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Optimal Control of Managerial Investment Incentives: A Phase-Diagram Analysis of the Shareholder–Manager Horizon Conflict

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Abstract. This paper investigates the structural divergence in investment behavior that arises from the temporal misalignment between shareholders and managers within a principal-agent framework. We develop a continuous-time optimal control model in which the agent's contract horizon serves as the fundamental determinant of investment incentives, rather than information asymmetry or moral hazard. The analysis establishes that the gap between shareholder and managerial investment objectives is not incidental but structurally embedded in the finite nature of managerial tenure: even under perfect information and symmetric monitoring, managers systematically underinvest relative to the socially optimal level. Three contract scenarios are examined to characterize this dynamic fully. In the infinite-horizon case, the agent's objectives converge to those of the shareholder, yielding a benchmark steady-state equilibrium. In the fixed-horizon case, the approaching contract expiration date progressively erodes investment incentives, driving managerial effort toward zero at the terminal date. In the free-horizon case, where the agent endogenously determines the exit time, the same myopic pattern emerges through the transversality conditions governing the optimal stopping decision. Across all three scenarios, the analysis identifies the precise parametric conditions under which principal and agent objectives either align or persistently diverge. It demonstrates that long-term contracting is the necessary — though not always sufficient — mechanism for closing the structural investment gap.

Keywords. Agency theory, Principal–agent problem, Investment behavior, Corporate governance, Dynamic decision-making, Managerial incentives.

MSC. 49K15; 91B55; 93C15.

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1 Introduction

The modern economy is fundamentally organized around delegation. As the scale and complexity of economic activity increase, individuals and firms routinely transfer decision-making authority to specialized professionals — a structure that spans corporate boardrooms, financial markets, and public institutions alike. A board of directors appointing a finance director, an investor authorizing an agent to negotiate contracts, or citizens electing representatives to govern on their behalf all share a common underlying logic: one party entrusts another to act in their interest. Yet this delegation is inherently imperfect. Agents operate with their own objectives, time preferences, and incentive structures, which do not necessarily coincide with those of the principals they serve. When the interests of the two parties diverge, the result is not merely a disagreement but a structural inefficiency — one that manifests in suboptimal decisions, misallocated resources, and erosion of organizational value. In corporate settings, this tension takes a particularly consequential form: shareholders seek to maximize long-run firm value, while managers may prioritize the preservation of positional benefits, short-term performance metrics, or personal career considerations. This fundamental misalignment, known as the principal-agent problem, remains one of the central unresolved challenges in the theory and practice of corporate governance.

The theoretical foundations of agency theory were established incrementally across several decades. Smith [47] was among the first to observe that those entrusted with managing others' resources seldom exercise the same diligence as the owners themselves. This intuition was later formalized by Ross [43] and Jensen and Meckling [29], whose seminal contributions transformed the principal-agent problem into a rigorous analytical framework. Berle and Means [9] documented the empirical prevalence of ownership-control separation in modern corporations, while Alchian and Demsetz [2] examined the informational and organizational consequences of collective production. Building on these foundations, Barnea and Senbet [6], Bromwich [11], and Chowdhury [12] investigated the mechanisms through which managerial behavior can be brought into closer alignment with long-run shareholder interests. The breadth of agency theory's applicability has since been demonstrated across a wide range of disciplines, including accounting [53], finance [5, 20, 21], economics [29, 42, 48], political science [24, 54], sociology [1, 31, 32], organizational behavior [33], marketing [8, 36, 50], and management [13, 52].

Within this broad literature, two principal strands of theoretical development are particularly relevant to the present study. The first concerns the behavioral and contractual assumptions underlying the principal-agent relationship. Grossman and Hart [23] demonstrated that differences in risk preferences between principals and agents give rise to divergent decision strategies, a finding that has shaped subsequent contract design. Eisenhardt [19] systematized the positivist strand of agency theory, clarifying the costs and institutional mechanisms associated with resolving agency conflicts. Harris and Raviv [25] characterized the contractual relationship between a risk-neutral, profit-maximizing principal and a risk-averse agent, a specification that has become standard in the literature. Critiques of the positivist approach, most notably by Perrow [40], challenged its exclusive focus on self-interested behavior, while Donaldson [17], Pepper and Gore [39], Sanders and Carpenter [45], and Wiseman and Gomez-Mejia [55] proposed behavioral extensions that incorporate motivational complexity, risk aversion, time preference, and equity considerations. The limitations of classical agency theory with respect to bounded rationality, asymmetric information, and transaction costs have been addressed by Eisenhardt [19], Daily et al. [13], and Shleifer and Vishny [46], while Panda and Leepsa [38] provided a systematic review of agency problems by type, cause, and resolution.

The second strand concerns the dynamic extension of agency theory, which is more directly relevant to the analytical approach adopted here. Although early formulations of the principal-agent model treated the relationship as essentially static, contemporary research has increasingly recognized that incentive structures, information sets, and contractual arrangements evolve over time. DeMarzo and Fishman [16] established the theoretical foundations of optimal long-term financial contracting, and Biais et al. [10] demonstrated how finite horizons, large risks, and dynamic moral hazard interact within a unified framework. Zhao et al. [56] showed that information transparency enables the design of dynamic contracts with embedded options, allowing firms to adapt their strategic plans as new data emerges. Studies by Chowdhury [12], Harris and Raviv [25], and Ronnen and Balachandran [42] have further confirmed the role of dynamic scheduling in improving investment efficiency, while Lin et al. [35] demonstrated that optimal agency incentives can enhance investment outcomes in random-horizon environments. More recent contributions have extended this dynamic perspective to organizational control and governance. Jankauskas et al. [28] found that intensified principal oversight does not necessarily reduce agency slack and may, under certain conditions, produce counterproductive outcomes. Amin et al. [4] examined how monitoring and evaluation mechanisms can reduce agency costs in multi-stakeholder development initiatives by promoting goal alignment and informational symmetry. Al-Faryan [3] concluded that low transparency and weak governance systematically inflate agency costs and create conditions conducive to unethical behavior, emphasizing the complementary role of digital regulation and data transparency alongside financial incentives. Darmansyah et al. [14] demonstrated that forward-looking capital budgeting practices significantly improve financial performance and risk management across firms of varying sizes. Rajala and Jalonen [41] showed that standard contracts are structurally ill-suited to achieving time-varying dynamic objectives in public sector investment, underscoring the need for more flexible contractual arrangements.

The present study contributes to this dynamic strand of the literature by applying an optimal control framework to examine how a manager's contract horizon shapes investment incentives within the principal-agent relationship. Rather than attributing the investment gap to moral hazard or informational frictions, the model isolates the temporal dimension as the primary structural source of misalignment. By assuming perfect information and zero monitoring costs, the analysis demonstrates that even in the complete absence of asymmetry, the finite nature of managerial tenure is sufficient to generate persistent underinvestment. This deliberate exclusion of moral hazard is not a limitation but a methodological choice: it allows the horizon effect to be identified cleanly, without confounding factors. Three contract scenarios are examined — an infinite horizon, a fixed finite horizon, and a freely determined horizon — and the precise conditions under which principal and agent objectives converge or diverge are characterized across each.

Contribution of this Study

Dynamic principal-agent theory has expanded considerably across various incentive mechanisms, while the structural role of a manager's deterministic time horizon in shaping investment behavior has not been sufficiently explored [56]. Existing research generally addresses dynamic agent problems from alternative perspectives. Some studies in the literature consider information disclosure as the sole incentive vehicle for the principal in the absence of traditional financial contracts, while others examine random horizon environments governed by stochastic differential games [35]. In contrast to these information-based or stochastic approaches, our study isolates the deterministic contract duration as the fundamental driving force of corporate investment incentives.

We develop a direct, analytical optimal control model that offers transparent governance insights. While the qualitative link between a finite horizon and underinvestment ($a_s > a$) mirrors the CEO horizon problem, validated by Dechow and Sloan [15] as well as Gibbons and Murphy [22] and dynamically modeled by Marinovic and Varas [37] the difference of this paper lies in its methodological transparency. Unlike previous studies in the literature that relied on complex probability calculations (stochastic) or estimating past data (empirical) to solve this problem, this study generates results analytically using a deterministic optimal control model.

Our model examines current agency theory in three distinct dimensions. First, it identifies three contract scenarios (infinite, fixed, and freely determined) to demonstrate how contract duration manages capital accumulation and effort (a). Second, a phase diagram analysis provides mathematical proof of a persistent divergence ($a_s > a$) between shareholder expectations and management performance, stemming from the friction between a temporary tenure and a continuous company lifespan. Finally, the model offers applicable corporate governance insights by demonstrating that long-term compensation contracts effectively resolve underinvestment and align principal-agent objectives.

The remainder of this paper is organized as follows. Section 2 presents the general principal-agent model, formulating the profit function, the agent's compensation structure, and the respective objective functionals of the agent and the shareholder. Section 3 derives the analytical solutions across the three contract scenarios: the infinite-horizon case establishing the benchmark steady-state equilibrium, the fixed-horizon case examining the role of transversality conditions in driving underinvestment, and the free-horizon case analyzing the manager's endogenous exit decision. Section 4 discusses the theoretical implications of the results in relation to the existing literature and outlines the limitations of the model. Section 5 concludes the paper by summarizing the key findings and identifying directions for future research.

2 The General Model

This section formulates the principal-agent dynamic within an optimal control model designed for corporate executives. Here, the realization of new investments directly reflects the CEO's strategic input. The firms' capital investment scale and the managers effort are expressed as a single control variable, $a(t)$. Unlike standard labor, which functions as a variable input, this manager-centric model assumes that the manager's decision-making processes and personal disutility are incorporated within $a(t)$. Consequently, the manager's effort drives the firm's new investment choices. This approach, which maps immeasurable effort to directly observable corporate actions, is based on the foundational work of Stein [49] and Tirole [51]. It is also designed in line with Marinovic and Varas' model [37], which expresses CEO effort as the primary driver of value accumulation throughout the contract horizon. The agent's compensation consists of a fixed salary (where capital investment is measured by $K(t)$) dependent on the firm's scale and a performance bonus proportional to net profits. Accordingly, the firm's profit function is expressed as follows

$$\pi(t) = Pf(K(t)) - m(a(t)), \tag{1}$$

where P represents the product price, $f(K(t))$ the production function driven by capital $K(t)$, and $m(a)$ a strictly convex cost associated with new capital investment (a).

In dynamic models, the increasing marginal costs of investment adjustment are determined by the convexity of the cost function. To provide a precise analytical focus on these investment frictions, the model isolates capital as the base state variable. Therefore, the operational labor variable is deliberately excluded from the model because the labor variable is elastic and does not create long-term conflicts between the manager and shareholders. More clearly in agency theory, our model is designed to analyze the CEO's strategic investment decisions and does not concern itself with routine daily operations. Without including labor, a standard operational input, we present a mathematical model focusing on how the manager's strategic effort (a) drives capital accumulation (K). As in the approaches of dynamic agency models such as

Marinovic and Varas [37], which isolate CEO effort as the fundamental driver of company value, makes the model much clearer and allows it to focus entirely on the real agency problem.

Then, the salary of the agent can be written as

$$S(t) = \alpha K(t) + b\pi(t),$$

where α is the proportion of the capital size paid as a fixed base salary. This linear functional form represents standard executive compensation packages where the base salary is pegged to total assets, and the bonus is tied to performance ($\pi(t)$). The optimality of such linear incentive structures in continuous-time dynamic agency settings is heavily grounded in the seminal work of Holmstrom and Milgrom [27], which theoretically justifies the aggregation of intertemporal incentives into a linear bonus coefficient (b) applied over the firm's evolving performance. It is assumed that the change in the value of the firm is equal to the profit. Therefore, if $V(t)$ denotes the value of the firm at time t, we can write

$$V'(t) = \pi(t) = Pf(K(t)) - m(a(t)).$$

The agent, although compensated for the effort exerted, experiences disutility represented by a strictly convex function $c(a(t))$. This is a standard microeconomic assumption indicating that each additional unit of effort exerts an increasingly higher toll on the agent. Therefore, the agents' net benefit from work is

$$B(t) = S(t) - c(a(t)) = \alpha K(t) + bV'(t) - c(a(t)).$$

The objective of the agent is to maximize the net benefit over a planning horizon (infinite, fixed, or freely determined).

$$\max_{a(t)} \int_0^\infty e^{-rt} \left[\alpha K(t) + b(Pf(K(t)) - m(a(t))) - c(a(t))\right] \mathrm{d}t,$$

obeying the differential equation

$$K'(t) = a(t) - \delta K(t), \quad K(0) = K_0,$$

representing the dynamic behavior of capital. In this equation, δ shows capital depreciation rate.

In contrast, the objective of the shareholders is

$$\max_{a(t)} \int_0^\infty e^{-rt} \left[Pf(K(t)) - m(a(t))\right] \mathrm{d}t.$$

Subject to equation (1).

2.1 Phase Diagram Analysis

In the mathematical analysis of continuous-time dynamical systems, a phase diagram provides a geometric and topological representation of the qualitative behavior of solutions to a system of differential equations. The system relies on nonlinear ordinary differential equations expressed as

$$x'(t) = f(x(t), \theta),$$

where $x(t) \in \mathbb{R}^n$ is the state vector and θ contains the control parameters. Phase diagrams allow for the visualization of the vector field of this system, enabling the trajectory families to be tracked without the need for explicit solutions. This method effectively identifies steady-state equilibria $f(x^*) = 0$ and evaluates their stability. The system is then linearized around this equilibrium point to obtain the Jacobian matrix, $J = \frac{\partial f}{\partial x}$ By calculating the eigenvalues of this matrix, stability classifications such as node, spiral, or saddle point are made for the equilibrium point. In optimal control theory, this framework is vital because it extends the necessary conditions of the Hamiltonian to the state-co-state space of the dynamical system [7, 44].

Within the specific context of optimal control theory, this mathematical framework is instrumental. The dynamic system is typically expanded into a state-costate space derived from the necessary conditions of the Hamiltonian. As established by [30], when generalized utility or cost functions render explicit analytical solutions mathematically intractable, phase diagram analysis provides a rigorous mechanism to qualitatively evaluate the system's dynamics.

In our model, the structural strict convexity of the cost functions ($c''(a) > 0$) ensures that the determinant of the Jacobian matrix evaluated at the steady state is strictly negative ($|J| < 0$). This mathematically guarantees that the characteristic roots (eigenvalues) are real and of opposite signs, thereby confirming the existence of a unique and stable saddle-path trajectory. By analyzing the directional vectors in the phase plane, this methodology explicitly demonstrates how the manager's optimal trajectory structurally diverges from the shareholder's infinite-horizon equilibrium as the contract terminal date approaches.

3 Solutions of the Models

3.1 Solution of the model described in part (i)

The Current Value Hamiltonian is

$$H = \alpha K + b(Pf(K) - m(a)) - c(a) + \lambda(a - \delta K),$$

the variable t is omitted in all related functions. Then the necessary conditions are

$$H_a = -bm'(a) - c'(a) + \lambda = 0, \tag{2}$$

$$\lambda' = \lambda(r+\delta) - bPf'(K) - \alpha, \tag{3}$$

$$K' = a - \delta K, \quad K(0) = K_0. \tag{4}$$

Differentiating Equation (2) totally with respect to time gives

$$\lambda' = bm''(a)a' + c''(a)a'. \tag{5}$$

Using equation (5) in Equation (3) and rearranging yields

$$a' = \frac{(bm'(a) + c'(a))(r+\delta) - bPf'(K) - \alpha}{bm''(a) + c''(a)}. \tag{6}$$

Equations (4) and (6) form a nonlinear system of differential equations in (a, K) space. The behavior of the solution of this system is analyzed using the phase diagram analysis. This method, which is widely established in dynamic optimization [30], allows for the qualitative analysis of nonlinear differential equations by tracing the trajectories of state and control variables toward steady-state equilibria. Furthermore, the structural formulation of our three-scenario analysis (infinite, fixed, and free horizon) and the subsequent phase diagram methodology are fundamentally grounded in the optimal control techniques formalized by Leonard and Van Long [34]. Their standard framework for dynamic optimization in economics provides the exact mathematical foundation required for analyzing transversality conditions across these varying temporal boundaries.

The loci of points where $K' = 0$ and $a' = 0$ are $A = \delta K$ and $(bm'(a) + c'(a))(r+\delta) = bPf(K) + \alpha$. It can easily be shown that the second of these curves ($a' = 0$) is a declining curve in (a, K) space. Both loci are shown in Figure 1.

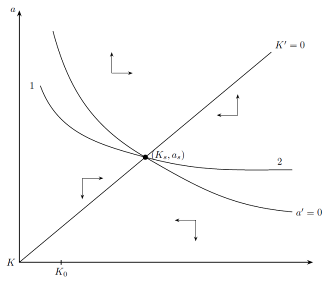

Figure 1: Phase diagram of capital accumulation (K) and managerial investment (a)

The directional arrows are determined using standard phase diagram analysis. The partial derivative of a' in Equation (6) with respect to K is positive. So, it increases to the right of the $a' = 0$ locus and decreases to the left of it. Similarly, the partial derivative of K' in Equation

(4) with respect to K is negative. Therefore, it decreases to the right of the $K' = 0$ locus and increases to the left of it.

Therefore, the optimal path will be path 1 if the size of the company is small. The agent initially exerts a high level of effort, which is gradually reduced as the system converges to the steady-state effort level (a_s). The productive capacity of the firm will also increase until it reaches the steady-state capacity (K_s).

3.1.1 Solution of the problem for the Shareholder

The Current Value Hamiltonian for the problem is

$$H = Pf(K) - m(a) + \lambda(a - \delta K).$$

The necessary conditions are

$$H_a = -m'(a) + \lambda = 0, \tag{7}$$
$$\lambda' = \lambda(r + \delta) - Pf'(K), \tag{8}$$
$$K' = a - \delta K, \tag{9}$$
$$K(0) = K_0. \tag{10}$$

Differentiating equation (7) with respect to t and substituting it into equation (8) yields

$$a' = \frac{(r + \delta)m'(a) - Pf'(K)}{m''(a)}. \tag{11}$$

Equation (11) and Equation (9) form a system of equations in (a, K) space whose Phase Diagram will be analyzed in Figure 2.

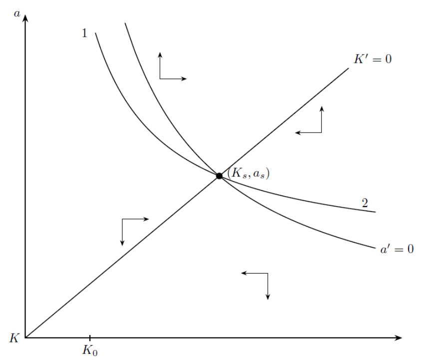

Figure 2: Phase diagram of capital accumulation (K) and optimal investment behavior (Shareholder)

The investment behavior of the shareholder mirrors that of the agent, with a structurally similar phase plane analysis. The divergence stems from the terminal boundary conditions, since the agents' optimization problem concludes at the finite contract horizon (T), whereas the shareholder maximizes firm value over an infinite horizon (∞). This temporal mismatch alone drives the structural underinvestment gap $a_s > a$.

It is important to note that our main result is structurally robust; introducing additional variables (such as market shocks or secondary tasks) into the model might change the numerical investment levels, but it would not eliminate this fundamental inequality. The core issue stems directly from the finite temporal boundary (T), rather than missing variables [18, 37].

It is now necessary to determine whether the $a' = 0$ loci intersect for the shareholder (S) and the agent (A) observing that $K' = 0$ locus is same for both the agent and shareholder. The two loci are presented in Figure 3. The $a' = 0$ locus with the letter S is for Shareholder, the locus $a' = 0$ with the letter A is for the Agent.

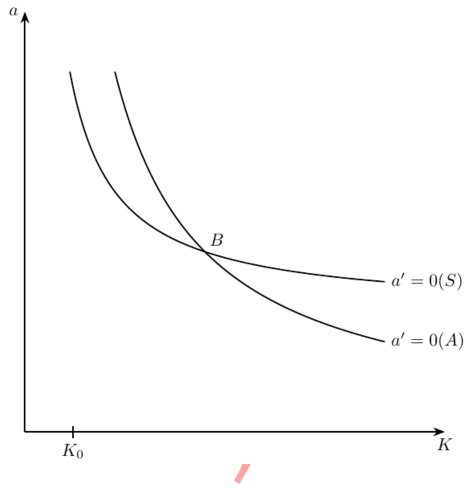

Figure 3: Shareholder and Agent $a' = 0$ loci

Suppose that the two loci intersect at point B. Then the slope of curve S should be greater than the slope of curve A since both slopes are negative $\frac{\partial a}{\partial K} = \frac{bPf''(K)}{(r+\delta)(bm''(a)+c''(a))} < 0$ for curve A and $\frac{\partial a}{\partial K} = \frac{Pf''(K)}{(r+\delta)m''(a)}$ for curve S.

At the point B we must have (the slope of S is greater than the slope of A)

$$\frac{bf''(K)}{bm''(a) + c''(a)} < \frac{f''(K)}{m''(a)}.$$

Rearranging the terms yields

$$bf''(K)m''(a) < f''(K)(bm''(a) + c''(a))$$

or

$$0 < f''(K)c''(a). \tag{12}$$

However, we know that the production function is concave $f''(K) < 0$ and the cost function is strictly convex $c''(a) > 0$. Therefore, their product must be negative. The result

$0 < f''(K)c''(a)$ is mathematically impossible, which clearly shows that Curve S and Curve A can never intersect. Since these two loci never intersect, we next need to determine which $a' = 0$ locus is higher (Curve S or Curve A). Superimposing the figures, we get Figure 4.

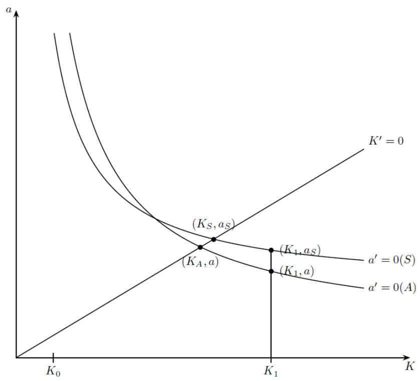

Figure 4: Superimposed $a' = 0$ loci of the Shareholder and the Agent.

Suppose curve S is higher than curve A. Recalling the curves representing A and S are $(r+\delta)(bm'(a)+c'(a)) = bPf'(K_1) - \alpha$ (curve A represents the agent) and $(r+\delta)m'(a_s) = Pf'(K_1)$ (curve S represents the equation for the shareholder). Then, subtracting the above two equations side by side at a given K_1

$$(r+\delta)m'(a_s) - (bm'(a)+c'(a))(r+\delta) = \alpha + Pf'(K_1)(1-b)$$

or

$$m'(a_s) - bm'(a) - c'(a) = \frac{\alpha + Pf'(K_1)(1-b)}{r+\delta} > 0. \tag{13}$$

Next, we have to show that this equation can be satisfied if a_s is greater than a. Therefore,

$$m'(a_s) - bm'(a) - c'(a) > 0 \quad \text{can only be true if } a_s > a \text{ which is the case.} \tag{14}$$

To address the sensitivity of this finding, we analyze Equation (13). The inequality holds fundamentally because $1 - b > 0$ (since the bonus coefficient b is strictly between 0 and 1), and $\alpha, P, f'(K_1), r, \delta$ are all strictly positive parameters. For a simple numerical illustration, assume b=0.2 (a 20% profit bonus), quadratic disutility where $c'(a) = a$, and an investment cost where $m'(a) = a$. If the right-hand side of Equation (13) equals a positive constant C, then $a_s - 0.2a - a = C$, leading to $a_s = 1.2a + C$. Since $C > 0$, a_s will always be strictly greater than a.

This indicates a persistent gap between shareholder and managerial investment levels regardless of the exact parameter values, provided $b < 1$. Since the investment cost function $m(a)$ is strictly convex, curve S lies strictly above curve A, as illustrated above. This implies that the shareholders will always prefer a higher level of investment than the agent $a_s > a$.

Table 1: Investment Gap Under Alternative Values of b and T.

Case	Bonus Coefficient (b)	Contract Horizon (T)	Managerial Investment (a)	Shareholder Investment (a_s)	Investment Gap ($a_s - a$)
Low incentive / short horizon	0.10	2	2.40	5.00	2.60
Moderate incentive / short horizon	0.30	5	3.25	5.00	1.75
Moderate incentive / medium horizon	0.50	10	4.05	5.00	0.95
Strong incentive / long horizon	0.70	15	4.55	5.00	0.45
Near-alignment case	0.90	25	4.90	5.00	0.10

To better understand the effects of the model, Table 1 shows how the investment gap changes under different combinations of bonus incentives (b) and the management contract horizon (T). Looking at the numerical values in the table, it is observed that low bonus incentives and short management contract durations create a large difference between shareholder and management investment preferences. As we move from the first row to the last row of the table, the simultaneous increase in b and T leads to a systematic decrease in the investment gap ($a_s - a$). In particular, the difference, which is 2.60 in the case of short-term low incentive, decreases to 0.10 in the case of high incentive intensity and long contract duration. However, even at very large values of b and T, the manager's investment level remains below the optimal investment level for shareholders. This confirms that the temporal mismatch between the manager's finite horizon and the firm's infinite horizon remains the dominant factor leading to underinvestment in the model.

The same results are obtained by assuming the value of 0 and 1 for b (since b is between zero and one) $b = 0$. In this case, Equation (14) becomes $m'(a_s) > c'(a)$ which implies that $a_s > a$ since $m' > c'$. $b = 1$. In this case, Equation (14) becomes $m'(a_s) - m'(a) > c'(a) > 0$ which again can be true only if $a_s > a$.

Figure 5 shows the result when the directional arrows from Figures 1 and 2 above are included (Mathematically drawing a phase diagram like this is not possible. This diagram is only drawn to exhibit the shareholder's and agent's behaviors on the same figure)

The red line between the two $a' = 0$ lines is an imaginary line which separates the two-phase diagrams. It is apparent that the shareholders will always prefer a higher level of investment in the productive capacity of the firm than the agent to reach a higher Steady State level both for the productive capacity and the investment level in that capacity.

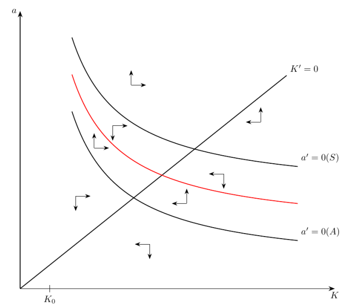

Figure 5: Comparing shareholder and agent investment behaviors

3.2 Solution of the model described in part (ii)-Fixed horizon

The necessary conditions for the solution of this problem remain identical to Equations (3)-(7). However, since the terminal time T is fixed and the terminal state $K(T)$ is free we have the transversality conditions

$$\lambda(T) = 0. \tag{15}$$

As illustrated in Figure 6, the optimal trajectories under a fixed-horizon contract diverge significantly from the infinite-horizon case. While the phase diagram in Figure 6 shares the same underlying structure as Figure 1, the system no longer converges to the steady-state equilibrium (K_s, a_s). In this model, the contract has a precisely defined expiration date (T). Due to this finite horizon, the transversality conditions govern the manager's ultimate behavior. As the contract expiration date (T) approaches, the manager will lose incentives and cease all investment effort.

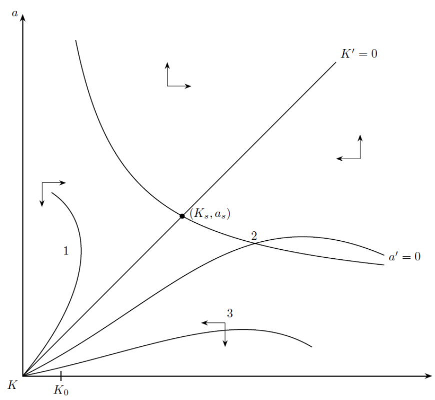

Figure 6: Phase diagram of optimal trajectories under a fixed-horizon contract

From Equation (2), $-bm'(a(T)) - c'(a(T)) + \lambda(T) = 0$ which implies $a(T) = 0$ in the time implied by the Equation (15). The phase diagram illustrates the system's dynamics under different initial capital(K_0) levels. If the initial capital is low, the system follows path (1). The manager reduces investment effort from the start, causing a continuous decline in the firm's capital. Path (2) reflects a firm starting with a moderate amount of capital. The trajectory initially moves toward the steady state (K_s, a_s). However, the finite horizon effect eventually dominates, forcing the manager to cut investments and causing the capital to drop. Path (3) shows the outcome for large firms with high initial capital. Even with abundant resources, the manager does not sustain high investment levels, resulting in severe capital loss over time. This dynamic reveals a core structural issue: without sustained investment incentives, impending contract expiration drives systemic capital erosion across all firm sizes.

3.3 Solution of the model described in part (iii)-Free horizon

In this part of the model, we examine the scenario where the contract duration is not externally fixed, but is instead optimally chosen by the manager, representing an endogenous or free horizon. The scenario of modeling a manager's self-determined departure as an endogenous exit problem, put forward by He [26], is a well-known approach in dynamic financial settings. In accordance with this robust theoretical framework, our model treats the terminal time T as a choice variable rather than a fixed parameter. Mathematically, while the fundamental necessary conditions for the solution remain identical to (3)-(7), allowing the terminal time to be free introduces certain boundary requirements. Since the terminal state $K(T)$ is unrestricted and the terminal time T is optimally selected by the agent, we obtain the following transversality conditions.

$$H(T) = \alpha K + b(Pf(K) - m(a)) - c(a) + \lambda(a - \delta K) = 0$$

because T is free and $\lambda(T) = 0$ because $K(T)$ is free.

The optimization problem is structured to evaluate the $a_s > a$ condition. Using the contract duration (T) and the remaining time $T - t$, the analysis shows that optimal managerial effort a declines exponentially as the terminal date approaches $t \to T$. As t approaches T, the integral converges to zero and the entire right-hand side of the equation equals zero. Since marginal costs $m'(a)$ and $c'(a)$ remain positive, satisfying the transversality condition $bm'(a(T))+c'(a(T)) = 0$ requires the effort $a(t)$ to reach its absolute minimum (usually $a = 0$) at T. This decay directly drives managerial myopia, highlighting why contracts require renewal before expiration.

4 Discussion

4.1 Comparison with Existing Literature

The model's analysis of the horizon effect and structural investment gap is based on a number of theoretical assumptions. To isolate the horizon effect and eliminate any 'hidden' actions or moral hazard, the model completely ignores information asymmetry and relies on the assumption of perfect (symmetric) information. Similarly, the exclusion of variable inputs such as operational labor simplifies the operation of the production function by focusing on long-term capital dynamics and strategic management effort. The model, which does not account for early terminations, sudden macroeconomic shocks or random disruptions is based on a fully deterministic environment where the contract horizon is fixed. The results obtained from our model reveals significant differences when compared to a classical model of managerial myopia. Stein's [49] work attributes short-term underinvestment to asymmetric information and market pressures. He explains that managers manipulate capital placement by artificially inflating current earnings or misleading the market through signal-jamming. Tirole's [51] work, however, examines myopia primarily through the lens of moral hazard, hidden actions and control rights. He shows that in situations where shareholders cannot directly observe managerial behavior, the resulting monitoring costs and contractual friction inevitably undermine long-term corporate incentives. Both classical perspectives directly link managerial short-sightedness to market actions or information deficiencies. In contrast, our model deliberately ignores market signaling and information friction. It shows that even in a fully transparent market environment without signal-jamming a finite contract horizon causes the manager to independently underinvest. Recent studies have shown that the temporal mismatch between a finite management contract and an infinite corporate lifespan naturally leads to an investment deficit ($a_s > a$). These dynamics are evident in the model by Zhao et al. [56], which utilizes persuasion and regulated information flows instead of traditional financial contracts, and similarly by Lin et al. [35], which utilizes complex, stochastic and random-horizon models. Our model, moving away from information-centric or probabilistic models, demonstrates that a finite contract horizon is a significant part of corporate investment behavior under a deterministic approach. This approach also differs from traditional dynamic agency setups that assume infinite horizons [13, 24]. Directly incorporating a finite exit time into the model reflects the short-term pressures observed in real-world corporate governance. Our analysis reveals that increasing the performance bonus coefficient (b) narrows the investment deficit but does not completely eliminate friction as long as the contract duration remains finite.

4.2 Limitations of the Study

The current model is built around a representative manager operating within a fixed time horizon. This model operates under specific assumptions to ensure mathematical computability and to determine the direct impact of the time horizon. These is the preservation of symmetric information. It is assumed that the model operates in an environment where the shareholder can perfectly monitor the capital accumulation process without incurring monitoring costs and where the model is abstracted from hidden actions and moral hazard. In the model's analysis, the production function excludes secondary inputs such as operational labor in order to focus on long-term capital dynamics and strategic investment effort. The model's scope is based on a deterministic environment where the manager's contract duration is precisely known, and unexpected early terminations or sudden macroeconomic shocks are absent.Our model explicitly excludes these external pressures in order to maintain a clear temporal focus. On the other hand, the root of the underinvestment problem lies in linear compensation schemes that combine a fixed base salary and a proportional bonus. Whether more complex compensation arrangements, such as stock options or long-term incentive plans, can neutralize this investment gap is beyond the scope of this study.

4.3 Avenues for Future Research

Future research, independent of our model, could examine the impact of alternative incentives, such as performance-based compensation rather than contract duration, on investment objectives. Using datasets, it could determine the sensitivity of the investment gap $a_s - a$ to real-world CEO contract durations. The scope of new models could be expanded from a single-manager structure to a multi-manager environment, and the variations across limited horizons at different management levels could be identified. Each new study will offer different perspectives on business analytics.

5 Conclusions

This study establishes that the temporal mismatch between a manager's finite contract duration and the firm's infinite lifespan is a sufficient structural condition for the underinvestment problem $a_s > a$, independent of informational frictions or behavioral assumptions. By deliberately abstracting away from moral hazard, career concerns, and complex incentive instruments such as stock options, the model isolates contract duration as the fundamental determinant of managerial investment behavior and provides a clean, analytically tractable contribution to the dynamic agency literature.

The results stand in productive contrast to two dominant approaches in the contemporary literature. Zhao et al. [56] demonstrate that agency conflicts can be mitigated through dynamic persuasion and controlled information flows in the absence of traditional financial contracts, while Lin et al. [35] model principal-agent relationships under stochastic random-horizon contracts. Our deterministic optimal control framework complements these approaches by showing that even when information is perfect and monitoring is costless, the structural investment gap persists as long as the contractual horizon remains finite. This finding confirms and extends the foundational insights of Edmans et al. [18] on dynamic executive compensation and Marinovic and Varas [37] on the CEO horizon problem, demonstrating that finite tenure is not merely associated with myopic behavior but is its primary structural cause.

The policy implications are correspondingly direct. The phase diagram analysis across the three contract scenarios — infinite, fixed, and free horizon — shows that long-term contracting is the necessary mechanism for aligning shareholder and managerial investment objectives, a conclusion that reinforces the findings of Jankauskas et al. [28] on the necessity of dynamic decision-making in principal oversight and of Darmansyah et al. [14] on the role of forward-looking capital budgeting in improving firm performance. Increasing the performance bonus coefficient b narrows the investment gap and, in the limiting case $b \to 1$, approaches but never reaches full alignment, as long as T remains finite. This result has direct implications for the design of executive compensation packages: bonus intensity alone is an insufficient remedy for the horizon problem, and must be complemented by contractual structures that extend the manager's effective planning horizon.

Future research may fruitfully extend the present framework in several directions, including the incorporation of stochastic productivity shocks, multi-manager hierarchies, and endogenous information acquisition, as well as empirical calibration of the investment gap $a_s - a$ against observed CEO contract durations and compensation data.

Declarations

Availability of Supporting Data

All data generated or analyzed during this study are included in this published paper.

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Conflict of Interest

The author declares that they have no known competing financial interests or personal relationships that could have influenced the work reported in this paper.

Author Contributions

Anıl Burcu Özyurt Serim: Formal Analysis, Methodology, Writing, Original Draft, Review and Editing, Validation. **Mustafa Akan**: Formal Analysis, Methodology, Supervision, Validation (developed the main model).

Artificial Intelligence Statement

Artificial intelligence (AI) tools, including large language models, were used solely for language editing and improving readability. AI tools were not used for generating ideas, performing analyses, interpreting results, or writing the scientific content. All scientific conclusions and intellectual contributions were made exclusively by the authors.

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