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Robust Switching Law Design for Uncertain Time-Delay Switched Linear Systems

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Abstract. Guaranteed cost control (GCC) is an impressive method of controlling nonlinear systems, incredibly uncertain switched systems. Most of the recent studies of GCC on uncertain switched linear systems have been concerned with asymptotic stability analysis. In this paper, a new robust switching law for time-delay uncertain switched linear systems is designed. First, the switching law is designed, and second, a state-feedback controller based on Lyapunov-Krasovskii Functional (LKF) is designed. Also, using Linear Matrix Inequality (LMI) particular condition for the existence of a solution of obtained switching law and controller is achieved. Consequently, in the presented theorems, the exponential stability of the overall system under switching law and controller is analyzed. Finally, theoretical results are verified via presenting an example.

Keywords. Uncertain switched linear systems, Time-delay, Guaranteed cost control, LKF, LMI, Exponential stability.

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1 Introduction

Switched systems, as a wide class of hybrid systems, are divided into two classes of systems: continuous and discrete-time subsystems. Generally, there is a switching strategy that selects a subsystem between other subsystems. In recent years, switching theory and its application have been extended to adaptive control to overcome disadvantages in the system's stability, where there are some difficulties in the proof of stability [1, 2, 3, 4, 5]. Some important problems in the concept of design procedures and stability analysis of switched systems have illustrated in [6].

There are many approaches in switched systems especially, looking for suitable switching; to stabilize the system even when the systems are unstable [7]. Also, dwell-time and its average concept have been studied for stabilization problems in the switched system with especial switching strategy has been performed [8, 6]. In recent past decades, time-delay systems have been concerned with expert researchers. These kinds of systems have many applications in electronics systems, transmission systems, chemical process systems, and power systems and, so on [9]. Delay mainly exists in some sensors and measurement units and frequently occurs in control systems [10]. Generally, since sensors and transducers are used in control systems to measure all or some important states, then, some delays may occur in these measurements. Switched systems with a time delay are a class of switched systems that has been focused on recent researches. In most studies on the time-delay switched systems, delay with a certain upper bound is assumed. Knowing such this upper bound can guarantee the stability of these kinds of systems. In this area, some rigorous researches have been achieved in recent years [11, 12, 13]. In [12], using Common Lyapunov Function (CLF), the stability of switching systems composed some finite linear subsystems which are described with time-delay differential equations has been performed. In [13], the Authors studied sufficient conditions for asymptotic stability analysis of a class of switched linear systems. Moreover, many types of research in the field of switched systems concentrate on the asymptotic behavior that reflects the system treatments in a limited interval time [14, 15]. In the concept of control a plant, designing a controller must guarantee not only the asymptotic stability of the system but also guarantee acceptable performance. Considering a quadratic performance index is a solution to formulate this problem. This method is named guaranteed cost control (GCC) [16]. In this approach, it is tried to provide an upper bound for a given cost function in the presence of uncertainties, and, based on this goal, the controller is designed [17, 18, 19, 20, 22]. Based on this approach, some significant researches have been reported on this topic in [20, 21, 23, 24, 25]. Some acceptable results have been reported for uncertain switched linear systems. In these studies, using CLF or Multiple Lyapunov Functions (MLFs), switching laws and state feedback controllers are designed. Moreover, for switching strategy design, a subsystem with minimum LF is chosen. When the switched system has only a switching signal to be designed, this approach provides asymptotic or exponential stability. Especially, to design switching laws using CLF, the designer must find some unknown matrices with solving some complex Linear Matrix Inequalities (LMIs) to be constructed via some theorems [26, 21, 23, 24, 25]. In more recent studies, some important researches on the exponential stability analysis and design of GCC for time delay switched systems has been performed [27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 41, 42, 43, 44]. In [31], using some extracted LMIs for switched time-delay systems, a sufficient condition for exponential stability analysis and GCC problem with the weighted form is obtained. Also, in [38] and based on dwell time and piecewise Lyapunov function approach exponential stability is studied, and its condition is derived. Besides, in [38], and based on the LKF method, to guarantee exponential stability and obtain the upper bound of the determined cost function, a new time delay condition is proposed. In this paper, by considering a complete form of uncertain time-delay switched systems containing delays both in states and control inputs, a new robust switching law is designed. To do this, motivated by the min-projection switching strategy [39] and Lyapunov-Krasovskii function (LKF), switching law and control are designed. The main contributions are listed in the following:

(i) Designing a new robust switching law to guarantee exponential stability of the switched system.

(ii) Proving that the proposed LKF satisfies the presented theorems.

Notation: Throughout the paper, m is an arbitrary positive integer that indicates the number of switched system's subsystems, and $\lambda(A)$ indicates eigenvalues of matrix A. the notation $P > 0$ denotes that P is a positive definite matrix.

2 Problem Formulation and Preparations

In this paper, the following general form of time-delay uncertain switched linear system is considered

$$\begin{aligned} \dot{x} &= (A_{\sigma(x,t)} + \Delta A_{\sigma(x,t)})x(t) + A_{d\sigma(x,t)}x(t-d) \\ &\quad + (B_{\sigma(x,t)} + \Delta B_{\sigma(x,t)})u(t) + W_{\sigma(x,t)}u(t-h), \\ x(t) &= \phi(t),\ t \in [-t_0,\ 0],\ t_0 \triangleq \max\{d,\ h\}, \end{aligned} \tag{1}$$

where, $x(t) \in R^n$ and $u(t) \in R^q$ are the state and control input vectors. $d > 0$ and $h > 0$ are delay constants in the states and inputs and $\sigma(x,t) \in \underline{m}$ is switching signal which is piecewise constant that determines the active subsystem. $A_i \in R^{n\times n}$, $B_i \in R^{n\times q}$, $A_{di} \in R^{n\times n}$ and $W_i \in R^{n\times n}$, $i \in \underline{m}$ are subsystem matrices and ΔA_i and ΔB_i, $i \in \underline{m}$, are additive uncertainties. The following notice shows the nature of uncertainties.

Notice 1. ΔA_i and ΔB_i in equation (1) are time-varying uncertain matrices and satisfy the following condition

$$[\Delta A_i \quad \Delta B_i] = N_i F_i [C_i \quad D_i], \qquad i \in \underline{m}, \tag{2}$$

where C_i, D_i and N_i are known matrices and F_i, $i \in \underline{m}$, are unknown matrices with Lebesgue measurable elements such that the following inequality holds

$$F_i^T(t) F_i(t) \leq I, \qquad i \in \underline{m}. \tag{3}$$

Throughout the paper, our goal is to minimize the following performance index for the uncertain system (1)

$$J = \int_0^\infty (x^T Q x + u^T R u) dt, \tag{4}$$

where $Q \in R^{n\times n}$ and $R \in R^{q\times q}$ are symmetric positive definite matrices. The main goal of the paper is to find switching law $\sigma(x,t)$ and state-feedback controller $u = K_i x(t)$, where $K_i \in R^{q\times n}$, $i \in \underline{m}$ such that, the the system (1) to be exponential stable and the cost function (4) satisfies $J \leq J^*$ where J^* is a guaranteed cost value, which is defined in Definition 1. Before presenting our main results, we introduce some necessary definitions, lemmas, and theorems.

Definition 1. [20] For all uncertainties satisfying (2) and (3), state-feedback control $u^*(t)$ and switching law $\sigma^*(x,t)$ are said to be guaranteed cost value (GCV) and guaranteed cost control law (GCCL), if the closed-loop system (1) to be asymptotic (or exponential) stable and the value of cost function (4) satisfies $J \leq J^*$, where J^* is a positive scalar.

Definition 2. [24, 33] The system (1) under switching law $\sigma(x,t)$ and control $u = K_i x(t)$ is said to be exponential stable if the norm of state vector $x(t)$ satisfies (5)

$$\|x(t)\| \leq k_1 e^{-k_2 t} \|x(0)\|, \tag{5}$$

where $k_1 > 0$ and $k_2 > 0$, and $\|x(0)\|$ is initial value at time $t = 0$.

Lemma 1. [26] For matrices L, P and $Q > 0$, the following inequality holds

$$\begin{bmatrix} P & L \\ L^T & -Q \end{bmatrix} < 0 \Longleftrightarrow P + L Q^{-1} L^T < 0. \tag{6}$$

Lemma 2. [33] Consider D, E, and F be real matrices, and matrix F satisfies $F^TF \leq I$. For any positive scalar ε, the following inequality holds

$$DFE+E^TF^TD^T \leq \varepsilon^{-1}DD^T+\varepsilon E^TE \tag{7}$$

Lemma 3. [25] For any symmetric matrix Y, arbitrary matrices M and N and for all F satisfying $F^TF \leq I, i \in \underline{m}$, the following inequality holds

$$Y + MFN + N^TF^TM^T < 0.$$

if and only if there exists positive scalar ε such that

$$Y + \varepsilon N^TN + \varepsilon^{-1}M^TM < 0,$$

Lemma 4. [40] For any real symmetric matrix $A \in R^{n\times n}$

$$\lambda_{\min}(A)\|x\|^2 \leq x^TAx \leq \lambda_{\max}(A)\|x\|^2, \tag{8}$$

where $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the smallest and largest eigenvalues of matrix A.

Theorem 1. For the system (1), if there exist matrices $P > 0$, $P_1 > 0$ and $P_2 > 0$, positive scalar α and positive definite scalar function $V(x(t))$ as a Lyapunov function for system (1) such that

$$\dot{V}(x(t)) \leq -\alpha\|x\|^2, \tag{9}$$

then, the switching law (10) can stabilize the switched system (1) exponentially.

$$\sigma(x,t) = arg \min_{i\in\underline{m}}\{x^TPf_i(x)\}. \tag{10}$$

Proof. In ([39]) using the min-projection switching strategy this theorem has been proved for nonlinear switched systems in the form of $\dot{x} = f_i(x)$, $i \in \underline{m}$ .To extend this theorem in switched systems (1), the following Lyapunov-Krasovskii function is proposed

$$V(x(t)) = x^T(t)Px(t) + \int_{-d}^{0} x^T(t+\tau)P_1x(t+\tau)d\tau$$
$$+ \int_{-h}^{0} x^T(t+\tau)P_2x(t+\tau)d\tau,$$

and it is proved to reach exponential stability, there exist positive scalars $k_1 = \sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}$ and $k_2 = \frac{\alpha}{2\lambda_{max}(P)}$ satisfy the exponential definition (5). □

3 Main Results

Theorem 2. System (1) under the following switching law is to be exponentially stable

$$\sigma(x,t) = arg\min_{i\in \underline{m}}\{\bar{x}^T Z_i \bar{x}\}, \tag{11}$$

where

$$Z_i = \begin{bmatrix} \theta_i & S_1 & PW_iK_i \\ S_1^T & -P_1 & 0 \\ K_i^T W_i^T P & 0 & -P_2 \end{bmatrix},$$
$$\bar{x} = \begin{bmatrix} x(t), x(t-d), x(t-h) \end{bmatrix}', \tag{12}$$

and

$$\chi_i = A_i + \Delta A_i + B_i K_i + \Delta B_i K_i,$$
$$\theta_i = \chi_i^T P + P\chi_i + P_1 + P_2 + Q + K_i^T R K_i,$$
$$S_1 = PA_{di},$$

if there exist symmetric positive-definite matrices P, P_1 and P_2, and matrices K_i, $i \in \underline{m}$, such that the following inequality holds:

$$\sum_{i=1}^{m}\Big[x^T(t)\theta_i x(t) + x^T(t)S_1 x(t-d) + x^T(t-d)S_1^T x(t)$$
$$+ x^T(t-h)K_i^T W_i^T P x(t) + x^T(t) P W_i K_i x(t-h)$$
$$- x^T(t-d)P_1 x(t-d) - x^T(t-h)P_2 x(t-h)\Big] < 0, \tag{13}$$

In addition, GSV is $J^* = \phi(0)^T P\phi(0) + \int_{-d}^{0}\phi^T(\tau)P_1\phi(\tau)d\tau + \int_{-h}^{0}\phi^T(\tau)P_2\phi(\tau)d\tau$.

Proof. Clearly from switching (11) and inequality (13), it is resulted that $\sum_{i=1}^{m} Z_i < 0$ and consequently, there exists an index $i \in \underline{m}$ such that $\bar{x}^T Z_i \bar{x} < 0$ for an augmented state vector $\bar{x} \in R^{3n}$, $\bar{x} \neq 0$. Now the following function is preposed as a Lyapunov-Krasovskii function, where P, P_1 and P_2 are symmetric positive definite matrices

$$V(x(t)) = x^T(t)Px(t) + \int_{-d}^{0} x^T(t+\tau)P_1 x(t+\tau)d\tau \tag{14}$$
$$+ \int_{-h}^{0} x^T(t+\tau)P_2 x(t+\tau)d\tau,$$

Time derivation of $V(x(t)$ and substituting $u(t) = K_i x(t)$ into system equations (1) and using Notice 1, results

$$\begin{aligned}\dot{V}(x(t)) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)P_1x(t) - x^T(t-d)P_1x(t-d)\\ &+ x^T(t)P_2x(t) - x^T(t-h)P_2x(t-h) = x^T(t)(A_i+\Delta A_i)^TPx(t)\\ &+ x^T(t)P(A_i+\Delta A_i)x(t) + x^T(t-d)A_{di}^TPx(t) + x^T(t)PA_{di}x(t-d)\\ &+ x^T(t)K_i^T(B_i+\Delta B_i)^TPx(t) + x^T(t)P(B_i+\Delta B_i)K_ix(t)\\ &+ x^T(t-h)K_i^TW_i^TPx(t) + x^T(t)PW_iK_ix(t-h) + x^T(t)P_1x(t)\\ &- x^T(t-d)P_1x(t-d) + x^T(t)P_2x(t) - x^T(t-h)P_2x(t-h)\\ &= x^T(t)\Big[P(A_i+B_iK_i) + (A_i+B_iK_i)^TP + PN_iF_i(C_i+D_iK_i)\\ &+ (C_i+D_iK_i)^TF_i^TN_i^TP + P_1 + P_2\Big]x(t) + x^T(t-d)A_{di}^TPx(t)\\ &+ x^T(t)PA_{di}x(t-d) + x^T(t-h)K_i^TW_i^TPx(t) + x^T(t)PW_iK_ix(t-h)\\ &- x^T(t-d)P_1x(t-d) - x^T(t-h)P_2x(t-h) \end{aligned} \tag{15}$$

Applying Lemma 2, we have

$$\begin{aligned}&PN_iF_i(C_i+D_iK_i) + (C_i+D_iK_i)^TF_i^TN_i^TP\\ &\quad\le \varepsilon_iPN_iN_i^TP + \varepsilon_i^{-1}(C_i+D_iK_i)(C_i+D_iK_i)^T.\end{aligned} \tag{16}$$

Rewritten equation (15) results

$$\begin{aligned}\dot{V}(x(t)) &\le x^T(t)\Big[P(A_i+B_iK_i) + (A_i+B_iK_i)^TP + \varepsilon_i^{-1}(C_i+D_iK_i)(C_i+D_iK_i)^T\\ &+ \varepsilon_iPN_iN_i^TP + P_1 + P_2\Big]x(t) + x^T(t-d)A_{di}^TPx(t) + x^T(t)PA_{di}x(t-d)\\ &+ x^T(t-h)K_i^TW_i^TPx(t) + x^T(t)PW_iK_ix(t-h) - x^T(t-d)P_1x(t-d)\\ &- x^T(t-h)P_2x(t-h).\end{aligned} \tag{17}$$

By defining

$$\begin{aligned}\theta_i &= P(A_i+B_iK_i) + (A_i+B_iK_i)^TP + \varepsilon_i^{-1}(C_i+D_iK_i)(C_i+D_iK_i)^T\\ &\quad + \varepsilon_iPN_iN_i^TP + P_1 + P_2 + Q + K_i^TRK_i\\ S_1 &= PA_{di},\end{aligned}$$

and adding $x^T(t)(Q+K_i^TRK_i)x(t)$ to (17), results

$$\begin{aligned}\dot{V}(x(t)) + x^T(t)(Q+K_i^TRK_i)x(t) &\le x^T(t)\theta_ix(t) + x^T(t)S_1x(t-d)\\ &+ x^T(t-d)S_1^Tx(t) - x^T(t-d)P_1x(t-d) + x^T(t-h)K_i^TW_i^TPx(t)\\ &+ x^T(t)PW_iK_ix(t-h) - x^T(t-h)P_2x(t-h).\end{aligned} \tag{18}$$

Consequently inequality (18) can be written as

$$\begin{aligned}&\dot{V}(x(t)) + x^T(t)(Q+K_i^TRK_i)x(t)\\ &\le \begin{bmatrix} x(t)\\ x(t-d)\\ x(t-h)\end{bmatrix}^T \begin{bmatrix}\theta_i & S_1 & PW_iK_i\\ S_1^T & -P_1 & 0\\ K_i^TW_i^TP & 0 & -P_2\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-d)\\ x(t-h)\end{bmatrix}\end{aligned}$$

$$= \bar{x}^T(t)\begin{bmatrix} \theta_i & S_1 & PW_iK_i \\ S_1^T & -P_1 & 0 \\ K_i^TW_i^TP & 0 & -P_2 \end{bmatrix}\bar{x}(t). \tag{19}$$

Now, it is concluded that there exist an $i \in \underline{m}$ such that $\bar{x}^T Z_i \bar{x} < 0$. Therefore, selecting switching law (11) for any time $t \in R$, results that $\bar{x}^T Z_i \bar{x} < 0$ and

$$\dot{V}(x(t)) + x^T(t)(Q + K_i^T RK_i)x(t) \leq \bar{x}^T(t)Z_i\bar{x}(t) < 0. \tag{20}$$

So,

$$\begin{aligned} \dot{V}(x(t)) &< -x^T(t)Qx(t) - x^T(t)(K_i^T RK_i)x(t) \\ &= -x^T(t)(Q + K_i^T RK_i)x(t), \end{aligned} \tag{21}$$

Obviously, it is concluded that $G_i = Q+K_i^T RK_i$ is positive-definite matrix for any $i \in \underline{m}$. Therefore, using Lemma 4, $\forall x \in R^n, i \in \underline{m}$ the following inequality holds:

$$-\lambda_{\max}(G_i)\|x\|^2 \leq -x^T G_i x \leq -\lambda_{\min}(G_i)\|x\|^2. \tag{22}$$

Now, by choosing

$$\gamma = \lambda_{\min}(G) = \min_{i\in\underline{m}}(\lambda_{\min}(G_i)), \tag{23}$$

Then, applying Theorem 1 results that switched system (1) is exponentially stable. □

Remark 1. We need to find unknown matrices P, P_1 ,P_2 and control gains K_i to realize the switching law (11). Also, positive scalars ε_i are designing constants and can be selected by the designer arbitrarily or by some optimization methods. In the Theorem 2 and using Lemma 1 it is shown that (13) is equal to a set of LMIs (24).

Theorem 3. If there exist invertible symmetric positive definite matrix X, P_2 and matrices M_i and V_i for some positive scalars ε_i, $i \in \underline{m}$, such that the following LMI to be satisfied

$$\begin{bmatrix} \Psi_i & A_{di} & W_iV_i & X^T & M_i^T & X^T & M_i^T & \bar{C}_i^{\,T} \\ A_{di}^T & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ V_i^TW_i & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ X & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ M_i & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 & 0 \\ X & 0 & 0 & 0 & 0 & -Q^{-1} & 0 & 0 \\ M_i & 0 & 0 & 0 & 0 & 0 & -R^{-1} & 0 \\ \bar{C}_i & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_i^{-1}I \end{bmatrix} < 0, \tag{24}$$

where

$$\Psi_i = (A_i X + B_i M_i)^T + A_i X + B_i M_i + \varepsilon_i^{-1} N_i N_i^T,$$
$$\bar{C}_i = C_i X + D_i M_i,$$

and then, inequality (13) holds and switching strategy (11) for the system (1) can be implemented.

Proof. Define the following matrix

$$Y = \begin{bmatrix} \bar{\Omega}_i & P^T A_{di} & P W_i V_i & I & K_i^T & I & K_i^T \\ A_{di}^T P & -P_1 & 0 & 0 & 0 & 0 & 0 \\ V_i^T W_i^T P & 0 & -I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & -P_1^{-1} & 0 & 0 & 0 \\ K_i & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -Q^{-1} & 0 \\ K_i & 0 & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix}, \tag{25}$$

where

$$\bar{\Omega}_i = (A_i + B_i K_i)^T P + P^T (A_i + B_i K_i).$$

Using Lemma 2, the matrix inequality (13) is equal to the following

$$\begin{aligned} Y &+ \begin{bmatrix} \Phi_i & 0_{1\times 6} \end{bmatrix}^T F_i^T(t) \begin{bmatrix} N_i^T P & 0_{1\times 6} \end{bmatrix} \\ &+ \begin{bmatrix} N_i^T P & 0_{1\times 6} \end{bmatrix}^T F_i(t) \begin{bmatrix} \Phi_i & 0_{1\times 6} \end{bmatrix} < 0 \end{aligned} \tag{26}$$

where

$$\Phi_i = C_i + D_i K_i.$$

By rewriting inequality (26), we have

$$\begin{bmatrix} \psi_{1i} & P^T A_{di} & P W_i V_i & I & K_i^T & I & K_i^T \\ A_{di}^T P & -P_1 & 0 & 0 & 0 & 0 & 0 \\ V_i^T W_i^T P & 0 & -I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & -P_1^{-1} & 0 & 0 & 0 \\ K_i & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -Q^{-1} & 0 \\ K_i & 0 & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix}$$
$$+ \begin{bmatrix} C_i^T + K_i^T D_i^T \\ 0_{6\times 1} \end{bmatrix} \begin{bmatrix} F_i^T(t) N_i^T P & 0_{1\times 6} \end{bmatrix}$$
$$+ \begin{bmatrix} P^T N_i \\ 0_{6\times 1} \end{bmatrix} \begin{bmatrix} F_i(t) C_i + F_i(t) D_i K_i & 0_{1\times 6} \end{bmatrix}$$

$$
= \begin{bmatrix} \psi_{2i} & P^T A_{di} & PW_i V_i & I & K_i^T & I & K_i^T \\ A_{di}^T P & -P_1 & 0 & 0 & 0 & 0 & 0 \\ V_i^T W_i^T P & 0 & -I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & -P_1^{-1} & 0 & 0 & 0 \\ K_i & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -Q^{-1} & 0 \\ K_i & 0 & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0, \tag{27}
$$

where

$$
\begin{aligned}
\psi_{1i} &= (A_i + B_i K_i)^T P + P^T (A_i + B_i K_i) \\
\psi_{2i} &= (A_i + B_i K_i)^T P + P^T (A_i + B_i K_i) + C_i^T F_i^T (t) N_i^T P \\
&\quad + K_i^T D_i^T F_i^T (t) N_i^T P + P^T N_i F_i(t) C_i + P^T N_i F_i(t) D_i K_i,
\end{aligned}
$$

By simple calculations in

$$
\begin{aligned}
& Y + \varepsilon_i \begin{bmatrix} \Phi_i & 0_{1\times 6} \end{bmatrix}^T \begin{bmatrix} \Phi_i & 0_{1\times 6} \end{bmatrix} \\
& \quad + \varepsilon_i^{-1} \begin{bmatrix} N_i^T P & 0_{1\times 6} \end{bmatrix}^T \begin{bmatrix} N_i^T P & 0_{1\times 6} \end{bmatrix} \\
& \quad = \begin{bmatrix} \psi_{1i} & P^T A_{di} & PW_i V_i & I & K_i^T & I & K_i^T \\ A_{di}^T P & -P_1 & 0 & 0 & 0 & 0 & 0 \\ V_i^T W_i^T P & 0 & -I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & -P_1^{-1} & 0 & 0 & 0 \\ K_i & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -Q^{-1} & 0 \\ K_i & 0 & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} \\
& \quad + \begin{bmatrix} \varepsilon_i (C_i + D_i K_i)^T (C_i + D_i K_i) & 0_{1\times 6} \\ 0_{6\times 1} & 0_{6\times 6} \end{bmatrix} \\
& \quad + \begin{bmatrix} \varepsilon_i^{-1} P^T N_i N_i^T P & 0_{1\times 6} \\ 0_{6\times 1} & 0_{6\times 6} \end{bmatrix} \\
& \quad = \begin{bmatrix} \psi_{3i} & P^T A_{di} & PW_i V_i & I & K_i^T & I & K_i^T \\ A_{di}^T P & -P_1 & 0 & 0 & 0 & 0 & 0 \\ V_i^T W_i^T P & 0 & -I & 0 & 0 & 0 & 0 \\ I & 0 & 0 & -P_1^{-1} & 0 & 0 & 0 \\ K_i & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -Q^{-1} & 0 \\ K_i & 0 & 0 & 0 & 0 & 0 & -R^{-1} \end{bmatrix} < 0,
\end{aligned} \tag{28}
$$

where

$$\begin{aligned}\psi_{3i} &= (A_i + B_iK_i)^T P + P^T (A_i + B_iK_i)\\ &+ \varepsilon_i (C_i + D_iK_i)^T (C_i + D_iK_i) + \varepsilon_i^{-1} P^T N_i N_i^T P.\end{aligned}$$

Now, from the Lemma 1, inequality (28) is equal to (29)

$$\begin{bmatrix} \Omega_i & P^T A_{di} & P^T W_i V_i & I & K_i^T & I & K_i^T & \Phi^T \\ A_{di}^T P & -P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ V_i^T W_i^T P & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & -P_1^{-1} & 0 & 0 & 0 & 0 \\ K_i & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -Q^{-1} & 0 & 0 \\ K_i & 0 & 0 & 0 & 0 & 0 & -R^{-1} & 0 \\ \Phi_i & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_i^{-1} I \end{bmatrix}, \tag{29}$$

where

$$\Omega_i = \bar{\Omega}_i + \varepsilon_i^{-1} P N_i N_i^T P,$$

Multiplying both sides of (29) by $diag\{P^{-T}, P_1^{-1}, I, I, I, I, I, I\}$ and $diag\{P^{-1}, P_1^{-1}, I, I, I, I, I, I\}$ yields

$$\begin{aligned} &= \begin{bmatrix} P^{-T} & 0 & 0 & \dots & 0 \\ 0 & P_1^{-1} & 0 & \dots & 0 \\ 0 & 0 & I & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \\ &\times \begin{bmatrix} \varphi_i & P^T A_{di} & \omega_i & I & K_i^T & I & K_i^T & \Phi_i^T \\ A_{di}^T P & -P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \omega_i & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ I & 0 & 0 & -P_1^{-1} & 0 & 0 & 0 & 0 \\ K_i & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & -Q^{-1} & 0 & 0 \\ K_i & 0 & 0 & 0 & 0 & 0 & -R^{-1} & 0 \\ \Phi_i & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_i^{-1} I \end{bmatrix} \\ &\times \begin{bmatrix} P^{-1} & 0 & 0 & \dots & 0 \\ 0 & P_1^{-1} & 0 & \dots & 0 \\ 0 & 0 & I & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \end{aligned}$$

where

$$\varphi_i = (A_i + B_iK_i)^T P + P^T (A_i + B_iK_i) + \varepsilon_i^{-1} P^T N_i N_i^T P,$$

$$\omega_i = P^T W_i V_i.$$

So we have

$$\begin{bmatrix} \begin{matrix} P^{-T}(A_i+B_iK_i)^T+(A_i+B_iK_i)P^{-1} \\ +\varepsilon_i^{-1}N_iN_i^T \end{matrix} & A_{di}T^{-1} & W_iV_i & I & P^{-T}K_i^T & P^{-T} & P^{-T}K_i^T & P^{-T}(C_i^T+K_i^TD_i^T) \\ T^{-1}A_{di} & -P_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ V_i^TW_i^T & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ P^{-1} & 0 & 0 & -P_1^{-1} & 0 & 0 & 0 & 0 \\ K_iP^{-1} & 0 & 0 & 0 & -P_2^{-1} & 0 & 0 & 0 \\ P^{-1} & 0 & 0 & 0 & 0 & -Q^{-1} & 0 & 0 \\ K_iP^{-1} & 0 & 0 & 0 & 0 & 0 & -R^{-1} & 0 \\ (C_i+D_iK_i)P^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_i^{-1}I \end{bmatrix} < 0. \tag{30}$$

□

In summary, to obtain $\sigma(x,t)$, $u(t)$ and J^*, the following steps are required to perform.

Step 1: Select positive scalars ε_i, $i \in \underline{m}$.

Step 2: Solve LMIs (24) in Theorem 3 (Via LMI commands in the Matlab software or YALMIP toolbox) and obtain invertible symmetric positive-definite matrices X, P_2 and matrices M_i, $i \in \underline{m}$. Note that $X = P^{-1}$, $M_i = K_iX$ and consequently. $P = X^{-1}$, $K_i = M_iX^{-1}$. Positive definite matrix P_1 can be given from inequality $\bar{x}^T Z_i \bar{x} < 0$ in Theorem 2.

Step 3: Obtain State feedback $u(t) = K_i x(t)$.

Step 4: Calculate Z_i, $i \in \underline{m}$ in Theorem 2.

Step 5: Obtain switching law $\sigma(x,t) = arg\min_{i\in\underline{m}}\{\bar{x}^T Z_i \bar{x}\}$.

Step 6: Calculate guaranteed cost control J^*.

4 Illustrative Example

Example 1. Consider the following uncertain time-delay switched linear system with two subsystems.

$$\begin{aligned} \dot{x} &= (A_{\sigma(x,t)} + \Delta A_{\sigma(x,t)})x(t) + A_{d\sigma(x,t)}x(t-d), \\ &\quad + (B_{\sigma(x,t)} + \Delta B_{\sigma(x,t)})u(t) + W_{\sigma(x,t)}u(t-h), \\ x(t) &= \phi(t),\ t \in [-t_0,\ 0],\ t_0 \triangleq \max\{d,\ h\}, \end{aligned} \tag{31}$$

for $i = 1, 2$, and the following matrices

$$A_1 = \begin{bmatrix} -2 & -1 \\ 3 & -4 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -4 & 1 \\ 0.5 & -1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \qquad B2 = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$W_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 0.4 & 0.3 \\ 0.3 & 0.5 \end{bmatrix},$$
$$N_1 = \begin{bmatrix} 0.4 & 0.3 \\ 0.3 & 0.6 \end{bmatrix}, \qquad N_2 = \begin{bmatrix} 0.2 & 0.7 \\ 0.6 & 0.3 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.6 & 0.2 \\ 0.3 & 0.6 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0.6 & 0.4 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0 & 0.4 \\ 0.4 & 0 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix},$$

$d = 2$ and $h = 1$ and

$$x(t) = [e^t \; -e^t]^T, \qquad t \in [-2 \; 0],$$

Also, weighted matrices Q and R are selected as

$$Q = R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{32}$$

Note that all subsystems of system (31) are stable and unknown matrices $F_i(t)$ in Notice 1 are considered as a diagonal random time-varying matrices such that $F_i^T(t)F_i(t) \leq I$. The aim is to find guaranteed cost controller $u{=}K_i x(\,t), i \in \{1, 2\}$, switching signal $\sigma(x, t)$ and guaranteed cost J^* of the switched system (31) with weighted matrices (32). We perform the following steps.

step 1: Scalars ε_1 and ε_2 are selected as

$$\varepsilon_1 = 0.1, \;\; \varepsilon_2 = 0.1,$$

step 2: Solving LMIs (24) we obtain

$$X = \begin{bmatrix} 2.8359 & -0.9024 \\ -0.9024 & 1.2247 \end{bmatrix},$$
$$M_1 = \begin{bmatrix} -2.5784 & 0.0004 \\ -0.0003 & -2.5774 \end{bmatrix},$$
$$M_2 = \begin{bmatrix} -0.5288 & -2.0491 \\ -3.1068 & 0.5289 \end{bmatrix},$$

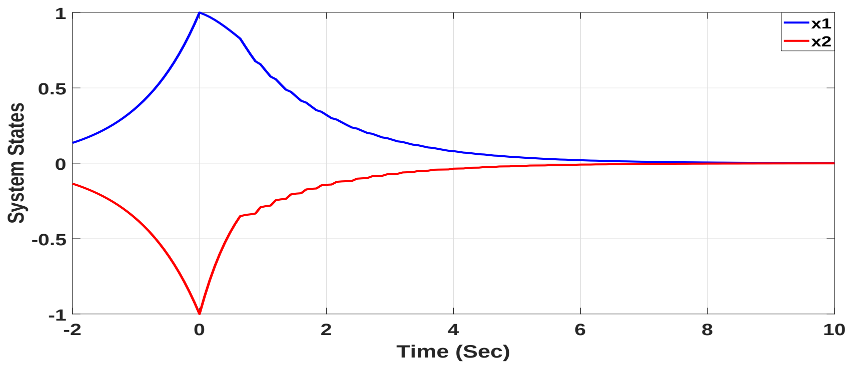

Figure 1: States $x_1(t)$ and $x_2(t)$.

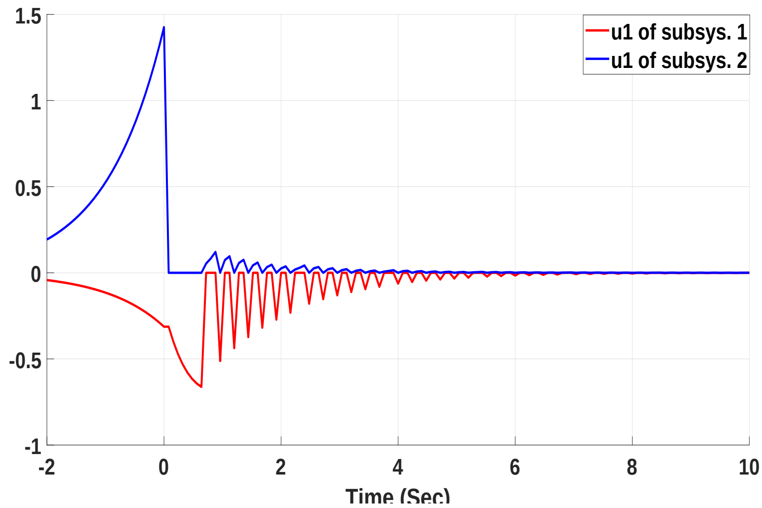

Figure 2: Control input $u_1(t)$ of each subsystem.

and thus

$$P = X^{-1} = \begin{bmatrix} 0.4606 & 0.3394 \\ 0.3394 & 1.0667 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -1.1876 & -0.8747 \\ -0.8750 & -2.7493 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.9391 & -2.3652 \\ -1.2516 & -0.4904 \end{bmatrix},$$

System states start from an initial condition x_0 and Figure 1 shows the state $x_1(t)$ and $x_2(t)$ and, Figure 2, Figure 3 and Figure 4 show control inputs $u_1(t)$ and $u_2(t)$ of each subsystem and switching signal $\sigma(x,t)$. It can be seen that theoretical results

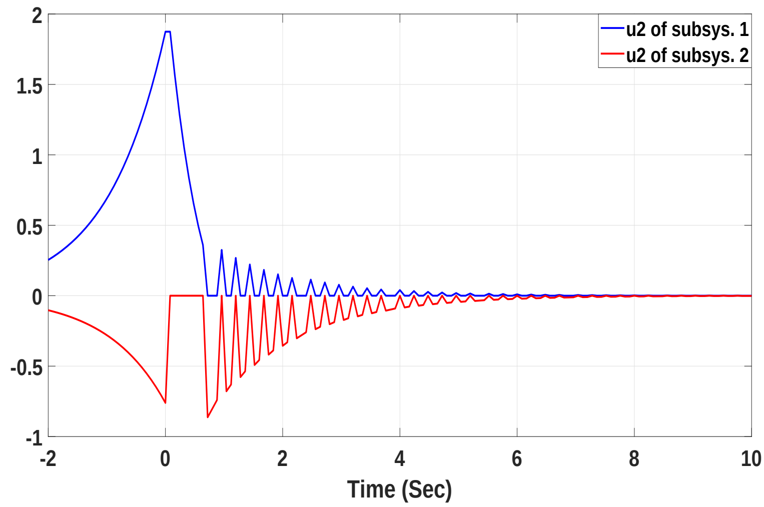

Figure 3: Control input $u_2(t)$ of each subsystem.

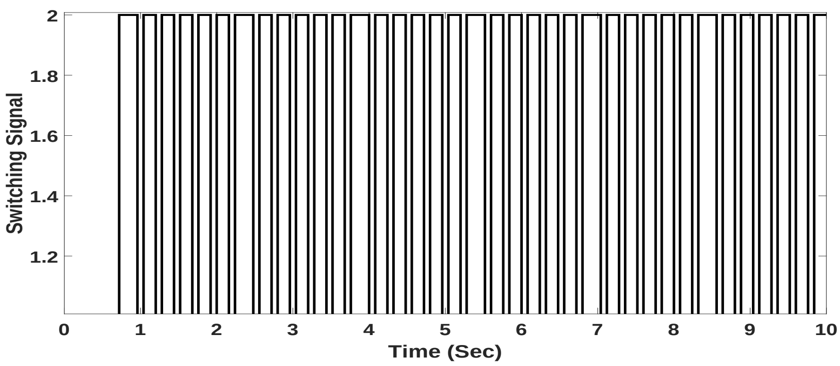

Figure 4: Switching signal $\sigma(x,t)$.

in the Theorem 2 and Theorem 3 which state that uncertain switched system (1) is exponentially stable under applying proposed switching strategy, are coincide with the simulation's results.

5 Conclusion

In this paper, a robust switching law for the GCC problem of a general form of uncertain time-delay switched system is designed. The presented method is based on using the LKF technique and extension of the min-projection switching strategy in this type of switched system. Also, uncertainties in each subsystem's dynamics are considered randomly and are additive. Besides switching law, guaranteed linear control is obtained via the solution of extracted LMIs in the presented theorems. Finally, simulation verifies the theorem's results.

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طراحی یک قانون کلیدزنی مقاوم برای سیستم‌های کلیدزنی خطی دارای عدم قطعیت و تاخیر زمانی

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چکیده

کنترل هزینه تضمینی یکی از روش‌های موثر در کنترل سیستم‌های غیرخطی به ویژه سیستم‌های کلیدزنی دارای عدم قطعیت است. بسیاری از تحقیقات اخیر در مسیله کنترل هزینه تضمینی سیستم‌های کلیدزنی دارای عدم قطعیت بر روی تحلیل پایداری مجانبی تمرکزیافته است. در این مقاله یک قانون سوییچ مقاوم جدید برای کنترل سیستم‌های کلیدزنی دارای عدم قطعیت و تاخیر زمانی ارایه می‌شود. در ابتدا قانون کلیدزنی و سپس کنترل کننده فیدبک حالت خطی بر اساس تابع لیاپانوف-کراسوفسکی طراحی می‌گردد. همچنین با استفاده از نامساوی‌های خطی ماتریسی، شرایط ویژه برای وجود جواب در قانون کلیدزنی و کنترل کننده خطی به دست می‌آید. همزمان و بر اساس قضایای ارایه شده، پایداری نمایی کل سیستم تحت اعمال قانون کلیدزنی و کنترل فیدبک حالت تحلیل و اثبات می‌گردد. در انتها و با شبیه‌سازی، نتایج تحلیلی پایداری نمایی نشان داده می‌شوند.

کلمات کلیدی

سیستم‌های کلیدزنی دارای عدم قطعیت، تاخیر زمانی، کنترل هزینه تضمینی، تابع لیاپانوف-کراسوفسکی، نامساوی خطی ماتریسی، پایداری نمایی.