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A KKT-Based Neurodynamic Framework for CCR-DEA: Lyapunov Stability Analysis and Application to Asian Games Efficiency Evaluation

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Abstract. Data envelopment analysis (DEA) is a widely used non-parametric method for evaluating the relative efficiency of peer decision-making units (DMUs). To address the computational demands of conventional solvers, this study develops a neurodynamic optimization framework derived from the Karush–Kuhn–Tucker (KKT) optimality conditions of the CCR-based, input-oriented DEA model. To date, only a few studies have applied neurodynamic approaches to DEA, and all of them rely on double-layer structures. Our primary contribution is the construction of a KKT-based dynamic system tailored to the CCR-based DEA formulation. Furthermore, we provide a rigorous theoretical foundation by establishing the Lyapunov stability of the proposed system, thereby ensuring its global convergence to an exact optimal solution. The model's practical effectiveness is validated through a comprehensive case study analyzing the efficiency of participating nations in the Asian Games. The results, obtained using our neurodynamic framework and evaluated in terms of CPU time, demonstrate its robustness and computational efficiency for real-world performance benchmarking.

Keywords. CCR model of DEA, Neurodynamic model, KKT conditions, Lyapunov stability, Global convergence.

MSC. 37N40; 90C05; 90C08.

1 Introduction

Data envelopment analysis (DEA) is widely regarded as an appropriate approach for assessing the efficiency of DMUs [4, 8, 11, 21]. DEA measures the relative efficiency of DMUs within an organization [26]. A DMU located on the efficiency frontier is classified as efficient, whereas a unit positioned below the frontier is regarded as inefficient. Various approaches have been introduced to incorporate the preference information of decision-makers (DMs) into DEA models [12].

DEA has been applied to DMUs operating in a wide range of sectors, such as police departments, banking systems, tax offices, hospitals, military units (army, navy, and air force), university faculties, and schools. For instance, Cikovic et al. [7] conducted a bibliometric review of 87 studies on the use of DEA in sustainable supplier selection. Tapia and Salvador [28] proposed a new approach for efficiency measurement based on stochastic quality indices. Zhang et al. [35] introduced a DEA-based feature selection method. Wu et al. [30] developed a DEA framework for Olympic ranking evaluation. Shen et al. [25] used DEA to analyze competitiveness in crowdsourcing through input–output efficiency assessment. Hwang et al. [15] evaluated research and application efficiency using DEA. Moreover, Emrouznejad et al. [9] provided a comprehensive compilation of DEA studies encompassing both theoretical advances and real-world applications (see also [17, 18, 22, 31]).

DEA is a non-parametric method that relies fundamentally on linear programming (LP). The primary aim of applying LP within DEA is to assess the performance of multiple DMUs. An additional key objective is to reduce computational time when handling large datasets in order to identify efficient DMUs [17]. Consequently, dynamic systems provide a valuable approach for solving such programming problems, and neurodynamic models, in particular, are effective in transforming programming tasks into dynamic-system formulations.

Neurodynamic models achieve the optimum of a given optimization problem by converting it into a dynamic system. In dynamic optimization, finding the best solution in real time can be challenging, particularly under conditions of uncertainty. In this context, neurodynamic models offer an effective alternative to traditional methods for solving such problems. This advantage arises from their distributed nature, biological plausibility, and capability for parallel information processing and hardware implementation. As a form of dynamic system, a neurodynamic model operates in real time, and the underlying problem admits a unique solution provided that the system satisfies the Lipschitz condition. Notably, the initial point can be selected outside the convergence region, a feature enabled by the global convergence property of the dynamic system. This characteristic distinguishes neurodynamic methods from conventional numerical approaches and highlights their advantage.

The application of neurodynamic approaches to solving optimization problems was first proposed by Tank and Hopfield [27]. Their contribution inspired subsequent research aimed at developing alternative neurodynamic frameworks for optimization. Consequently, many studies have addressed both linear and nonlinear programming problems using such models [5, 10, 14, 20, 23, 32, 34]. Abbasi and Ghomashi [1] introduced a neurodynamic approach for the CCR-based DEA model, employing a double-layer structure based on projection mapping. More recently, Bani Hassan et al. [2] presented a neurodynamic model for solving DEA problems. The approach developed in the present study achieves faster performance than the model reported in [2]; a comparison between the two models, based on CPU time, is provided later in this paper.

Building on this prior research, the present study proposes a novel neurodynamic approach for solving the DEA problem. To the best of our knowledge, this work is among the first to apply a neurodynamic model to DEA. We reformulate the CCR-based DEA model as an LP problem and derive the corresponding neurodynamic model from the KKT optimality conditions. The proposed model is then used to compute the solution of the resulting dynamic system, and its stability and convergence are rigorously established. Finally, we demonstrate the model's practical applicability through a case study on Iran's Asian Sports Caravan. It is worth noting that the proposed model is formulation-based rather than data-dependent, and can therefore be applied to any dataset that can be represented within the DEA linear-programming structure. In summary, the contributions of this paper are as follows:

- A new, single-layer neurodynamic model for LP problems is presented.
- The proposed neurodynamic model is directly applicable to the LP structure of DEA.
- The results indicate that the neurodynamic model is stable and possesses the global convergence property.
- A real-world case study (Iran's Asian Sports Caravan) is presented to demonstrate the applicability of the proposed model and solution approach.

The remainder of this paper is organized as follows. Section 2 presents the problem formulation, including a general model of sports caravans based on the DEA-CCR model. Section 3 introduces the proposed neurodynamic model and analyzes its stability. Section 4 reports the simulation results, and Section 5 concludes the paper with a discussion of the main findings.

2 Problem Formulation

Consider a set of n decision-making units (DMUs), where each DMU uses m inputs to generate s outputs. Let

$$y = (y_{1j}, y_{2j}, \dots, y_{sj}) \in \mathbb{R}^{s\times n}, \qquad x = (x_{1j}, x_{2j}, \dots, x_{mj}) \in \mathbb{R}^{m\times n}, \qquad j = 1, 2, \dots, n,$$

represent the matrices of observed outputs and inputs, respectively, for all DMUs. Here, y_j (the jth column of y) and x_j (the jth column of x) denote the output and input vectors of DMU_j. We assume that all outputs and inputs are non-negative, with at least one element in each vector being strictly positive. The DMU under evaluation is denoted DMU_0.

In the standard CCR model [4], efficiency is measured as the maximum ratio of weighted outputs to weighted inputs, subject to the constraint that this ratio does not exceed unity for any DMU. However, DEA becomes computationally intensive—requiring significant CPU time and memory—when applied to large datasets with numerous inputs and outputs. Thus, the development of fast, low-complexity computational methods remains a critical challenge. Based on the preceding discussion, we state the DEA-CCR model used in this study as follows [4]:

$$\begin{aligned} \text{min } z_0 = & \sum_{i=1}^{m} v_i x_{i0} \\ \text{s.t. } & \sum_{r=1}^{s} \eta_r y_{r0} = 1, \\ & \sum_{r=1}^{s} \eta_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, 2, \dots, n \\ & v_i \ge 0, \quad i = 1, 2, \dots, m, \\ & \eta_r \ge 0, \quad r = 1, 2, \dots, s. \end{aligned} \tag{1}$$

The variable z_0 represents the technical efficiency score of the target DMU_0. In this model, y_{rj} denotes the quantity of the rth output produced by DMU_j; x_{ij} denotes the quantity of the ith input consumed by DMU_j; η_r is the weight assigned to the rth output; and v_i is the weight assigned to the ith input. Although the envelopment form is more commonly presented first in the DEA literature, the CCR model is stated here in its multiplier (dual) form, since this form provides direct insight into the input weights and facilitates the neurodynamic solution approach developed below.

2.1 General DEA-CCR Model for Sports-Caravan Evaluation

We present a general model for evaluating the Asian Games caravan of a fixed country. Suppose that a country participates in the Asian Games n times (see Table 1), and consider the following definitions, for $i = 1, 2, \dots, n$:

- λ_i: the year of the ith Asian Games;
- β_{i1}: the number of gold medals won in year λ_i;

- β_{i2}: the number of silver medals won in year λ_i;
- β_{i3}: the number of bronze medals won in year λ_i;
- α_{i1}: the GDP of the country in year λ_i;
- α_{i2}: the population of the country in year λ_i.

Table 1: General input–output structure for the DEA evaluation of sports-caravan DMUs.

		Output		Input	
Asian Games	Gold medals	Silver medals	Bronze medals	GDP (billion USD)	Population
λ_1	β_{11}	β_{12}	β_{13}	α_{11}	α_{12}
λ_2	β_{21}	β_{22}	β_{23}	α_{21}	α_{22}
λ_3	β_{31}	β_{32}	β_{33}	α_{31}	α_{32}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
λ_n	β_{n1}	β_{n2}	β_{n3}	α_{n1}	α_{n2}

Note that the inputs are population and GDP in year λ_i, and the outputs are the numbers of gold, silver, and bronze medals won. The CCR model of DEA for DMU$_i$ can then be written as:

$$\begin{aligned}
\min\ z_i &= \beta_{i1}v_1 + \beta_{i2}v_2 + \beta_{i3}v_3 \\
\text{s.t. } & \alpha_{i1}\eta_1 + \alpha_{i2}\eta_2 = 1, \\
& \alpha_{11}\eta_1 + \alpha_{12}\eta_2 - (\beta_{11}v_1 + \beta_{12}v_2 + \beta_{13}v_3) \leq 0, \\
& \alpha_{21}\eta_1 + \alpha_{22}\eta_2 - (\beta_{21}v_1 + \beta_{22}v_2 + \beta_{23}v_3) \leq 0, \\
& \vdots \\
& \alpha_{n1}\eta_1 + \alpha_{n2}\eta_2 - (\beta_{n1}v_1 + \beta_{n2}v_2 + \beta_{n3}v_3) \leq 0, \\
& v_1, v_2, v_3, \eta_1, \eta_2 \geq 0.
\end{aligned} \tag{2}$$

The matrix form of (2) is given by the following LP:

$$\begin{aligned}
\min\ z_0 &= x^T v \\
\text{s.t. } & y^T\eta = 1, \\
& A\eta - Bv \leq 0, \\
& \eta, v \geq 0.
\end{aligned} \tag{3}$$

Replacing the equality constraint $y^T\eta = 1$ with the equivalent pair of inequalities $y^T\eta \leq 1$ and $-y^T\eta \leq -1$, we obtain:

$$\begin{aligned} \min\ z_0 &= c^T w \\ \text{s.t. } Gw &\leq g \\ w &\geq 0, \end{aligned} \tag{4}$$

where

$$c = \begin{bmatrix} 0_{2\times1} \\ x \end{bmatrix}_{5\times1}, \quad w = \begin{bmatrix} \eta \\ v \end{bmatrix}_{5\times1}, \quad G = \begin{bmatrix} y^T & 0_{1\times3} \\ -y^T & 0_{1\times3} \\ A_{n\times2} & -B_{n\times3} \end{bmatrix}_{(n+2)\times5}, \quad g = \begin{bmatrix} 1 \\ -1 \\ 0_{n\times1} \end{bmatrix}_{(n+2)\times1}. \tag{5}$$

Rewriting the non-negativity constraint $w \geq 0$ as $-w \leq 0$ in (4), we obtain the following general LP:

$$\begin{aligned} \min\ z_0 &= c^T w \\ \text{s.t. } Hw - k &\leq 0, \end{aligned} \tag{6}$$

where

$$H = \begin{bmatrix} G \\ -I \end{bmatrix}_{(n+7)\times5}, \quad k = \begin{bmatrix} g \\ 0_{5\times1} \end{bmatrix}_{(n+7)\times1}. \tag{6}$$

Thus, (6) constitutes a general LP formulation of the DEA-CCR model, which we solve in the next section using a neurodynamic approach.

3 Neurodynamic Model

Here, we propose a novel neurodynamic model to solve LP (6). The KKT optimality conditions of (6) can be stated as the following system of equations [3]:

$$\text{KKT conditions:} \begin{cases} u^{*T} \geq 0, \quad u^{*T}(Hw^* - k) = 0, \\ c + Hu^* = 0, \quad Hw^* - k \leq 0, \end{cases} \tag{7}$$

where u denotes the dual variable. Here, our goal is to develop a neurodynamic model that yields the solution of the original problem (6). First, it is easy to show the following result.

Lemma 1. $(u \geq 0, Hw - k \leq 0) \Longleftrightarrow (u + Hw - k)^+ = u$, where $(\cdot)^+ = \max\{0, \cdot\}$.

The neurodynamic model that satisfies the KKT conditions of (6) can then be designed as the following dynamic system:

$$\frac{dw}{dt} = -(c + H(u + Hw - k)^+), \tag{8}$$

$$\frac{du}{dt} = (u + Hw - k)^+ - u, \tag{9}$$

where c is the column vector defined in (5), u is the dual variable, and all terms on the right-hand side of (8) are dimensionally consistent column vectors. By defining $z = (w, u)$ and

$$\phi(z) = \begin{pmatrix} -(c + H(u + Hw - k)^+) \\ (u + Hw - k)^+ - u \end{pmatrix},$$

the proposed neurodynamic model reduces to the following equation:

$$\frac{dz}{dt} = \xi\phi(z), \qquad z(t_0) = z_0, \tag{10}$$

where $\xi > 0$ denotes the convergence rate.

Remark 1. The neurodynamic model (10) is a first-order differential equation. The variables w and u are obtained by solving model (10), which will be shown below to yield the optimal solution of LP (6).

The next result shows that the neurodynamic model (10) and the LP (6) have the same solution.

Theorem 1. The neurodynamic model (10) and the LP (6) have the same solution.

Proof. Satisfying the KKT conditions guarantees optimality in LP problems, since these conditions are both necessary and sufficient. First, we show that if (w^*, u^*) is an equilibrium point of (10), then the KKT conditions hold. Suppose that (w^*, u^*) is an equilibrium point of the neurodynamic model (10); therefore,

$$c + H(u^* + Hw^* - k)^+ = 0, \tag{11}$$

$$(u^* + Hw^* - k)^+ = u^*. \tag{12}$$

Now, if $u^* + Hw^* - k \geq 0$, then from (12) and the definition of "+" we obtain $u^* + Hw^* - k = u^*$, and hence $Hw^* - k = 0$. Thus, using (11) gives $c + Hu^* = 0$, since $Hw^* - k = 0$. On the other hand, if $u^* + Hw^* - k < 0$, then from (12) we get $u^* = 0$, and hence $Hw^* - k \leq 0$. These two cases show that the KKT conditions hold. Conversely, if the KKT conditions hold, then applying Lemma 1 completes the proof. □

3.1 Stability Analysis

This section establishes the stability of the proposed neurodynamic model (10). First, we recall some preliminaries from [16].

Definition 1. Consider the function $h : \mathbb{R}^n \to \mathbb{R}^n$ and the dynamic system

$$\dot{z} = h(z(t)), \quad z(t_0) = z_0 \in \mathbb{R}^n. \tag{13}$$

Then:

1. z^* is called an equilibrium point if $h(z^*) = 0$.

2. Let $\theta : \mathbb{R}^n \longrightarrow \mathbb{R}$ be a continuously differentiable function. If

$$\begin{cases} \theta(z^*) = 0,\ \theta(z) > 0, & \text{for all } z \in \Omega \backslash \{z^*\}, \\ \dfrac{d\theta(z(t))}{dt} = \nabla\theta(z(t))^T h(z(t)) \leq 0, & \text{for all } z \in \Omega, \end{cases} \tag{14}$$

then θ is called a Lyapunov function for (13), where $\Omega \subseteq \mathbb{R}^n$ is an open neighborhood of z^*.

Definition 2. A real-valued function $h : \mathbb{R} \to \mathbb{R}$ is called Lipschitz continuous if there exists a positive real constant l such that, for all real x and y, $|h(x) - h(y)| \leq l|x - y|$.

We now calculate the Jacobian of $\phi(z)$ in the following lemma.

Lemma 2. The Jacobian matrix $\nabla\phi$ is negative semi-definite.

Proof. Assume that there exists an index p ($p = 0, 1, 2, \ldots, n + 7$) such that

$$(u + Hw - k)^+ = (u_1 + Hw_1 - k_1, u_2 + Hw_2 - k_2, \ldots, u_p + Hw_p - k_p, 0, 0, \ldots, 0).$$

Therefore,

$$\nabla\phi(z) = \begin{bmatrix} \begin{pmatrix} -H_p^2 \\ 0 \end{pmatrix} & \begin{pmatrix} -H_p^T \\ 0 \end{pmatrix} \\ \begin{pmatrix} H_p \\ 0 \end{pmatrix} & \begin{pmatrix} I_{p\times p} & 0 \\ 0 & 0 \end{pmatrix} - I \end{bmatrix}, \tag{15}$$

where

$$H_p = \begin{bmatrix} h_{11} & h_{12} & \ldots & h_{1,n+2} \\ h_{21} & h_{22} & \ldots & h_{2,n+2} \\ \vdots & \vdots & & \vdots \\ h_{p1} & h_{p2} & \ldots & h_{p,n+2} \end{bmatrix}. \tag{16}$$

Clearly, $\begin{pmatrix} -H_p^2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} I_{p\times p} & 0 \\ 0 & 0 \end{pmatrix} - I$ are negative semi-definite matrices; thus, $\nabla\phi(z)$ is negative semi-definite as well. In particular, for the boundary cases $p = 0$ and $p = n + 7$, we have the following observations. If $p = 0$ (all components of $(u + Hw - k)^+$ are zero), then the Jacobian simplifies to $-I$, which is trivially negative definite. If $p = n+7$ (all components are active), then the full matrix H is used, and the Jacobian again simplifies to $-I$, which is negative definite. □

The next result demonstrates that the neurodynamic model (10) has a unique solution.

Lemma 3. The neurodynamic model (10) has a unique solution.

Proof. Each term on the right-hand side of (10) is locally Lipschitz continuous. Thus, the result follows from Theorem 3.2 in [16]. □

Lemma 4 ([20])**.** A differentiable mapping $G : \mathbb{R}^n \to \mathbb{R}^n$ is monotonic (i.e., for all $x, y \in \mathbb{R}^n$, $(x-y)^T(G(x)-G(y)) \geq 0$) if and only if the Jacobian matrix $\nabla G(x)$ is positive semi-definite for all $x \in \mathbb{R}^n$.

The following result establishes the stability of the proposed neurodynamic model (10).

Theorem 2. The neurodynamic model (10) is stable in the sense of Lyapunov.

Proof. The existence and uniqueness of the solution of the developed neurodynamic model are established in Theorem 1 and Lemma 3. Now, consider the energy function

$$\theta(z) = \|\phi(z)\|^2 + \frac{1}{2}\|z - z^*\|^2. \tag{17}$$

It is obvious that $\theta(z^*) = 0$ and $\theta(z) > 0$ for $z \neq z^*$. Also, $\frac{d\phi}{dt} = \frac{d\phi}{dz}\frac{dz}{dt} = \nabla\phi(z)\phi(z)$. Thus, for any $z \neq z^*$, we have:

$$\frac{d\theta(z)}{dt} = \left(\frac{d\phi}{dt}\right)^T \phi + \phi^T \frac{d\phi}{dt} + (z - z^*)^T \frac{dz}{dt} \tag{18}$$

$$= \phi^T \left(\nabla\phi^T(z) + \nabla\phi(z)\right)\phi + (z - z^*)^T \phi(z). \tag{19}$$

According to Lemma 2, $\nabla\phi(z)$ is a negative semi-definite matrix, so $\phi^T(\nabla\phi^T(z)+\nabla\phi(z))\phi \leq 0$. Also, based on Lemma 4, $-\phi$ is monotone, so $(z - z^*)^T(-\phi(z) + \phi(z^*)) \geq 0$; and since $\phi(z^*) = 0$, we have:

$$(z - z^*)^T \phi(z) = (z - z^*)^T(\phi(z) - \phi(z^*)) \leq 0.$$

Hence, inequality (18) gives:

$$\frac{d\theta(z)}{dt} \leq 0.$$

Thus, the Lyapunov stability of the neurodynamic model (10) is established. □

4 Numerical Example and Results

This section presents an illustrative example and an application to Iran's Asian Games sports caravan. MATLAB (R2023a) on a system with 4 GB RAM and an Intel Core i7 dual-core processor, 2 GHz, was used to run the codes. All initial points were selected randomly. The MATLAB `ode45` solver was used with its default tolerance settings; specifically, relative tolerance RelTol $= 10^{-3}$ and absolute tolerance AbsTol $= 10^{-6}$.

4.1 Simple Example

For this example, Table 2 lists the data for six DMUs. It shows that each of the six DMUs produces a single output in the amount y by using two inputs in the amounts x_1 and x_2. The DEA model for DMU$_A$, based on the CCR formulation, is stated as:

$$
\begin{aligned}
\text{min} \quad & z_A = 4v_1 + 3v_2, \\
\text{s.t.} \quad & u = 1, \\
& 4v_1 + 3v_2 - u \geq 0, \\
& 7v_1 + 3v_2 - u \geq 0, \\
& 8v_1 + v_2 - u \geq 0, \\
& 4v_1 + 2v_2 - u \geq 0, \\
& 2v_1 + 4v_2 - u \geq 0, \\
& 10v_1 + v_2 - u \geq 0, \\
& v_1, v_2, u \geq 0.
\end{aligned}
$$

Table 2: Input and output data for the illustrative example.

	DMU	A	B	C	D	E	F
Input	x_1	4	7	8	4	2	10
	x_2	3	3	1	2	4	1
Output	y	1	1	1	1	1	1

Using the neurodynamic model (10) to solve this problem yields the results shown in Table 3. Figure 1 illustrates the transient behavior of DMU$_A$, showing convergence to $u = 1$ and $v_1 = v_2 = 0.166$.

Table 3: Solution of the illustrative example.

	v_1	v_2	u	z
DMU$_A$	0.166	0.166	1	1.16
DMU$_B$	0.083	0.333	1	1.58
DMU$_C$	0.059	0.529	1	1
DMU$_D$	0.153	0.193	1	1
DMU$_E$	0.240	0.129	1	1
DMU$_F$	0	1	1	1

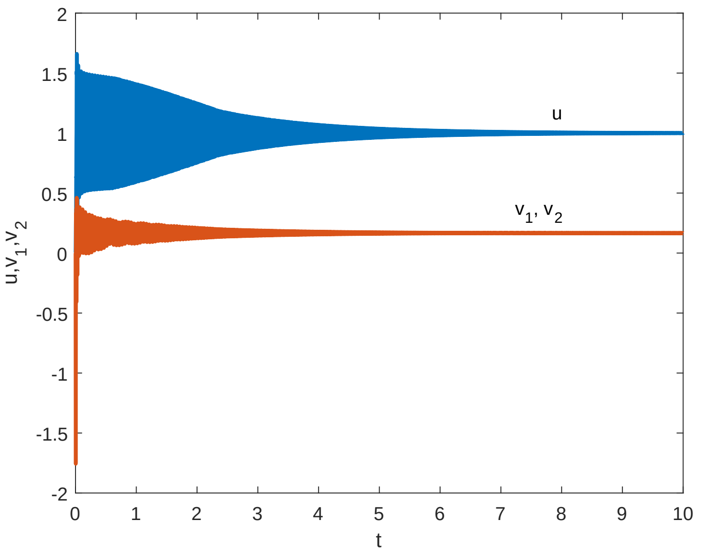

Figure 1: Convergence trajectories of DMU_A.

For a convergence-rate comparison, we computed the norm $\|z(t) - z^*\|$ for this example. Figure 2 shows the behavior of $\|z(t) - z^*\|$ for different values of the convergence-rate parameter ξ.

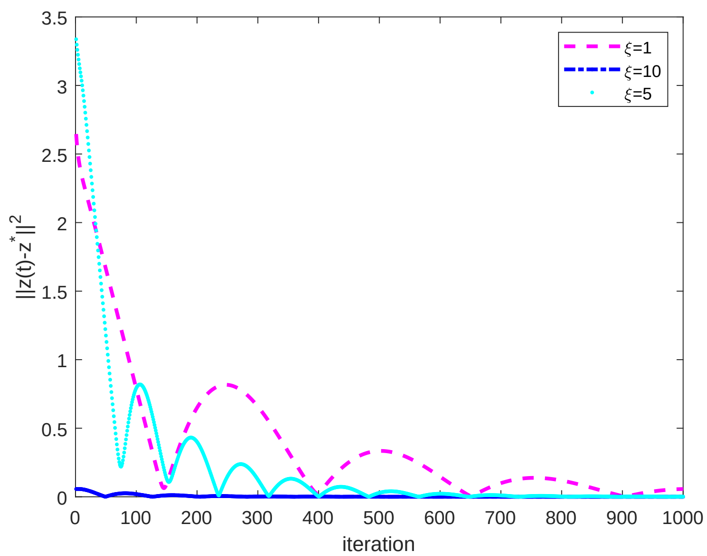

Figure 2: Behavior of $\|z(t) - z^*\|$.

4.2 Iran’s Asian Sports Caravan

The Asian Games are among the most important sporting events in the world, and Asian countries compete intensely to achieve a strong ranking by the end of the tournament. Iran has consistently sought to improve its standing among other countries at this event. Table 4 presents Iran’s GDP, as reported by the World Bank [29], together with the numbers of gold, silver, and bronze medals won, as reported by the National Olympic Committee of Iran [24].

Table 4: Iran’s medal counts, GDP, and population across editions of the Asian Games.

Asian Games	Output			Input	
	Gold Medals	Silver Medals	Bronze Medals	GDP* (Billion USD)	Population
Bangkok 1966	6	8	16	90.67	25 624 373
Bangkok 1970	9	7	7	147.87	28 513 866
Tehran 1974	38	28	17	218.83	31 786 471
New Delhi 1982	4	4	4	175.13	41 869 236
Seoul 1986	6	6	10	166.02	49 260 255
Beijing 1990	4	6	8	187.66	56 366 217
Hiroshima 1994	9	9	8	211.63	60 590 614
Bangkok 1998	10	11	13	238.45	63 971 836
Busan 2002	8	14	14	284.89	67 284 796
Doha 2006	11	15	22	349.89	70 554 760
Guangzhou 2010	20	14	25	405.41	73 762 519
Incheon 2014	21	18	18	414.11	77 465 770
Jakarta 2018	20	20	22	446.16	81 800 188
Hangzhou 2022	13	21	20	394.40	84 873 346

*GDP: Gross Domestic Product.

As shown in Table 4, Iran’s population and GDP both display an increasing trend over the period considered. The population grew from approximately 26 million in 1966 to approximately 82 million in 2018, while GDP rose from 90.67 billion USD in 1966 to 446.16 billion USD in 2018. The number of medals won, particularly gold medals, likewise shows an overall upward trend. Based on bronze medals, Iran ranked first among all editions in Guangzhou in 2010. Iran’s best overall result occurred at the 1974 Asian Games, held in Tehran; this edition was an exception, since Iran was the host country. Hosting the event can increase a country’s medal count, as the host nation is typically entitled to participate in all disciplines and may benefit from home-field advantages. The highest GDP in the period considered was recorded in 2018, whereas the highest population was recorded in 2022.

This study employs the CCR model of DEA to evaluate Iran's competitive efficiency in the Asian Games. The analysis uses the following variables: input variables are national GDP and population size, and output variables are medal achievements (gold, silver, and bronze medals). The neurodynamic model (10) was used for this purpose, and the resulting efficient units, based on the results in Table 5, were determined accordingly.

Table 5: Iran's efficiency scores and rankings in the Asian Games.

Year	**1966**	**1970**	**1974**	**1982**	**1986**	**1990**	**1994**	**1998**	**2002**	**2006**	**2010**	**2014**	**2018**	**2022**
Efficiency	0.623	0.785	1.000	0.224	0.426	0.311	0.601	0.698	0.531	0.821	0.923	0.897	0.875	0.801
Rank	9	7	1	14	12	13	10	8	11	5	2	3	4	6

The results in Table 5 indicate that 1974 was the most efficient year for Iran in the Asian Games, whereas all other years were classified as inefficient. Separately, an analysis of medal trends from 1966 to 2022 reveals a generally upward trajectory in medal counts; the efficiency score, by contrast, peaked unexpectedly in 1974, the host year, independently of this medal trend. The data further suggest a positive association between population growth and medal count in most years.

Table 6: Comparison of CPU time across neurodynamic models.

Model	**Neurodynamic model** (10)	**RNN** [2]	**Model** [20]	**Model** [10]
CPU time (sec.)	4.23	4.60	5.53	4.67

To validate the proposed approach, we implemented alternative neurodynamic models for comparison. As shown in Table 6, the developed neurodynamic model achieves a lower CPU time than the competing methods, owing to its simpler computational structure. All four models compared in Table 6 were reimplemented by the authors and run on identical hardware and software (the same machine, the same MATLAB R2023a version, and the same operating system).

5 Discussion and Conclusions

This study aimed to evaluate the efficiency of sports delegations using DEA, with a specific focus on Iran's performance in the Asian Games across 14 participation periods from 1966 to 2022. The DEA approach has been widely recognized as an effective tool for assessing national performance in multi-sport events, as demonstrated in prior research [6, 19, 26, 30]. A distinctive feature of our analysis is the deliberate exclusion of medal valuations from the efficiency calculations. While some studies [19, 30] have addressed this limitation by assigning weighted coefficients to different medal types (gold, silver, and bronze), such approaches fundamentally alter the evaluation framework from a pure gold-medal standard to a composite medal-value system. Our study offers an alternative perspective by maintaining the conventional DEA scoring scheme without medal weighting, thereby providing a baseline for comparative efficiency assessments in international sports competitions. While several studies have applied a DEA approach to evaluate countries in a single edition of the Asian Games, the present study analyzes Iran's performance across 14 editions. As an extension of this study, we recommend applying the proposed model to evaluate individual participating countries over multiple competition periods; this approach would enable more granular efficiency assessments and meaningful cross-country comparisons. The findings offer valuable insights for coaching staff and sports administrators, helping them identify (i) key strengths to maintain and build upon, and (ii) critical weaknesses that require improvement.

The developed single-layer neurodynamic system proved effective in solving the DEA model, and its stability and convergence were rigorously established. An application of the model to Iran's Asian Sports Caravan was presented to demonstrate both its applicability and the effectiveness of the solution method. In summary, the key quantitative findings of this study are as follows. The results in Table 5 show that Iran achieved its highest efficiency score (1.000) in 1974, the year it hosted the Asian Games, while all other years from 1966 to 2022 remained inefficient, with scores ranging from 0.224 (1982) to 0.923 (2010). Efficiency notably improved after 1990, reaching 0.923 in 2010 and consistently exceeding 0.8 in 2014, 2018, and 2022. Regarding computational performance, Table 6 shows that the proposed neurodynamic model achieved a CPU time of 4.23 seconds, outperforming the RNN model of [2] (4.60 sec.) and the other benchmark models (5.53 sec. and 4.67 sec.). These results confirm a clear computational-time advantage for the proposed approach, owing to its simpler structure.

For future research, several promising directions emerge. First, integrating hybrid machine-learning models with DEA could enhance predictive accuracy and help uncover nonlinear relationships in efficiency patterns. Second, developing further neurodynamic models to solve DEA represents a valuable avenue for methodological advancement.

Declarations

Availability of Supporting Data

All data generated or analyzed during this study are included in this published paper.

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Conflict of Interest

The author declares that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Authors Contributions

Amin Mansoori: Conceptualization, Methodology, Writing – Original Draft, Writing – Review and Editing, Validation. **Mohammad Eshaghnezhad**: Conceptualization, Methodology, Writing – Original Draft, Writing – Review and Editing, Validation. **Niloufar Kamkar**: Conceptualization, Methodology, Writing – Original Draft, Writing – Review and Editing, Validation. **Jalal A. Nasiri**: Conceptualization, Methodology, Writing – Review and Editing, Validation.

Artificial Intelligence Statement

Artificial intelligence (AI) tools, including large language models, were used solely for language editing and improving readability. AI tools were not used for generating ideas, performing analyses, interpreting results, or writing the scientific content. All scientific conclusions and intellectual contributions were made exclusively by the authors. Also, authors confirm that all mathematical derivations, proofs, and numerical results are entirely the product of human intellectual work.

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