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Best Proximity Point Result for New Type of Contractions in Metric Spaces with a Graph

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Abstract. In this paper, we introduce a new type of graph contraction using a special class of functions and give a best proximity point theorem for this contraction in complete metric spaces endowed with a graph under two different conditions. We then support our main theorem by a non-trivial example and give some consequences of best proximity point of it for usual graphs.

Keywords. Best proximity point, G-continuous mapping, G-φ-contraction.

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1 Introduction and Preliminaries

The most important graph theory approach to metric fixed point theory introduced so far is attributed to Jachymski [9]. In this approach, the underlying metric space was endowed with a directed graph and the Banach contraction was formulated in a graph language. In 2012, Aghanians et al. [2] generalized Banach G-contractions by defining (p,φ)-G-contractions in uniform spaces associated with an $\mathcal{E}$-distance and endowed with a graph. They also discussed the existence of fixed points for these contractions. For further works in graph metric fixed point theory, see e.g., [1, 3, 6, 11, 15].

The main goal of the best proximity point theory is to provide sufficient conditions assuring the existence of such points. Numerous works on best proximity point theory were done and in the past decade, several authors have investigated the existence of best proximity points for different types of contractions in metric and partially ordered metric spaces as well as metric spaces endowed with a graph (see e.g., [4, 5, 8, 12, 14, 16, 17]).

In this paper, we introduce the notion of a G-φ-contraction in a metric space endowed with a graph and then establish a result on the existence and uniqueness of best proximity points for this contraction. We next give some consequences of our main result for particular choices of graphs as well as the fixed-point-version of it.

We start by reviewing a few basic notions in graph and best proximity point theory which are frequently used in this paper. For more details on graphs, the reader is referred to [7].

In an arbitrary graph G, by a link, it is meant an edge of G with distinct ends and by a loop, an edge of G with identical ends. Two or more links of G with the same pairs of ends are called parallel edges of G.

Let (X,d) be a metric space and G be a directed graph with vertex set $V(G)=X$ such that the edge set $E(G)$ contains all loops and G has no parallel edges. Under these hypotheses, the graph G can be easily denoted by a pair $(V(G),E(G))$ and it is said that the metric space (X,d) is endowed with the graph G.

Considering a pair (A,B) of nonempty subsets of (X,d), we will use the following notations in the paper:

$$\begin{aligned} d(A,B) &= \inf\big\{d(x,y) : x\in A,\ y\in B\big\},\\ A_0 &= \big\{x\in A:\ d(x,y)=d(A,B) \text{ for some } y\in B\big\},\\ B_0 &= \big\{y\in B:\ d(x,y)=d(A,B) \text{ for some } x\in A\big\}.\end{aligned}$$

Definition 1 ([13])**.** Let (A,B) be a pair of nonempty subsets of a metric space (X,d) and $f:A\to B$ be a non-self mapping. An element $x\in A$ is said to be a best proximity point for T if

$$d(x,fx)=d(A,B).$$

By the above notations, if x is a best proximity point for f, then we have $x\in A_0$ and $fx\in B_0$.

Definition 2 ([10])**.** A pair (A,B) of nonempty subsets of a metric space (X,d) is said to have the P-property if

$$\left.\begin{array}{r} d(x_1,y_1)=d(A,B) \\ d(x_2,y_2)=d(A,B) \end{array}\right\} \Longrightarrow d(x_1,x_2)=d(y_1,y_2)$$

for all $x_1, x_2 \in A_0$ and $y_1, y_2 \in B_0$.

By making a little change in Jachymski's definition [9, Definition 2.3], we formulate the notion of G-continuity in metric spaces endowed with a graph for non-self mappings as follows:

Definition 3 ([9]). Let (X,d) be a metric space endowed with a graph G. A mapping $f : A \to B$ is said to be G-continuous on A if $x_n \to x$ in A implies $fx_n \to fx$ in B for all sequences $\{x_n\}$ in A with $(x_n, x_{n+1}) \in E(G)$ for $n = 1, 2, \ldots$.

2 Main Results

In this section, we assume that (X,d) is a metric space endowed with a graph G and (A,B) is a pair of nonempty closed subsets of X unless otherwise stated. We also consider a class Φ consisting of all nondecreasing functions $\varphi : [0,\infty) \to [0,1)$.

First, motivated form the idea of Sadiq Basha [13], we introduce the concept of a G-proximal mapping in a metric space endowed with a graph as follows:

Definition 4. We say that a non-self mapping $f : A \to B$ is G-proximal if f satisfies

$$\left.\begin{array}{r} (y_1,y_2)\in E(G) \\ d(x_1,fy_1)=d(A,B) \\ d(x_2,fy_2)=d(A,B) \end{array}\right\} \Longrightarrow (x_1,x_2)\in E(G)$$

for all $x_1, x_2, y_1, y_2 \in A$.

Following the idea of Jachymski [9] and Aghanians et al. [2], we introduce G-φ-contractions in metric spaces endowed with a graph.

Definition 5. We say that a non-self mapping $f : A \to B$ is a G-φ-contraction if

- $\mathbf{C_1}$: f is G-proximal;
- $\mathbf{C_2}$: The weights of the edges of G are φ-decreased by f, that is,

$$d(fx,fy) \le \varphi\big(d(x,y)\big)d(x,y), \tag{1}$$

for all $x, y \in A$ with $(x,y) \in E(G)$, where $\varphi \in \Phi$.

Theorem 1. Let (X,d) be complete and $f : A \to B$ be a G-φ-contraction satisfying the following properties:

- (i) $f(A_0) \subseteq B_0$ and the pair (A,B) has the P-property;
- (ii) There exist elements $x_0, x_1 \in A_0$ such that $(x_0,x_1) \in E(G)$ and $d(x_1, fx_0) = d(A,B)$.

Then f has a best proximity point in A if one of the following statements hold:

1) f is G-continuous on A;

2) The triple (X, d, G) satisfies the following property:

   (∗) If $x_n \to x$ and $(x_n, x_{n+1}) \in E(G)$ for all $n \geq 1$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $(x_{n_k}, x) \in E(G)$ for all $k \geq 1$.

Furthermore, if for any two best proximity points $u, v \in A$ we have $(u, v) \in E(G)$, then f has a unique best proximity point in A.

Proof. From $x_1 \in A_0$ and $f(A_0) \subseteq B_0$ in (i), there exists an $x_2 \in A$ such that $d(x_2, fx_1) = d(A, B)$. In particular, $x_2 \in A_0$. Since by (ii), $d(x_1, fx_0) = d(A, B)$ and $(x_0, x_1) \in E(G)$, it follows from the G-proximality of f that $(x_1, x_2) \in E(G)$. Continuing this process, we obtain a sequence $\{x_n\}$ in A_0 such that

$$(x_n, x_{n+1}) \in E(G) \quad \text{and} \quad d(x_{n+1}, fx_n) = d(A, B) \qquad n = 0, 1, \dots . \tag{2}$$

Since the pair (A, B) satisfies the P-property, it follows that

$$\left.\begin{array}{l} d(x_n, fx_{n-1}) = d(A, B) \\ d(x_{n+1}, fx_n) = d(A, B) \end{array}\right\} \Longrightarrow d(x_n, x_{n+1}) = d(fx_{n-1}, fx_n)$$

for all $n \in \mathbb{N}$. On the other hand, if $n \in \mathbb{N}$, then from $(x_{n-1}, x_n) \in E(G)$ and the contractive condition (1) we get

$$\begin{aligned} d(x_n, x_{n+1}) &= d(fx_{n-1}, fx_n) \\ &\leq \varphi\big(d(x_{n-1}, x_n)\big) d(x_{n-1}, x_n) \\ &= \varphi\big(d(x_{n-1}, x_n)\big) d(fx_{n-2}, fx_{n-1}) \\ &\leq \varphi\big(d(x_{n-1}, x_n)\big) \varphi\big(d(x_{n-2}, x_{n-1})\big) d(x_{n-2}, x_{n-1}) \\ &\vdots \\ &\leq \prod_{k=1}^{n} \varphi\big(d(x_{k-1}, x_k)\big) d(x_0, x_1). \end{aligned} \tag{3}$$

Furthermore, because the values of φ do not exceed 1, one has

$$d(x_n, x_{n+1}) \leq \varphi\big(d(x_{n-1}, x_n)\big) d(x_{n-1}, x_n) \leq d(x_{n-1}, x_n) \leq \cdots \leq d(x_0, x_1) \qquad n = 0, 1, \dots$$

and because φ is nondecreasing, (3) yields

$$d(x_n, x_{n+1}) \leq r^n d(x_0, x_1) \qquad n = 0, 1, \dots$$

where $r = \varphi(d(x_0, x_1)) < 1$. Therefore, for all $m \geq n \geq 1$, we get

$$\begin{aligned} d(x_n, x_m) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{m-1}, x_m) \\ &\leq r^n d(x_0, x_1) + r^{n+1} d(x_0, x_1) + \cdots + r^{m-1} d(x_0, x_1) \\ &\leq (r^n + r^{n+1} + \cdots + r^{m-1}) d(x_0, x_1) \\ &\leq \Big(\frac{r^n}{1 - r}\Big) d(x_0, x_1). \end{aligned}$$

Hence $\{x_n\}$ is a Cauchy sequence in $A_0 \subseteq A$ and since (X,d) is complete, there exists an $x^* \in X$ such that $x_n \to x^*$. Moreover, since A is closed, it follows that $x^* \in A$.

We next show that x^* is a best proximity point for f. To this end, if f is G-continuous on A, since $x_n \to x^*$ and $(x_n, x_{n+1}) \in E(G)$ for $n = 0, 1, \ldots$, we obtain $fx_n \to fx^*$. Also, the joint continuity of the metric function d implies that $d(x_n, fx_n) \to d(x^*, fx^*)$. On the other hand, (2) shows that the sequence $\{d(x_n, fx_n)\}$ is a constant sequence converging to $d(A,B)$. Thus, from the uniqueness of the limits of converging sequences in metric spaces, we get $d(x^*, fx^*) = d(A,B)$, that is, x^* is a best proximity point for f.

Otherwise, if the triple (X,d,G) satisfies $(*)$, then there exists a strictly increasing sequence $\{n_k\}$ of positive integers such that $(x_{n_k}, x^*) \in E(G)$ for all $k \geq 1$. Hence from the contractive condition (1) we have

$$d(fx_{n_k}, fx^*) \leq \varphi\big(d(x_{n_k}, x^*)\big)d(x_{n_k}, x^*) \leq d(x_{n_k}, x^*) \to 0 \quad \text{as} \quad k \to \infty,$$

that is, $fx_{n_k} \to fx^*$. Again, the joint continuity of the metric function d implies that $d(x_{n_k}, fx_{n_k}) \to d(x^*, fx^*)$. Now, a similar argument to that appeared above shows x^* is a best proximity point for f.

Finally, for uniqueness, suppose that $x^{**} \in A$ is a best proximity point for f such that $(x^*, x^{**}) \in E(G)$. Since by (i), the pair (A,B) satisfies the P-property, it follows that

$$\left.\begin{array}{r} d(x^*, fx^*) = d(A,B) \\ d(x^{**}, fx^{**}) = d(A,B) \end{array}\right\} \Longrightarrow d(x^*, x^{**}) = d(fx^*, fx^{**}).$$

Hence by the contractive condition (1), we have

$$d(x^*, x^{**}) = d(fx^*, fx^{**}) \leq \varphi\big(d(x^*, x^{**})\big)d(x^*, x^{**})$$

which is a contradiction unless $d(x^*, x^{**}) = 0$ or equivalently, $x^* = x^{**}$. □

We next give an example to support our main theorem and to show that the graph case is a real generalization of the usual case.

Example 1. Let $X = \mathbb{R}^2$ be equipped with the usual Euclidean metric d and put

$$A = \big\{(x,1) : \ x \in [0,1]\big\} \quad \text{and} \quad B = \big\{(x,0) : x \in [0,1]\big\}.$$

Clearly, A and B are nonempty closed subsets of $(\mathbb{R}^2, d)$. Let a mapping $f : A \to B$ be defined by

$$f(x,1) = \begin{cases} (0,0) & 0 \leq x < 1, \\ (\frac{4}{5}, 0) & x = 1 \end{cases} \qquad \big(x \in [0,1]\big).$$

Observe that for elements $(1,1)$ and $(\frac{1}{2}, 1)$ of $\mathbb{R}^2$ one has

$$\begin{aligned} d\big(f(1,1), f(\tfrac{1}{2},1)\big) &= d\big((1,0),(\tfrac{1}{2},1)\big) = \frac{4}{5} > \frac{1}{2}\varphi(\frac{1}{2}) \\ &= \varphi\Big(d\big((1,1),(\tfrac{1}{2},1)\big)\Big)d\big((1,1),(\tfrac{1}{2},1)\big), \end{aligned}$$

for all $\varphi \in \Phi$. So f does not satisfy the usual version (non-graph version) of (1).

Now, define a graph G_4 by $V(G_4) = \mathbb{R}^2$ and

$$E(G_4) = \big\{\big((x_1, x_2), (x_1, x_2)\big) : (x_1, x_2) \in \mathbb{R}^2\big\} \cup \big\{\big((0,1), (1,1)\big), \big((1,1), (0,1)\big)\big\},$$

and suppose that $(\mathbb{R}^2, d)$ is endowed with G_4. Clearly, $d(A, B) = 1$, $A = A_0$ and $B = B_0$. In particular A_0 is nonempty. Moreover, it can be easily shown that $f(A_0) \subseteq B_0$ and the pair (A, B) and the triple $(\mathbb{R}^2, d, G_4)$ satisfies the P-property and $(*)$, respectively.

To show that f is a G_4-φ-contraction, define a function $\varphi : [0, \infty) \to [0, 1)$ by the

$$\varphi(t) = \begin{cases} \frac{5}{6}t & 0 \le t < 1, \\ \frac{5}{6} & t \ge 1 \end{cases} \qquad \big(t \in [0, +\infty]\big).$$

It is obvious that $\varphi \in \Phi$ and we have

$$d\big(f(x,1), f(x,1)\big) = d\big((0,0), (0,0)\big) = 0 \le \varphi\Big(d\big((x,1), (x,1)\big)\Big)d\big((x,1), (x,1)\big)$$

for all $x \in [0, 1]$ and also

$$\begin{aligned} d\big(f(0,1), f(1,1)\big) &= d\big((0,0), (\frac{2}{3}, 0)\big) = \frac{4}{5} < \frac{5}{6} = \varphi(1) \cdot 1 \\ &= \varphi\Big(d\big((0,1), (1,1)\big)\Big)d\big((0,1), (1,1)\big), \end{aligned}$$

that is, f is a G_4-φ-contraction. Hence all hypotheses of Theorem 1 are satisfied and therefore, f has a best proximity point $x^* = (0, 1)$.

Finally, let $x^{**} = (x, 1) \in A$ with $x \in [0, 1]$ be a best proximity point for f. If $x \in [0, 1)$, then

$$d\big((x,1), f(x,1)\big) = d\big((x,1), (0,0)\big) = \sqrt{x^2 + 1} > d(A, B),$$

and else if $x = 1$, then

$$d\big((1,1), f(1,1)\big) = d\big((1,1), (\frac{4}{5}, 0)\big) = \sqrt{\frac{1}{25} + 1} > d(A, B).$$

Since both above cases yield contradictions, it follows that $(0, 1)$ is the unique best proximity point for f in A.

Various consequences of Theorem 1 follow now for particular choices of the graph G. First, consider the complete graph G_0 whose vertex set coincides with X and $E(G_0) = X \times X$. Clearly, $f : A \to B$ is G_0-proximal and the triple (X, d, G_0) satisfies $(*)$ obviously. Hence we get the following result:

Corollary 1. Let (X, d) be a complete metric space, (A, B) be a pair of nonempty closed subsets of (X, d) and $f : A \to B$ be a G_0-φ-contraction satisfying the following properties:

(i) $f(A_0) \subseteq B_0$ and the pair (A, B) satisfies the P-property;

(ii) There exist elements $x_0, x_1 \in A_0$ such that $d(x_1, fx_0) = d(A, B)$.

Then f has a unique best proximity point in A.

Suppose next that $(X, \sqsubseteq)$ is a poset and consider the poset graph G_1 defined by $V(G_1) = X$ and $E(G_1) = \{(x, y) \in X \times X : x \sqsubseteq y\}$. If we set $G = G_1$ in Theorem 1, then we obtain the following best proximity point result in partially ordered complete metric spaces:

Corollary 2. Let $(X, \sqsubseteq)$ be a poset and (X, d) be a complete metric space. Suppose that (A, B) is a pair of nonempty closed subsets of (X, d) and $f : A \to B$ is a G_1-φ-contraction satisfying the following properties:

(i) f is a G_1-proximal with $f(A_0) \subseteq B_0$ and the pair (A, B) satisfies the P-property;

(ii) There exist elements $x_0, x_1 \in A_0$ such that $x_0 \sqsubseteq x_1$ and $d(x_1, fx_0) = d(A, B)$.

Then f has a best proximity point in A if one of the following statements hold:

1) f is G_1-continuous on A;

2) The triple $(X, d, \sqsubseteq)$ satisfies the following property:

> If $x_n \to x$ and $x_n \sqsubseteq x_{n+1}$ for all $n \geq 1$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \sqsubseteq x$ for all $k \geq 1$.

Then f has a best proximity point in A. Furthermore, if for any two best proximity points $u, v \in A$ we have $u \subseteq v$, then f has a unique best proximity point in A.

For the next corollary, suppose again that $(X, \sqsubseteq)$ is a poset and consider the other poset graph G_2 defined by $V(G_2) = X$ and $E(G_2) = \{(x, y) \in X \times X : x \sqsubseteq y \ \vee \ y \sqsubseteq x\}$. If we set $G = G_2$ in Theorem 1, then we obtain another interesting best proximity point result in partially ordered complete metric spaces.

Corollary 3. Let $(X, \sqsubseteq)$ be a poset and (X, d) be a complete metric space. Suppose that (A, B) is a pair of nonempty closed subsets of (X, d) and $f : A \to B$ is a G_2-φ-contraction satisfying the following properties:

(i) f is G_2-proximal with $f(A_0) \subseteq B_0$ and the pair (A, B) satisfies the P-property;

(ii) There exist comparable elements $x_0, x_1 \in A_0$ with respect to $\sqsubseteq$ such that $d(x_1, fx_0) = d(A, B)$.

Then f has a best proximity point in A if one of the following statements hold:

1) f is G_2-continuous on A;

2) The triple $(X, d, \sqsubseteq)$ satisfies the following property:

> If $x_n \to x$ and x_n and x_{n+1} are comparable with respect to $\sqsubseteq$ for all $n \geq 1$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that x_{n_k}'s and x are comparable with respect to $\sqsubseteq$.

Furthermore, if any two best proximity points are comparable with respect to $\sqsubseteq$, then f has a unique best proximity point in A.

Finally, let $\varepsilon > 0$ be a fixed number and consider the graph G_ε defined by $V(G_\varepsilon) = X$ and $E(G_\varepsilon) = \{(x, y) \in X \times X : \ d(x, y) < \varepsilon\}$. If we set $G = G_\varepsilon$ in Theorem 1, then an easy argument shows that the triple (X, d, G_ε) satisfies $(*)$ and so we get the following result in complete metric spaces:

Corollary 4. Let $\varepsilon > 0$ be fixed and (X, d) be complete. Suppose that (A, B) is a pair of nonempty closed subsets of (X, d) and $f : A \to B$ is a G_ε-φ-contraction satisfying the following properties:

(i) f is G_ε-proximal with $f(A_0) \subseteq B_0$ and the pair (A, B) satisfies the P-property;

(ii) There exist elements $x_0, x_1 \in A_0$ such that $d(x_0, x_1) < \varepsilon$ and $d(x_1, fx_0) = d(A, B)$.

Then f has a best proximity point in A. Furthermore, if for any two best proximity points $u, v \in A$ we have $d(u, v) < \varepsilon$, then f has a unique best proximity point in A.

Beside the above consequences in best proximity point theory, setting $A = B = X$ in Theorem 1, one can easily see that $d(A, B) = 0$ and find the following interesting consequence in graph metric fixed point theory:

Corollary 5. Let (X, d) be a complete metric space endowed with a graph G and a mapping $f : X \to X$ satisfy the following properties:

(i) f preserves the edges of G, that is, $(x, y) \in E(G)$ implies $(fx, fy) \in E(G)$ for all $x, y \in X$;

(ii) There exists an $x_0 \in X$ such that $(x_0, fx_0) \in E(G)$;

(iii) There exists a $\varphi \in \Phi$ such that

$$d(fx, fy) \leq \varphi\big(d(x, y)\big)d(x, y)$$

for all $x, y \in X$ with $(x, y) \in E(G)$.

Then f has a fixed point in X if either f is G-continuous on X or the triple (X, d, G) satisfies $(*)$. Furthermore, if for any two fixed points $u, v \in X$ we have $(u, v) \in E(G)$, then f has a unique fixed point in X.

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نتایج بهترین نقطه تقریب برای نوعی از انقباض ها در فضاهای متریک مجهز به یک گراف

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چکیده

در این مقاله، ما کلاس خاصی از نوعی جدید از انقباض گرافی را معرفی کرده و بهترین نقطه تقریب را برای این انقباض در فضاهای متریک کامل مجهز به گراف تحت دو شرط متفاوت مورد بررسی قرار می‌دهیم. سپس به کمک یک مثال اهمیت و وجود قضیه اصلی را نشان داده و برخی از نتایج بهترین نقطه تقریب را برای گراف‌های معمولی بیان می‌کنیم.

کلمات کلیدی

بهترین نقطه تقریب، نگاشت G-پیوسته، انقباض $G-\varphi$.