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Solving System of Nonlinear Equations by using a New Three-Step Method

H. Esmaeili[1*], R. Erfanifar[2], M. Ahmadi[3]

[1,2]Department of Mathematics, Bu-Ali Sina University, Hamedan, Iran
[3]Department of Mathematics, Malayer University, Malayer, Iran.

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Abstract. In this paper, we suggest a fifth order convergence three-step method for solving system of nonlinear equations. Each iteration of the method requires two function evaluations, two first Fréchet derivative evaluations and two matrix inversions. Hence, the efficiency index is $5^{1/(2n+4n^2+\frac{4}{3}n^3)}$, which is better than that of other three-step methods. The advantages of the method lie in the feature that this technique not only achieves an approximate solution with high accuracy, but also improves the calculation speed. Also, under several mild conditions the convergence analysis of the proposed method is provided. An efficient error estimation is presented for the approximate solution. Numerical examples are included to demonstrate the validity and applicability of the method and the comparisons are made with the existing results.
Keywords. Nonlinear equations, Iterative method, Convergence order, Efficiency index.
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∗ Corresponding author
esmaeili@basu.ac.ir, rerfanifar92@basu.ac.ir, mehdi.ahmadi@malayeru.ac.ir
http://mathco.journals.pnu.ac.ir

1 Introduction

The system of nonlinear equations is ubiquitous in many areas of applied mathematics and plays vital roles in a number of applications such as science and engineering. Most physical problems, such as biological applications in population dynamics and genetics where impulses arise naturally or are caused by control, can be modelled by nonlinear equations or a system of them. The system of nonlinear equations is usually difficult to solve analytically, therefore a numerical method is needed. Construction of iterative methods to approximate solution for system of nonlinear equations is one of the most important tasks in the applied mathematics.

Consider the following system of nonlinear equations:

$$F(x) = 0, \qquad F : D \to \mathbb{R}^n, \tag{1}$$

in which $F(x) = (f_1(x), f_2(x), \ldots, f_n(x))$ is a Fréchet differentiable function, $D \subseteq \mathbb{R}^n$, and $x = (x_1, x_2, \ldots, x_n)^T$ is an unknown vector. Suppose that $F(x) = 0$ has a solution $a \in D$.

Newton method is undoubtedly the most famous iterative method to find a by using

$$x^{(k+1)} = x^{(k)} - F'(x^{(k)})^{-1} F(x^{(k)})\,, \tag{2}$$

that converges quadratically in some neighbourhoods of a ([8], [10]). In recent years, several iterative methods have been proposed to improve the order of convergence and efficiency of Newton method (2) to solve the system of nonlinear equations (1), by using essentially Taylor's polynomial, decomposition, homotopy perturbation method, quadrature formulas and other techniques ([1], [2], [4], [7], [10], [14], [15]).

In this paper, we introduce an iterative three-step method for solving (1). It is proved that our method is fifth order convergence and each of its iterations requires two function evaluations, two first Fréchet derivative evaluations and two matrix inversions. Two numerical examples are given to illustrate the efficiency and the performance of the new iterative method. The obtained results suggest that this newly improvement technique introduces a promising and powerful tool for solving system of nonlinear equations.

This paper is organized as follows. In Section 2, we provide our new method to solve (1). It is proved that the method is fifth order convergence and efficiency analysis will be discussed. In Section 3, the proposed method is applied to several types of examples and the comparisons with the existing numerical solvers are made as reported in the other published works in this area. Finally, some conclusions are given in Section 4.

2 Solution procedure

Before discussing on the numerical solution for solving (1), we need to consider some definitions and one theorem.

Definition 1. Let $\{x^{(k)}\}_{k \geqslant 0}$ be a sequence in $\mathbb{R}^n$, which converge to a. Then, the sequence $\{x^{(k)}\}_{k \geqslant 0}$ is said to be convergence of order p to a if there exist a constant c and a natural number N such that $\|x^{(k+1)} - a\| \leq c\|x^{(k)} - a\|^p$, for all $k > N$.

Definition 2. Let a be a zero of function $F(x)$ and suppose that $x^{(k-1)}$, $x^{(k)}$ and $x^{(k+1)}$ are three consecutive iterations close to a. Then, the computational order of convergence ρ (denoted by COC) is defined by

$$\rho = \frac{\ln(\|x^{(k+1)} - a\|/\|x^{(k)} - a\|)}{\ln(\|x^{(k)} - a\|/\|x^{(k-1)} - a\|)}.$$

It is well-known that the computational order of convergence ρ can be approximated by means of

$$\rho \approx \frac{\ln(\|x^{(k+1)} - x^{(k)}\|/\|x^{(k)} - x^{(k-1)}\|)}{\ln(\|x^{(k)} - x^{(k-1)}\|/\|x^{(k-1)} - x^{(k-2)}\|)},$$

which was used by Cordero and Torregrosa [4].

Definition 3. Let $e^{(k)} = x^{(k)} - a$ be the error in the k-th iteration of an iterative method. We call the relation

$$e^{(k+1)} = Ce^{(k)^p} + O(e^{(k)^{p+1}})$$

as the error equation.

If we obtain the error equation for any iterative method, then the value of p is its convergence order.

For the sake of improving the local order of convergence, many modified methods have been proposed in literatures. In the sequel, we mention some three-step methods.

Darvishi and Barati [5] provided the following fourth order convergence method, in which each iteration requires two function evaluations, three first Fréchet derivative evaluations and two matrix inversions:

$$\begin{aligned} y^{(k)} &= x^{(k)} - F'(x^{(k)})^{-1}F(x^{(k)}), \\ z^{(k)} &= x^{(k)} - F'(x^{(k)})^{-1}(F(x^{(k)}) + F(y^{(k)})), \\ x^{(k+1)} &= x^{(k)} - \Big[\frac{1}{6}F'(x^{(k)}) + \frac{2}{3}F'\big(\frac{x^{(k)} + z^{(k)}}{2}\big) + \frac{1}{6}F'(z^{(k)})\Big]^{-1}F(x^{(k)}). \end{aligned} \tag{3}$$

Cordero et al. [3] provided the following fifth order convergence method, in which each iteration requires two function evaluations, two first Fréchet derivative evaluations and three matrix inversions:

$$\begin{aligned} y^{(k)} &= x^{(k)} - F'(x^{(k)})^{-1}F(x^{(k)}), \\ z^{(k)} &= x^{(k)} - 2\Big[F'(y^{(k)}) + F'(x^{(k)})\Big]^{-1}F(x^{(k)}), \\ x^{(k+1)} &= z^{(k)} - F'(y^{(k)})^{-1}F(z^{(k)}). \end{aligned} \tag{4}$$

Cordero et al. [2] provided the following sixth order convergence method, in which each iteration requires two function evaluations, two first Fréchet derivative evaluations and two matrix inversions:

$$\begin{aligned} y^{(k)} &= x^{(k)} - \tfrac{2}{3}F'(x^{(k)})^{-1}F(x^{(k)}), \\ z^{(k)} &= x^{(k)} - \tfrac{1}{2}(3F'(y^{(k)}) - F'(x^{(k)}))^{-1}(3F'(y^{(k)}) + F'(x^{(k)}))F'(x^{(k)})^{-1}F(x^{(k)}), \\ x^{(k+1)} &= z^{(k)} - 2(3F'(y^{(k)}) - F'(x^{(k)}))^{-1}F(z^{(k)}). \end{aligned} \tag{5}$$

Soleymani et al. [13] provided the following sixth order convergence method, in which each iteration requires two function evaluations, two first Fréchet derivative evaluations and two matrix inversions:

$$\begin{aligned} y^{(k)} &= x^{(k)} - \tfrac{2}{3}F'(x^{(k)})^{-1}F(x^{(k)}), \\ z^{(k)} &= x^{(k)} - \tfrac{1}{2}(3F'(y^{(k)}) - F'(x^{(k)}))^{-1}(3F'(y^{(k)}) + F'(x^{(k)}))F'(x^{(k)})^{-1}F(x^{(k)}), \\ x^{(k+1)} &= z^{(k)} - \Big[\tfrac{1}{2}(3F'(y^{(k)}) - F'(x^{(k)}))^{-1}(3F'(y^{(k)}) + F'(x^{(k)}))\Big]^2 F'(x^{(k)})^{-1}F(z^{(k)}). \end{aligned} \tag{6}$$

Beside above methods, we introduce our three-step method as follows:

$$\begin{aligned} y^{(k)} &= x^{(k)} - F'(x^{(k)})^{-1}F(x^{(k)}), \\ z^{(k)} &= x^{(k)} - \frac{1}{2}\Big[F'(x^{(k)})^{-1} + F'(y^{(k)})^{-1}\Big]F(x^{(k)}), \\ x^{(k+1)} &= z^{(k)} - F'(y^{(k)})^{-1}F(z^{(k)}). \end{aligned} \tag{7}$$

It is clear that each iteration of (7) requires two function evaluations, two first Fréchet derivative evaluations and two matrix inversions.

Now, we analyze the convergence order of the recently described method. The following theorem shows that the order of convergence is 5.

Theorem 1. Let $F(x)$ be sufficiently differentiable in a neighbourhood of $a \in D$ which is a solution of the nonlinear system (1). Let us suppose that $F'(x)$ is continuous and nonsingular in a. If $x^{(0)}$ is sufficiently close to a, then the three-step method defined by (7) is fifth-order, and satisfies the error equation

$$e^{(k+1)} = C_2^2 C_3 {e^{(k)}}^5 + O({e^{(k)}}^6),$$

in which $e^{(k)} = x^{(k)} - a$, and $C_k = \dfrac{1}{k!}F'(a)^{-1}F(a),\ k = 2, 3, \ldots.$

Proof. Using Taylor's expansion we have

$$F(x^{(k)}) = F'(a)\Big[e^{(k)} + C_2{e^{(k)}}^2 + C_3{e^{(k)}}^3 + C_4{e^{(k)}}^4 + C_5{e^{(k)}}^5\Big] + O({e^{(k)}}^6), \tag{8}$$

and

$$F'(x^{(k)}) = F'(a)\Big[I + 2C_2e^{(k)} + 3C_3{e^{(k)}}^2 + \cdots + 6C_6{e^{(k)}}^5\Big] + O({e^{(k)}}^6). \tag{9}$$

From (9) we have

$$F'(x^{(k)})^{-1} = F'(a)^{-1}\Big[I + d_1e^{(k)} + d_2{e^{(k)}}^2 + d_3{e^{(k)}}^3 + d_4{e^{(k)}}^4\Big] + O({e^{(k)}}^5), \tag{10}$$

where

$$\begin{aligned} d_1 &= -2C_2, \\ d_2 &= 4C_2^2 - 3C_3, \\ d_3 &= -8C_2^3 + 12C_2C_3 - 4C_4, \\ d_4 &= 16C_2^4 - 36C_2^2C_3 + 16C_2C_4 + 9C_3^2 - 5C_5. \end{aligned}$$

Then, from (8) and (10), we have:

$$F'(x^{(k)})^{-1}F(x^{(k)}) = e^{(k)} + n_2{e^{(k)}}^2 + \cdots + n_5{e^{(k)}}^5 + O({e^{(k)}}^6), \tag{11}$$

in which

$$\begin{aligned} n_2 &= -C_2, \\ n_3 &= 2C_2^2 - 2C_3, \\ n_4 &= -4C_2^3 + 7C_2C_3 - 3C_4, \\ n_5 &= 8C_2^4 - 20C_2^2C_3 + 10C_2C_4 + 6C_3^2 - 4C_5. \end{aligned}$$

Now, for $y^{(k)}$ we have:

$$y^{(k)} - a = -n_2 e^{(k)^2} - n_3 e^{(k)^3} - \cdots - n_5 e^{(k)^5} + O(e^{(k)^6}),$$

and

$$\begin{aligned} F'(y^{(k)}) &= F'(a)\Big[I + 2C_2(y^{(k)} - a) + \cdots\Big] \\ &= F'(a)\Big[I + N_2 e^{(k)^2} + N_3 e^{(k)^3} + N_4 e^{(k)^4}\Big] + O(e^{(k)^5}), \end{aligned} \tag{12}$$

where

$$\begin{aligned} N_2 &= -2n_2C_2, \\ N_3 &= -2n_3C_2, \\ N_4 &= -2n_4C_2 + 3n_2^2C_3. \end{aligned}$$

From (12) we have

$$F'(y^{(k)})^{-1} = F'(a)^{-1}\Big(I - N_2e^{(k)^2} - N_3e^{(k)^3} + (N_2^2 - N_4)e^{(k)^4}\Big) + O(e^{(k)^5}). \tag{13}$$

From (8) and (13) we have

$$F'(y^{(k)})^{-1}F(x^{(k)}) = e^{(k)} + L_2e^{(k)^2} + L_3e^{(k)^3} + L_4e^{(k)^4} + L_5e^{(k)^5} + O(e^{(k)^6}), \tag{14}$$

where

$$\begin{aligned} L_2 &= C_2, \\ L_3 &= C_3 - N_2, \\ L_4 &= C_4 - C_2N_2 - N_3, \\ L_5 &= C_5 - C_3N_2 - C_2N_3 - N_3 + (N_2^2 - N_4). \end{aligned}$$

Now, for $z^{(k)}$ we have

$$z^{(k)} - a = M_3e^{(k)^3} + M_4e^{(k)^4} + M_5e^{(k)^5} + O(e^{(k)^6}), \tag{15}$$

where

$$M_i = -\frac{L_i + n_i}{2} \quad i = 3, 4, 5.$$

Taylor expansion of $F(z^{(k)})$ is equal to

$$\begin{aligned} F(z^{(k)}) &= F'(a)\Big[(z^{(k)} - a) + C_2(z^{(k)} - a)^2\Big] \\ &= F'(a)\Big[M_3e^{(k)^3} + M_4e^{(k)^4} + M_5e^{(k)^5}\Big] + O(e^{(k)^6}). \end{aligned} \tag{16}$$

From (13) and (16) we have

$$F'(y^{(k)})^{-1}F(z^{(k)}) = \Big[q_3e^{(k)^3} + q_4e^{(k)^4} + q_5e^{(k)^5}\Big] + O(e^{(k)^6}), \tag{17}$$

where

$$q_3 = M_3, \qquad q_4 = M_4, \qquad q_5 = M_5 - N_2M_3.$$

Now, for $x^{(k+1)}$ we have

$$x^{(k+1)} - a = N_2M_3e^{(k)^5} + O(e^{(k)^6}) = C_2^2C_3e^{(k)^5} + O(e^{(k)^6}). \tag{18}$$

Therefore,

$$e^{(k+1)} = C_2^2C_3e^{(k)^5} + O(e^{(k)^6}), \tag{19}$$

which shows that the convergence order of the method (7) is five. □

In order to compare the introduced methods, we use the index of efficiency defined by $I = p^{1/Op}$, where p is convergence order and "Op" is the number of operations per iteration in terms of the number of arithmetic computations. To this end, evaluation of any scalar function is considered as an operation. We display index of efficiency of fifth-order Cordero et al. method (4) by I_{CM5}, sixth-order Cordero et al. method (5) by I_{CM6}, Soleymani et al. method (6) by I_{SOM}, and the proposed method (7) by I_{EAM}.

To calculate the computational cost per iteration of any method, we notice the following facts: Every computation of $F(x^{(k)})$ requires n evaluations of component scalar functions $f_i(x^{(k)})$, $i = 1, \ldots, n$; Every computation of $F'(x^{(k)})$ requires n^2 evaluations of scalar functions. A LU decomposition for solving the linear systems involved requires $\frac{2}{3}n^3$ floating point arithmetic, while the floating point operations for solving a triangular system is n^2. Also, n^3 and n^2 operations need to compute a matrix-matrix and a matrix-vector multiplication, respectively. According to above enumerations, index of efficiency of different methods are as follows:

$$I_{EAM} = 5^{\frac{1}{2n+4n^2+\frac{4}{3}n^3}}, \qquad I_{DM} = 4^{\frac{1}{2n+6n^2+\frac{4}{3}n^3}}, \qquad I_{CM5} = 5^{\frac{1}{2n+8n^2+2n^3}},$$
$$I_{CM6} = 6^{\frac{1}{2n+6n^2+\frac{7}{3}n^3}}, \qquad I_{SOM} = 6^{\frac{1}{2n+6n^2+\frac{7}{3}n^3}}.$$

In Figure 1, efficiency indices of various methods are compared (for $n = 2, 3, 4, 5, 6$). We notice that larger n result in exponential decrease in efficiency of the method. Furthermore, our method (7) has the highest efficiency index.

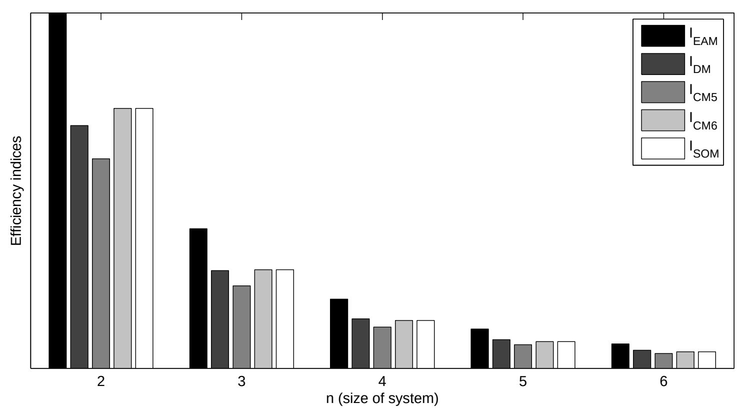

Figure 1: Comparison of efficiency indices

3 Applications and numerical results

In this section, several numerical examples are given to illustrate the accuracy and effectiveness of our method. All computations are done by using a PC with CPU of 2.5 GHz and RAM 4 GB, and all codes have been carried out in MATLAB, with variable precision arithmetic that uses floating point representation of 4096 decimal digits of mantissa. We accept an approximate solution as the exact root, depending on the precision (ε) of the computer. We use the following stopping criterion for computer programs:

$$Error = \|x^{(k+1)} - x^{(k)}\| + \|F(x^{(k)})\| < \varepsilon,$$

or the maximum number of iterations is $maxiter$. So, when the stopping criterion is satisfied, $x^* := x^{(k+1)}$ is taken as the exactly computed root a. For numerical illustrations in this section, we used the fixed stopping criterion $\varepsilon = 10^{-200}$ and $maxiter = 250$.

Example 1. Consider the mixed Hammerstein integral equation [6]

$$x(s) = 1 + \frac{1}{5}\int_0^1 G(s,t)x(t)^3 dt,$$

where $x \in C[0,1]$, $s, t \in [0,1]$ and the kernel G is

$$G(s,t) = \begin{cases} (1-s)t & t \leq s, \\ s(1-t) & s \leq t. \end{cases}$$

We transform the above equation into a finite-dimensional problem by using the Gauss-Legendre quadrature formula given as

$$\int_0^1 f(t)dt \approx \sum_{j=1}^{10} \omega_j f(t_j),$$

in which the abscissas t_j and the weights ω_j are determined. Denoting the approximation of $x(t_i)$ by x_i, $i = 1, 2, \ldots, 10$, we obtain the system of nonlinear equations

$$x_i - 1 - \frac{1}{5}\sum_{j=1}^{10} a_{ij} x_j^3 = 0, \qquad i = 1, 2, \ldots, 10,$$

in which

$$a_{ij} = \begin{cases} \omega_j t_j (1-t_i) & j \leq i, \\ \omega_j t_i (1-t_j) & i < j. \end{cases}$$

The abscissas t_j and the weights ω_j are shown in Table 1. The initial approximation is $x^{(0)} = [1.5, 1.5, \ldots, 1.5]^t$ and the solution of this problem is as follows:

$$\begin{aligned} a = \ & [1.0013772322535366\cdots, 1.0067577156846859\cdots, 1.0145195809538557\cdots, \\ & 1.0219909920721493\cdots, 1.0265429339811655\cdots, 1.0265471862815878\cdots, \\ & 1.0220033146356359\cdots, 1.0145387536672509\cdots, 1.0146449605572704\cdots, \\ & 1.0146499054957095\cdots]^t. \end{aligned}$$

Numerical results of the various methods are given in Table 2.

Example 2. To illustrate advantage of our method (7), consider the following nonlinear system:

$$\begin{aligned} x^2 + y^2 - 4 &= 0 \\ -e^x + y - 1 &= 0. \end{aligned} \tag{20}$$

There are two exact solutions for this system: $a = [0, 2]$ and $b = [-1.5983..., 1.2022...]$. The real dynamical planes in $\mathbb{R}^2$ and the two solutions (black dots) of the system (20) are represented in Figure 2. According to the figure, for any starting point arise from the white or gray regions, the method (7) is converged to the solution in that region, while starting points from other region fails to converge (The point $a = [0, 2]$ is in the gray region).

Table 1: Abscissas and weights for $n = 10$

j	t_j	ω_j
1	$0.01304673574141413996101 79\cdots$	$0.03333567215434406879678440\cdots$
2	$0.06746831665550774463395 16\cdots$	$0.07472567457529029657288817\cdots$
3	$0.16029521585048779688283 63\cdots$	$0.10954318125799102199776746\cdots$
4	$0.28330230293537640460036 70\cdots$	$0.13463335965499817754561346\cdots$
5	$0.42556283050918439455758 70\cdots$	$0.14776211235737643508694649\cdots$
6	$0.57443716949081560544241 30\cdots$	$0.14776211235737643508694649\cdots$
7	$0.71669769706462359539963 30\cdots$	$0.13463335965499817754561346\cdots$
8	$0.83970478414951220311716 37\cdots$	$0.10954318125799102199776746\cdots$
9	$0.93253168334449225536604 83\cdots$	$0.07472567457529029657288817\cdots$
10	$0.98695326425858586003898 20\cdots$	$0.03333567215434406879678440\cdots$

Table 2: Numerical results

Function	Method	Iter	Solution	ρ	Error	Time(sec.)
Example 1	EAM	9	a	4.9734	$1.75e-228$	61.79
	NM	16	a	2.3067	$2.26e-201$	52.20
	DM	14	a	4.3952	$3.53e-203$	116.8
	$CM5$	9	a	4.6762	$1.24e-201$	65.28
	$CM6$	9	a	5.9946	$2.52e-225$	88.52
	SOM	9	a	4.7066	$1.07e-232$	94.42

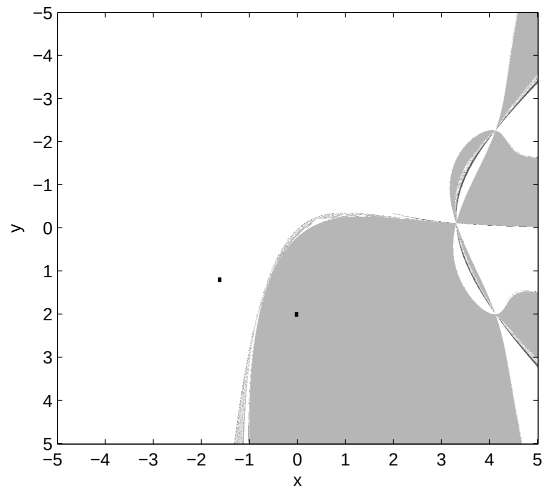

Figure 2: Real dynamical planes for Example 2.

4 Conclusion

In this paper, we have proposed an efficient iterative method for finding real roots of nonlinear systems. One of the advantages of proposed method is that the convergence order of this method is five. The calculation speed Improves greatly and it is another considerable advantage of this method. Moreover, satisfactory results of illustrative examples with respect to the several other methods were used to demonstrate the application of this method.

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حل دستگاه معادلات غیرخطی با استفاده از یک روش سه گامی جدید

اسماعیلی ح .
دانشیار ریاضی کاربردی-نویسنده مسئول
ایران، همدان، دانشگاه بوعلی‌سینا، گروه ریاضی.
esmaeili@basu.ac.ir
عرفانی‌فر ر.
دانشجوی کارشناسی ارشد ریاضی کاربردی
ایران، همدان، دانشگاه بوعلی‌سینا، گروه ریاضی.
rerfanifar92@basu.ac.ir
احمدی م.
دانشجوی دکتری ریاضی کاربردی
ایران، ملایر، دانشگاه ملایر، گروه ریاضی.
mehdi.ahmadi@malayeru.ac.ir

چکیده
در این مقاله، یک روش سه گامی مرتبه پنج برای حل دستگاه معادلات غیرخطی ارائه می دهیم. که در آن هر تکرار روش مستلزم محاسبه دو تابع، دو مشتق فرشه تابع و دو ماتریس معکوس می باشد. بنابراین اندیس کارایی روش فوق برابر $5^{\frac{1}{2n+4n^2+\frac{4}{3}n^3}}$ می باشد که اندیس کارایی روش فوق نسبت به روش های سه گامی دیگر بهتر است. از مزیت‌های روش می‌توان به تعداد تکرار، سرعت و دقت بالا اشاره کرد. نتایج عددی به دست آمده نشان از برتری روش فوق نسبت به دیگر روش‌های سه گامی می‌باشد.

کلمات کلیدی
معادلات غیرخطی، روش‌های تکراری، مرتبه همگرایی، اندیس کارایی.