition into linguistic if-then rules without requiring a precise mathematical model of the system [1, 22]. Benbouabdallah and Zhu [3] employs a fuzzy controller for moving-target tracking in which the

Particle Swarm Optimisation (PSO) algorithm is used to tune the fuzzy membership functions, improving performance at the expense of a high offline computational cost. Tolossa et al. [29] similarly propose a fuzzy logic controller with optimally tuned parameters for trajectory tracking, demonstrating that systematic tuning can improve accuracy, but noting that the added optimisation overhead limits real-time applicability. Vision-based approaches provide rich perceptual information but introduce significant computational load and require additional hardware independent of the robot kinematic model [4, 9, 13, 20]. The literature also reflects considerable diversity in the types of controllers proposed for mobile robot tracking. These include adaptive control [7, 20], model predictive control [33], neural network-based methods [6], reinforcement learning [27], and deep reinforcement learning [9, 11, 34], as well as hybrid combinations of these strategies [35, 38]. Although each approach can be effective, all carry their own trade-offs in terms of computational load, model dependency, sensor requirements, and generalisability.

In this paper, a fuzzy control-based tracking method is proposed and a formal Lyapunov-based stability proof is provided. Unlike many existing studies, which consider only the pursuit of another nonholonomic robot following a structured path, the proposed approach handles arbitrary moving targets whose motion is unconstrained. This generalisation substantially increases the method's applicability to real-world objects such as pedestrians, vehicles, and rolling objects, which has great practical significance for engineering. A key novelty of this work is the explicit incorporation of a minimum safety distance constraint to prevent collision: the fuzzy rules are designed so that both the robot-target distance and the line-of-sight bearing angle converge rapidly and simultaneously to their desired values. Furthermore, the controller is designed to operate on minimal sensor inputs—only the distance and heading angle to the target—so as to reduce hardware cost and complexity. This simplicity is especially valuable in energy-constrained or resource-limited platforms such as those used in military reconnaissance or disaster-response scenarios, where structural minimality directly translates to extended operational endurance [30].

In this paper, we propose a moving-target tracking method using a Mamdani fuzzy controller formulated in polar coordinates, without relaxing the nonholonomic constraint of the robot. The control laws for linear velocity and angular velocity are adjusted to ensure that both the robot-target distance and the azimuth angle converge to their ideal tracking values. Simulation results confirm the effectiveness of the approach. The main contributions of this paper are:

i. Development of a Mamdani-type fuzzy controller for tracking moving targets with safety distance enforcement.

ii. Extension of moving-target tracking from nonholonomic robot pursuit to the pursuit of arbitrary targets with unconstrained motion.

iii. Formal Lyapunov-based stability analysis of the closed-loop tracking system.

iv. Demonstration of controller robustness through multiple simulation scenarios, including a high-noise measurement case.

The remainder of the paper is arranged as follows. Section 2 presents the problem formulation and robot model. Section 3 describes the fuzzy controller design. Section 4 provides simulation results. Finally, conclusions, limitations, and directions for future work are presented in Section 5.

2 Problem Statements

This section formalises the moving-target tracking problem for a nonholonomic mobile robot. Moving-target tracking differs fundamentally from conventional point-to-point routing. In conventional routing, the destination is fixed and the robot can reach it via a pre-planned path or a stabilising control law [10, 16, 17]. In the tracking problem, however, the destination evolves continuously in time, causing both position error and heading error to vary without bound unless actively regulated. The robot must simultaneously approach the target, maintain a predefined safe distance to avoid collision, satisfy its own kinematic constraints, and respond appropriately to sudden changes in the target's trajectory.

When the target is stationary, it suffices for the robot's Cartesian coordinates to converge to the fixed target coordinates (x_T, y_T). For a moving target, however, both the instantaneous target position and its direction of motion must be incorporated into the controller at every time step. This becomes especially important when the robot is nonholonomic. Nonholonomic robots, such as two-wheeled differential-drive platforms, cannot translate freely in all directions: they have no velocity component along the axis perpendicular to the wheel rotation axis (i.e., they cannot slide sideways). Consequently, the robot's configuration in the plane is described by three state variables (x, y, θ), yet motion can only be commanded through two inputs—linear velocity V and angular velocity Ω. This discrepancy between the dimension of the configuration space and the number of independent control inputs renders the tracking problem inherently nonlinear and constrained [5, 32]. From a mathematical standpoint, the resulting constraint is non-integrable, meaning it cannot be reduced to a holonomic constraint on the coordinates alone, which substantially increases the complexity of modelling and control [18, 25].

2.1 Kinematic Model of the Mobile Robot

The mobile robot considered in this study is a differential-drive platform. It comprises two fixed wheels driven by independent motors and a passive omnidirectional caster for balance, as

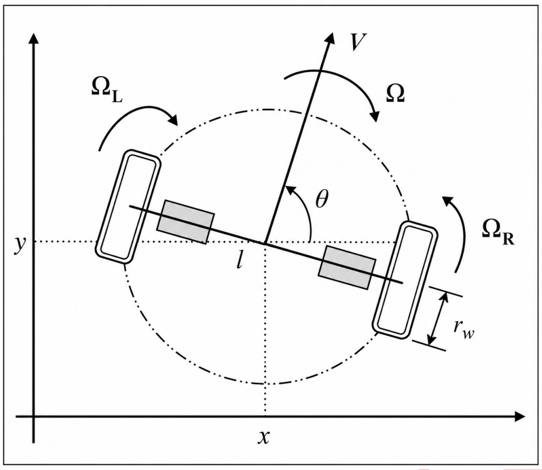

Figure 1: Kinematics of the mobile robot.

shown in Figure 1. The angular velocities of the right and left wheels are denoted Ω_R and Ω_L, respectively, and constitute the only directly controllable variables in this system.

Let l denote the track width (centre-to-centre distance between the two wheels) and r_w the wheel radius. The robot's linear velocity $V(t)$ and angular velocity $\Omega(t)$ are then given by [5]:

$$V(t) = \frac{r_w}{2}\big(\Omega_L(t) + \Omega_R(t)\big), \tag{1}$$

$$\Omega(t) = \frac{r_w}{l}\big(\Omega_L(t) - \Omega_R(t)\big). \tag{2}$$

The configuration of the robot at time t is described by the state vector $X = [x,\, y,\, \theta]^T$, whose time evolution satisfies:

$$\dot{X}(t) = f\big(X(t),\, V(t),\, \Omega(t)\big). \tag{3}$$

Substituting (1)–(2) into the standard unicycle kinematics yields the following system of differential equations [5]:

$$\begin{cases} \dot{x}(t) = V(t)\cos\theta(t), \\ \dot{y}(t) = V(t)\sin\theta(t), \\ \dot{\theta}(t) = \Omega(t), \end{cases} \tag{4}$$

which can be written compactly in matrix form as:

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V \\ \Omega \end{bmatrix}, \tag{5}$$

where $[x,\, y]^T \in \mathbb{R}^2$ are the Cartesian coordinates of the robot in the plane Ψ and θ is its heading angle.

Since the robot has no velocity component perpendicular to the wheel axis, the following constraint holds at all times:

$$\begin{bmatrix} -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = 0. \tag{6}$$

Equation (6) is non-integrable and constitutes the nonholonomic constraint of the system.

Remark 1. If multiple robots are present, the index i identifies each robot, with V_i and θ_i denoting its linear velocity and heading angle, respectively. For clarity, this paper considers a single tracking robot ($n = 1$).

Remark 2. The proposed controller requires knowledge of the robot's current configuration and the target's relative distance and bearing. In this study these quantities are assumed to be provided reliably by an onboard sensor suite through standard data-fusion techniques [21, 30].

2.2 Target Motion Model

The motion of the moving target is described by:

$$\begin{cases} r(t) = \big(r_x(t),\, r_y(t)\big) = (x_T,\, y_T), \\ \|\dot{r}(t)\| \le \nu_T, \end{cases} \tag{7}$$

where $r(t)$ is the target position, $[x_T,\, y_T]^T \in \mathbb{R}^2$ are its coordinates in Ψ, and $\nu_T > 0$ is an upper bound on its speed. The velocity components along the X^d and Y^d axes, denoted ν_x and ν_y, are both bounded. It is assumed throughout that the robot's maximum speed exceeds that of the target, i.e. $\nu_R > \nu_T$, which is a necessary condition for guaranteed tracking.

Figure 2 illustrates the geometric relationship between the robot and the moving target.

As shown in Figure 2, the Euclidean distance between the robot and the target is:

$$d = \sqrt{(x - x_T)^2 + (y - y_T)^2}. \tag{8}$$

Defining $d_x = x - x_T$ and $d_y = y - y_T$, the absolute bearing angle β from the robot to the target satisfies:

$$\beta = \tan^{-1}\left|\frac{d_y}{d_x}\right|. \tag{9}$$

Because the signs of d_x and d_y change as the robot and target move relative to each other, the four-quadrant `atan2` function is preferred for computing β without ambiguity:

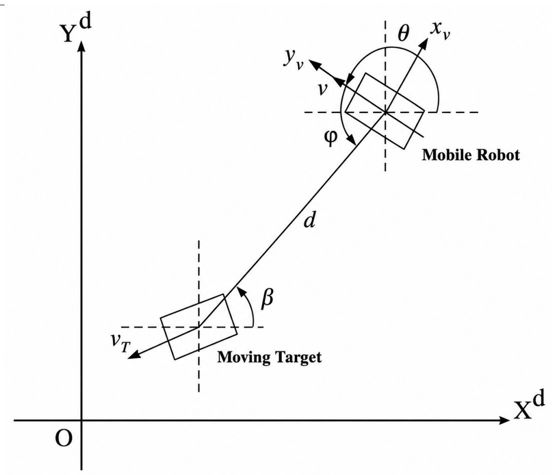

Figure 2: Tracking a moving target at time t.

$$\operatorname{atan2}(y,x)=\begin{cases}\arctan\left(\frac{y}{x}\right), & x>0,\\ \arctan\left(\frac{y}{x}\right)+\pi, & y\geq 0,\ x<0,\\ \arctan\left(\frac{y}{x}\right)-\pi, & y<0,\ x<0,\\ +\frac{\pi}{2}, & y>0,\ x=0,\\ -\frac{\pi}{2}, & y<0,\ x=0,\\ \text{undefined}, & y=0,\ x=0.\end{cases} \tag{10}$$

Given Figure 2 and knowledge of the robot heading θ, the line-of-sight angle φ can be computed directly in all scenarios and always satisfies $\varphi \in (-\pi, \pi]$.

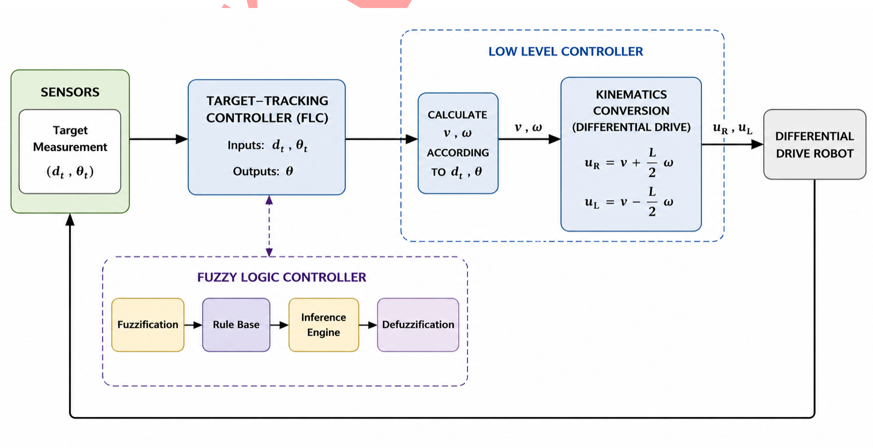

Figure 3: Mobile robot target tracking control principle scheme.

3 Fuzzy Controller Design

The overall structure of the control system is shown in Figure 3. This section presents the design of the main target-seeking controller; low-level wheel-speed regulation and hardware interfacing are outside its scope. A fuzzy control strategy is adopted for the tracking task. The central idea of fuzzy control is the encoding of human knowledge and intuition about a process into a set of linguistic if-then rules, which are then evaluated through membership functions and a fuzzy inference engine [1, 31]. In contrast to model-based methods [7, 26], fuzzy control does not require an explicit mathematical model of the system dynamics: it is sufficient to specify the qualitative relationship between the controller inputs and the desired output. For example, if the target is far away and to the left, the robot should accelerate and simultaneously turn left. Such rules are intuitive and readily implementable as membership functions and rule tables [22, 31].

We assume that the robot's maximum linear speed is bounded by $V_{\max}(t) = \nu_R$, and that the angular velocity Ω is generated by a Mamdani-type fuzzy controller. The moving target $r(t)$ is assumed to travel with speed bounded by $\nu_T < \nu_R$. The relative position and bearing variables are taken as measurable via onboard sensors [21, 30]. The control objective is to determine appropriate laws for V and Ω such that the robot converges to and maintains a prescribed tracking distance from the target.

For the subsequent analysis, two primary error variables are defined:

- $d(t)$: the Euclidean distance from the robot to the target, and
- $\varphi(t)$: the line-of-sight (LOS) angle of the target relative to the robot's heading direction.

These are given explicitly by:

$$\begin{cases} d(t) = \|r(t) - X(t)\|, \\ \varphi(t) = \operatorname{atan2}(r_y - X_y,\ r_x - X_x) - \theta, \end{cases} \tag{11}$$

where $\varphi \in (-\pi, \pi]$ represents the rotation the robot must execute to face the target directly.

3.1 Error Dynamics

The dynamics of $d(t)$ and $\varphi(t)$ are derived from the kinematic model (4)–(5). Differentiating d gives:

$$\dot{d} = -\nu \cos\varphi + \hat{\nu}_T(t), \tag{12}$$

where $\hat{\nu}_T(t)$ is the projection of the target velocity onto the robot-target line, satisfying $|\hat{\nu}_T(t)| \leq \nu_T$. The general upper bound on the distance dynamics is therefore:

$$\dot{d} \le -\nu \cos\varphi + \nu_T. \tag{13}$$

To derive $\dot{\varphi}$, let $\Delta_x = r_x - X_x$ and $\Delta_y = r_y - X_y$. Differentiating (11) with respect to time using the chain rule for `atan2`, and substituting $\dot{\theta} = \Omega$ from (4) together with $\dot{\Delta}_x = \nu_T \cos\theta_T - \nu_R \cos\theta$ and the analogous expression for $\dot{\Delta}_y$, yields:

$$\dot{\varphi} = \frac{\Delta_x(\nu_T \sin\theta_T - \nu_R \sin\theta) - \Delta_y(\nu_T \cos\theta_T - \nu_R \cos\theta)}{\Delta_x^2 + \Delta_y^2} - \Omega. \tag{14}$$

From Figure 2, $\Delta_x = d\cos\beta$ and $\Delta_y = d\sin\beta$. Substituting into (14):

$$\dot{\varphi} = \frac{d\cos\beta(\nu_T \sin\theta_T - \nu_R \sin\theta) - d\sin\beta(\nu_T \cos\theta_T - \nu_R \cos\theta)}{d^2} - \Omega. \tag{15}$$

Applying the identity $\cos\beta\sin\theta - \sin\beta\cos\theta = \sin(\theta-\beta)$ and the geometric relation $\theta+\varphi-\beta = \pi$ (see Figure 2), equation (15) simplifies to:

$$\dot{\varphi} = \frac{\nu}{d}\sin\varphi - \Omega + \Omega_T(t), \tag{16}$$

where $\Omega_T(t)$ captures the effect of the target's lateral motion on the LOS angle and satisfies $|\Omega_T(t)| \le \nu_T/d$. This term acts as a bounded disturbance that the controller must reject.

Collecting (12) and (16), and defining the error state $(e_1, e_2) = (d, \varphi)$, the full error dynamics for $d > 0$ are:

$$\begin{cases} e_1 = d = \|r - X\| = \sqrt{(x_T - x)^2 + (y_T - y)^2}, \\ e_2 = \varphi = \text{atan2}((y_T - y), (x_T - x)) - \theta, \end{cases} \tag{17}$$

$$\begin{cases} \dot{e}_1 = \dot{d} = -\nu_R \cos\varphi + \hat{\nu}_T(t), \\ \dot{e}_2 = \dot{\varphi} = \Omega - \dfrac{\nu_R}{d}\sin\varphi + \Omega_T(t), \end{cases} \tag{18}$$

where $|\hat{\nu}_T(t)| \le \nu_T$ and $|\Omega_T(t)| \le \nu_T/d$.

3.2 Stability Analysis

Lemma 1 (Continuity of Mamdani fuzzy controller mapping, [31]). Consider a Mamdani fuzzy controller with n inputs and one output satisfying the following conditions:

1. The membership functions of all inputs and outputs, for every rule i and input j, are continuous and bounded.

2. The fuzzy AND operator is of the min or product type.

3. Fuzzy inference is of the Mamdani type (max-min or max-product).

4. Rule outputs are aggregated using the max operator.

5. Defuzzification is performed by the centroid method, with the denominator of the centroid integral non-zero for all $x \in \Omega \subset \mathbb{R}^n$.

Then the controller mapping $u = F(x) : \Omega \subset \mathbb{R}^n \to \mathbb{R}$ is continuous with respect to the input variables.

By Lemma 1, the output of the fuzzy controller Ω_F is a continuous function of the inputs (d, φ), and can therefore be written as:

$$\Omega_F = -k(d, \varphi)\, \varphi, \tag{19}$$

for some positive continuous gain function $k(d, \varphi) > 0$.

Theorem 1. Consider the mobile robot tracking system (17)–(18). Suppose the fuzzy control law (19) is applied as the angular velocity input. If $\nu_R > \nu_T$, the closed-loop system is asymptotically stable and the tracking errors converge to zero:

$$\lim_{t\to\infty} d(t) = 0, \qquad \lim_{t\to\infty} \varphi(t) = 0.$$

Proof. Consider the Lyapunov function candidate:

$$V(d, \varphi) = \tfrac{1}{2}\varphi^2 + \lambda d, \qquad \lambda > 0.$$

Its time derivative along trajectories of (18) is $\dot{V} = \varphi\dot{\varphi} + \lambda\dot{d}$. Substituting (18) and (19):

$$\dot{V} = \frac{\nu_R}{d}\varphi \sin\varphi - k(d, \varphi)\varphi^2 + \varphi\Omega_T - \lambda\nu_R \cos\varphi + \lambda\hat{\nu}_T.$$

For $|\varphi| < \pi/2$, the following bounds hold:

$$\cos\varphi \geq 1 - \frac{\varphi^2}{2}, \quad \varphi\sin\varphi \leq \varphi^2, \quad |\varphi\Omega_T| \leq \frac{\nu_T}{d}|\varphi|, \quad |\lambda\hat{\nu}_T| \leq \lambda\nu_T.$$

Applying these inequalities yields:

$$\dot{V} \leq \left(\frac{\nu_R}{d} - k_{\text{min}} + \frac{\lambda\nu_R}{2}\right)\varphi^2 + \frac{\nu_T}{d}|\varphi| - \lambda(\nu_R - \nu_T).$$

Choosing λ and k_{min} sufficiently large so that the coefficient of φ^2 is negative and $\lambda(\nu_R - \nu_T)$ dominates the linear term in $|\varphi|$, we obtain:

$$\dot{V} \leq -c_1\varphi^2 - c_2, \qquad c_1, c_2 > 0.$$

Since $\dot{V}$ is not globally negative definite (it does not depend on d alone), we invoke LaSalle's Invariance Principle. The largest invariant set contained in $\{\dot{V} = 0\}$ is $\{(d, \varphi) = (0, 0)\}$, and hence the system is asymptotically stable. □ □

Remark 3. It can be shown more generally that if the Mamdani fuzzy controller approximates the reference law $\Omega^* = -k\varphi$ with a uniformly bounded error $\epsilon > 0$, and if $\nu_R > \nu_T$, then $d(t)$ and $\varphi(t)$ remain bounded and converge to an $O(\epsilon)$-neighbourhood of zero [31]. The following practical guidelines follow directly from this result.

Practical Considerations:

- Increasing the number of fuzzy rules and refining the membership function design reduces the approximation error ϵ and improves steady-state tracking accuracy.
- A larger gain k accelerates convergence but may introduce oscillations; a trade-off between speed and smoothness must be maintained.
- The condition $\nu_R \geq \nu_T$ is necessary for guaranteed tracking; if the target's speed exceeds the robot's maximum speed, the controller cannot prevent the target from escaping.
- In hardware implementation, actuator saturation limits and motor torque constraints must be incorporated into the control allocation stage.

3.3 Fuzzy Controller Details

The target-seeking module is implemented as a two-input, single-output Mamdani fuzzy system. The first input is the Euclidean distance d from the robot's centre to the moving target, and the second input is the LOS angle φ between the target direction and the robot's current heading. The single output is a steering correction angle $\Delta\theta$, commanding the robot to rotate relative to its current heading. The overall architecture is illustrated in Figures 4–6.

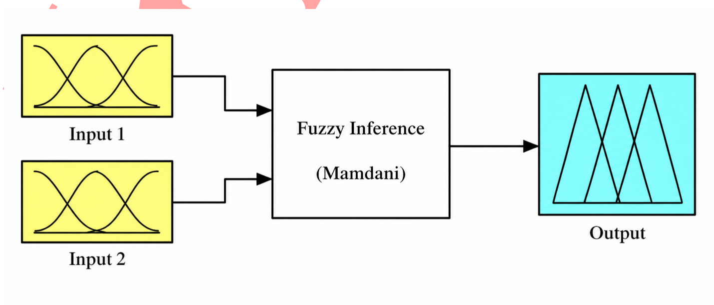

Figure 4: Internal structure of the fuzzy system.

Membership functions play a central role in determining the controller's behaviour at the boundaries between linguistic regions [1, 31]. For the distance input d, three membership functions are defined over the range $[0, 20]$ m:

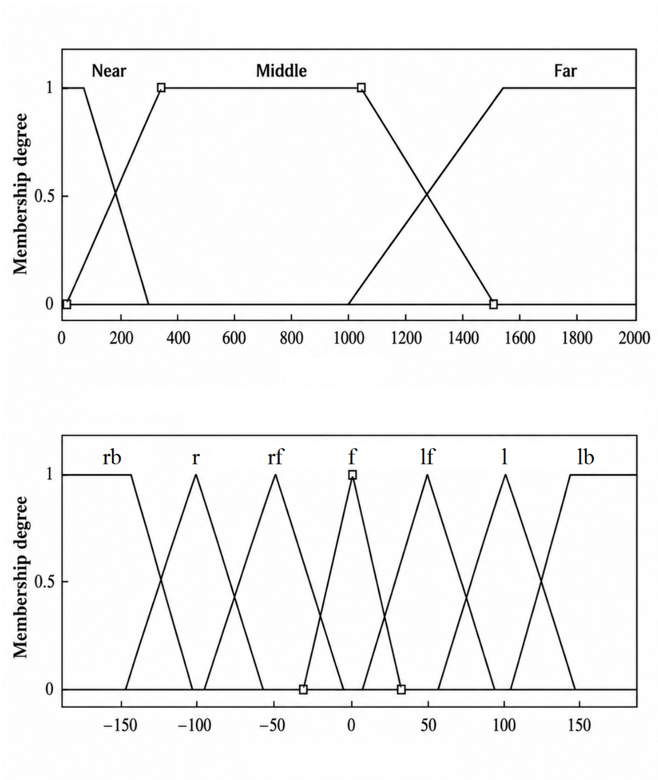

Figure 5: Membership functions of the fuzzy system inputs: distance to target (top) and LOS angle relative to the robot heading (bottom).

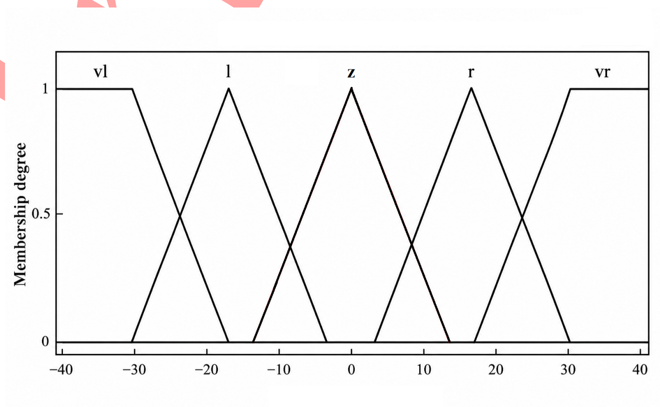

Figure 6: Output membership functions of the fuzzy system for steering angle correction (range: $-45°$ to $+45°$).

- *near*: active when d is below the prescribed safe distance, prompting the robot to decelerate or maintain separation;
- *middle*: active when d is close to the desired tracking distance;
- *far*: active when d is large, commanding the robot to approach the target rapidly.

For the angle input φ, a finer partition of the range $[-180°, +180°]$ into seven linguistic regions is used: right-back (*rb*), right (*r*), right-front (*rf*), front (*f*), left-front (*lf*), left (*l*), and left-back (*lb*). A finer angular partition is warranted because the required corrective action is highly sensitive to the target's angular position relative to the robot: a target slightly to the left-front requires only a gentle turn, whereas a target to the left-back demands a sharp rotation. For the output $\Delta\theta$, five membership functions spanning $[-45°, +45°]$ are employed: very-right (*vr*), right (*r*), straight (*z*), left (*l*), and very-left (*vl*). This range is sufficient for effective steering while preventing excessively sharp or destabilising commands.

With three distance sets and seven angle sets, a total of $3 \times 7 = 21$ rules are required. The rules are formulated by analogy with human driving behaviour and refined during simulation. Due to left-right symmetry, Table 1 lists only the rules for the front and right half-plane; the corresponding left-side rules are obtained by reflection.

Table 1: Representative rules of the proposed fuzzy controller (right and front quadrants; left-side rules follow by symmetry).

Distance	Target angle (LOS)	Steering output
near	rb	vr
near	r	vr
near	rf	r
near	f	z
middle	rb	vr
middle	r	r
middle	rf	r
middle	f	z
far	rb	vr
far	r	r
far	rf	z
far	f	z

Because the controller inputs are the robot-target distance and bearing—quantities that are sensor-agnostic and independent of sensor placement—the proposed method is transferable to any robotic platform capable of providing these measurements, whether through ultrasonic ranging, laser scanners, GPS differencing, or vision-based estimation [4, 30]. Furthermore, since the fuzzy system is model-free, it is inherently robust to wheel slip, parameter variations, and unmodelled dynamics: any resulting tracking error is corrected within one or a small number of control cycles [3, 29]. The primary practical challenge is the accurate real-time estimation of d and φ; in accuracy-critical applications this can be addressed by fusing multiple sensing modalities [21, 30].

4 Simulation and Results

To evaluate the performance of the proposed fuzzy tracking controller, simulations were conducted in MATLAB/Simulink across three scenarios involving stationary and moving targets with distinct motion patterns. Controller performance was refined by iteratively adjusting membership function parameters and rule weights during the design phase. Throughout the simulations, the robot's configuration and the robot-target distance and bearing are assumed to be computed without error, isolating the evaluation to the controller itself; the effect of measurement noise is addressed separately in Scenario 3.

The discrete-time kinematic equations of the mobile robot, obtained by applying the Euler forward method to (4) with a uniform sampling period T, are:

$$\begin{bmatrix} x(k+1) \\ y(k+1) \\ \theta(k+1) \end{bmatrix} = \begin{bmatrix} x(k) \\ y(k) \\ \theta(k) \end{bmatrix} + \begin{bmatrix} V(k)\cos\theta(kT)\cdot T \\ V(k)\sin\theta(kT)\cdot T \\ \Omega(k)\cdot T \end{bmatrix}, \tag{20}$$

where $V(k)$ and $\Omega(k)$ are the linear and angular velocities at time step k, respectively. The virtual environment is obstacle-free. Wheel velocities $\Omega_L(k)$ and $\Omega_R(k)$ are recovered from $V(k)$ and $\Omega(k)$ via (1)–(2), using $r_w = 10\,\text{cm}$ and $l = 80\,\text{cm}$.

Scenario 1: Circular Target Path

The moving target follows a circular path, enabling direct comparison with the results of Benbouabdallah and Zhu [3], Cui et al. [7] and Petrović et al. [26]. Simulation results for several distinct starting configurations are shown in Figures 7–10.

As shown in Figure 7, when the robot starts far from the target it initially moves at maximum speed to close the distance. As the robot-target separation approaches the prescribed safe distance, the controller reduces speed, locks onto the target orbit, and maintains the separation with minimal steady-state error. The time histories in Figure 8 confirm that the distance error e_1 converges to zero rapidly under the proposed control law, with smooth velocity profiles throughout.

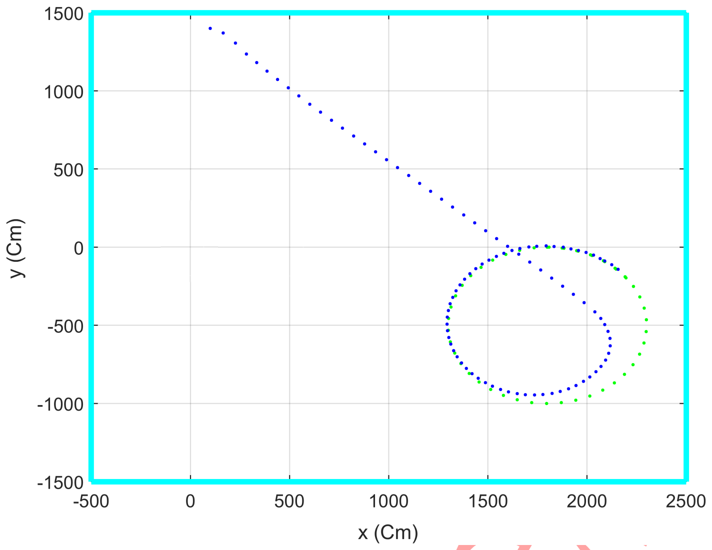

Figure 7: Scenario 1: robot and target trajectories when the robot starts far from the target. The robot converges rapidly to the tracking orbit and maintains the prescribed safe distance thereafter.

Figures 9 and 10 demonstrate that the controller's convergence behaviour is independent of the robot's initial distance and heading relative to the target: successful tracking is achieved from every tested starting configuration. Compared with the methods of [3], [7] and [26], which simulate qualitatively similar circular pursuit scenarios, the proposed controller achieves comparable or superior tracking accuracy while requiring a strictly simpler sensor setup and lower computational load (see Table 2).

Scenario 2: Target with Sudden Heading Changes

The target follows a path that violates the nonholonomic constraint, incorporating abrupt heading changes; this scenario mirrors the evaluation in Petrović et al. [26]. Simulation results are shown in Figure 11.

Sharp turns in the target path produce transient tracking errors, as expected given the nonholonomic constraints of the robot. However, the fuzzy controller detects the angular deviation rapidly and issues corrective steering commands that restore accurate tracking within a small number of sampling periods. No persistent steady-state error is observed following each heading change.

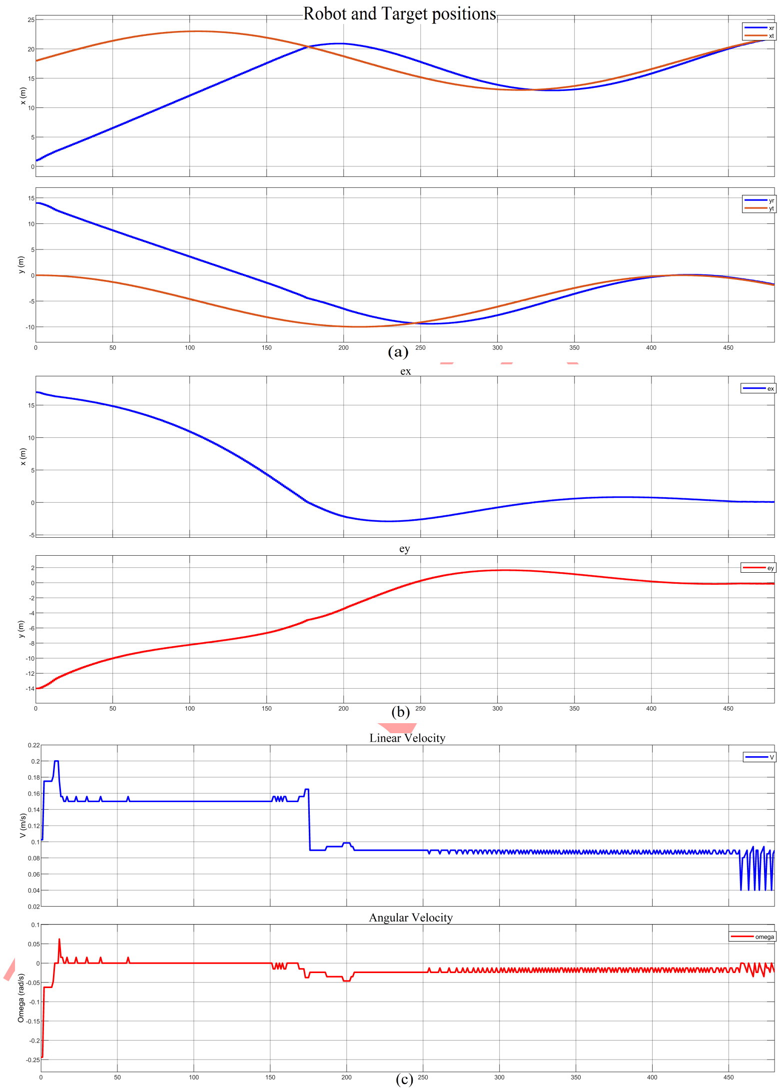

Figure 8: Scenario 1 time histories: (top) robot and target positions; (middle) position error $e_1 = d$; (bottom) linear velocity $V(k)$ and angular velocity $\Omega(k)$.

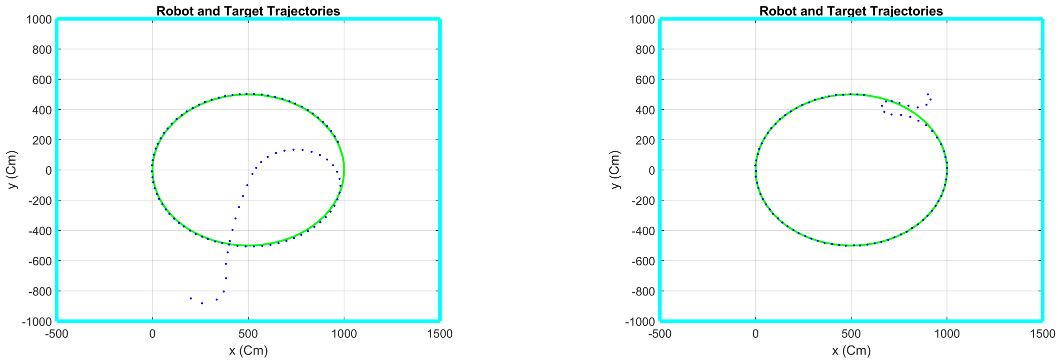

Figure 9: Scenario 1: robot and target trajectories for two additional starting configurations (circular path).

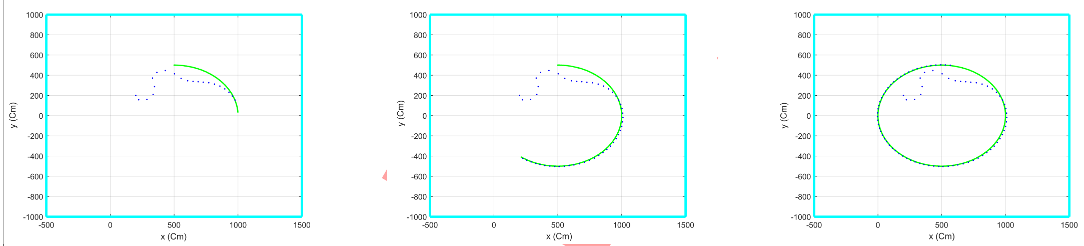

Figure 10: Scenario 1: robot and target trajectories for three further starting configurations, illustrating initialisation-independence of the controller.

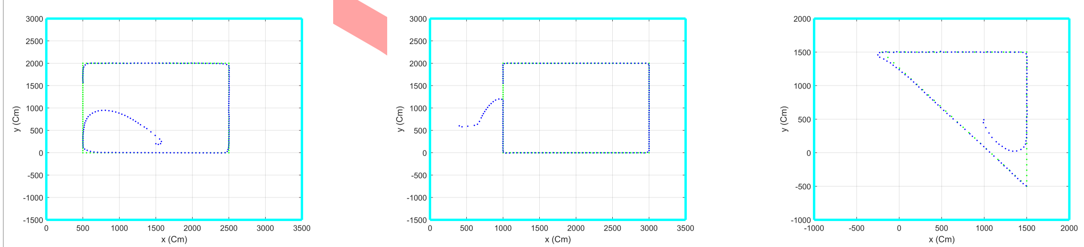

Figure 11: Scenario 2: robot and target trajectories for three starting configurations when the target undergoes sudden heading changes.

Scenario 3: Spiral Target Path and Noise Robustness

The target follows an expanding spiral path, analogous to the trajectory considered by Asif et al. [2]. This scenario also serves as a robustness test: since real sensor measurements are contaminated by noise whose magnitude typically scales with distance [30], the controller inputs (d and φ) were corrupted with distance-dependent additive noise of progressively increasing amplitude. Trajectory results for two starting points (clean measurements) are shown in Figure 12. The noisy input signals and the resulting trajectory under high-amplitude noise ($\approx$50% of the true value) are presented in Figures 13 and 14, respectively.

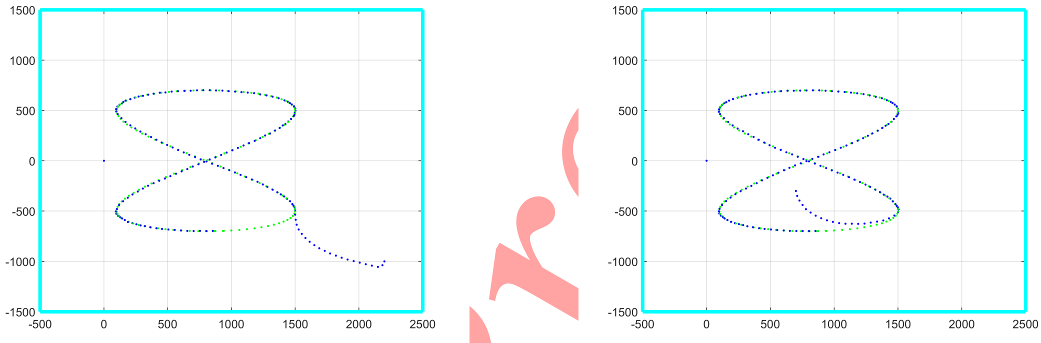

Figure 12: Scenario 3: robot and target trajectories on the spiral path from two different starting points (clean measurements).

Under low-to-moderate noise levels the tracking behaviour was visually indistinguishable from the clean-measurement case. At the elevated noise amplitude shown in Figures 13 and 14, the robot maintains the prescribed safe tracking distance and achieves acceptable error levels, but the trajectory exhibits visible oscillations as the controller continuously compensates for the corrupted bearing input. This result confirms the robustness of the model-free fuzzy architecture to sensor noise, while also highlighting that highly accurate distance and angle estimation—achievable through sensor fusion [21, 30]—is the principal prerequisite for smooth operation in practice.

Comparative Evaluation

A qualitative comparison of the proposed controller against the benchmark methods is summarised in Table 2. The proposed method achieves good tracking accuracy with low computational load, model-free operation, and demonstrated robustness to sudden target heading changes—combining advantages that no single benchmark method offers simultaneously.

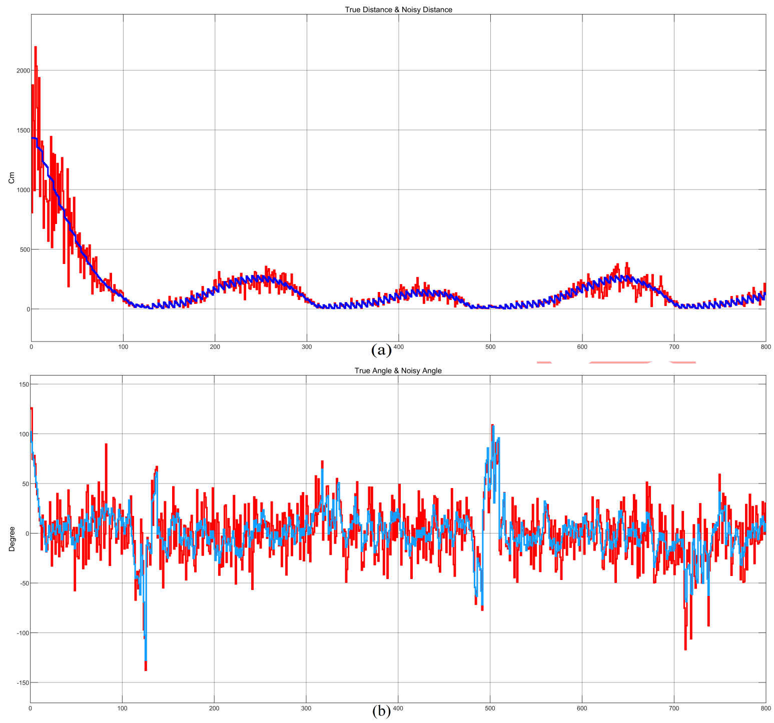

Figure 13: Scenario 3 (noisy case): (top) true and noise-contaminated robot-target distance; (bottom) true and noise-contaminated LOS angle.

Table 2: Qualitative comparison of the proposed method with existing approaches.

Ref.	Control Method	Comp. Load	Virtual Env.	Controller Type	Sudden Change	Opt. Routing	Tracking Acc.
[26]	Fuzzy & PD	Low	Obstacle-free	Model-free	Supported	No	Medium
[2]	Feedforward & Feedback Kinematic	Medium	Obstacle-free	Model-free	Not tested	No	Medium
[3]	PSO-Tuned Fuzzy	High	Obstacle-free	Model-based	Not tested	Yes	Good
[7]	Adaptive (UKF-based)	High	Single fixed obstacle	Model-free	Not tested	No	Medium
Proposed	**Mamdani Fuzzy**	**Low**	**Obstacle-free**	**Model-free**	**Supported**	**No**	**Good**

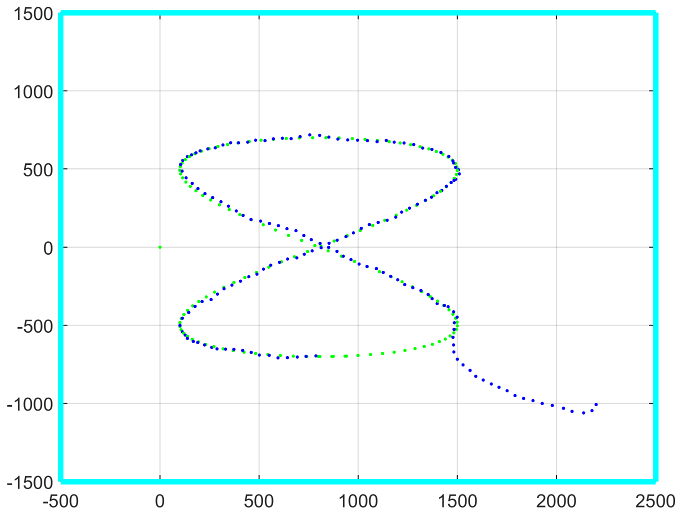

Figure 14: Scenario 3 (noisy case): robot trajectory under distance-dependent noise with amplitude approximately 50% of the true measurement.

5 Conclusion

This paper presented a model-free Mamdani-type fuzzy logic controller for moving-target tracking with safety distance enforcement on a non-holonomic differential-drive mobile robot. The controller takes only the robot-target distance and line-of-sight angle as inputs, requires no dynamic model of the robot, and generates angular velocity commands through a compact set of linguistically interpretable if-then rules. A formal Lyapunov-based stability proof was provided, establishing that the closed-loop system is asymptotically stable whenever the robot's maximum speed exceeds that of the target. Simulation results across multiple scenarios—including circular paths, spiral paths, paths with sudden heading changes, and high-amplitude measurement noise—confirm that the controller achieves accurate and smooth tracking behaviour with minimal error. Unlike many existing approaches [7, 26], which restrict the target to follow another nonholonomic robot on a structured path, the proposed method imposes no constraints on the target's motion, extending applicability to pedestrians, vehicles, and other real-world objects. Compared to optimisation-based fuzzy methods [3, 29] and vision-dependent or model-based controllers [6, 9, 13, 20], the proposed architecture offers a favourable balance between tracking accuracy, computational simplicity, and sensor cost, making it well suited for energy-constrained and resource-limited platforms [30].

Limitations

- The controller's performance is sensitive to the quality of membership function design. Poorly chosen intervals or insufficient overlap between fuzzy sets degrades tracking accuracy, and no systematic or learning-based tuning procedure is currently employed [29].
- The method assumes obstacle-free environments. In cluttered or dynamic settings, a dedicated obstacle-avoidance layer must be integrated before deployment [15, 17, 22, 38].
- Tracking is not guaranteed when the target's speed exceeds the robot's maximum linear velocity, as established by the stability condition $\nu_R > \nu_T$.
- The paper assumes perfect knowledge of the robot's position and of the robot-target distance and angle. In practice, these quantities are subject to estimation errors arising from sensor noise, calibration drift, and occlusion [30]. The impact of state estimation uncertainty on closed-loop performance requires further analysis.
- Only a single output (angular velocity) is generated by the fuzzy system; linear velocity is governed by a simple hand-crafted distance rule rather than a fuzzy law, which limits the richness of the controller's behaviour.
- The proposed system has been validated in simulation only. Real hardware introduces unmodelled effects—wheel slip, actuator saturation, motor torque limits, and communication latency—that are not captured in the current evaluation [7].

Future Work

- *Obstacle avoidance integration.* Extending the controller to dynamic environments containing static and moving obstacles is the most immediate priority. Reactive fuzzy obstacle-avoidance layers [22, 17, 38] or behaviour-based architectures [14] provide natural candidates for integration with the present target-seeking module.
- *Fuzzy velocity control.* Adding a second fuzzy output for linear velocity would enable the robot to modulate its speed based on both distance and angle, yielding richer and more energy-efficient behaviour. The resulting increase in rule-base size must be balanced against the computational budget of the target platform.
- *Systematic membership function tuning.* Replacing hand-tuned membership functions with a systematic optimisation procedure—such as PSO [3], genetic algorithms, or gradient-based learning—would improve generalisation across scenarios and reduce the dependency on expert knowledge [29].
- *Multi-robot cooperative tracking.* Extending the framework to teams of robots performing cooperative target tracking or formation-based pursuit [6, 19, 21] represents a prac-

tically significant direction, particularly for surveillance and search-and-rescue applications.

- *State estimation under noisy sensing.* Integrating an onboard state estimator—such as an Extended Kalman Filter or an Unscented Kalman Filter [7]—to provide robust distance and angle estimates from low-cost sensors [30] would bring the system significantly closer to practical deployment.
- *Hardware validation.* Experimental validation on a physical differential-drive platform such as a TurtleBot or Pioneer 3-DX is essential to characterise the effects of wheel slip, actuator saturation, and real-world sensor noise on the controller's closed-loop performance [2, 26].

Declarations

Availability of Supporting Data

All data generated or analyzed during this study are included in this published paper.

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Conflict of Interest

The author declares that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Author Contributions

Mehdi Ramezanifard performed simulations and wrote the initial draft. **Naser Pariz** and **M.B. Naghibi Sistani** supervised the research. **Majid Akbarian** contributed to writing, revised and edited the manuscript. All authors reviewed and approved the final manuscript.

Artificial Intelligence Statement

The authors used AI-based tools (ChatGPT, GPT-4) only to polish the English language and enhance readability. These tools were applied to the Introduction, Methodology, Simulation Results, and Conclusion sections. English is not our native language, so this assistance was helpful. However, AI was not used to generate ideas, perform analyses, interpret results, prove theorems, or produce scientific content. All scientific reasoning, mathematical proofs, stability analyses, and intellectual contributions are solely the authors' own.

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