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Research Article

On Optimal Identification of Distributions for Two Independent Markov Chains to the Subject Reliability Criterion

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Abstract. In this paper, the problem of identification of distributions for two independent objects via simple homogeneous stationary Markov chains with a finite number of states is studied. This problem is introduced by Ahlswede and Haroutunian on the identification of hypotheses under reliability requirements. The problem of identification of distributions for one object via Markov chains was studied by Haroutunian and Navaei in 2009.

Keywords. Identification, Error probability, Two independent objects, Markov chain.

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1 Introduction

Applications of information-theoretic methods in mathematical statistics are reflected in the monographs by Kullback [10] , Csiszár and Körner [3], Csiszár and Shields [4] and Dembo and Zeitouni [5].

Ahlswede and Haroutunian in [1] formulated an ensemble of problems on multiple hypotheses testing for many objects and on the identification of hypotheses under reliability requirement. The problem of many ($L > 2$) hypotheses testing on distributions of a finite state Markov chain is studied in [13] via large deviations techniques and also, identification of distributions for one object via Markov chains is studied in [9].

Notice that the application of large deviations techniques for error exponents to multiple hypotheses testing is studied in [12]. Application of hypotheses testing in steganography systems is discussed in [14].

In this paper, we solve the problem of identifying the distributions of many hypotheses for two independent objects by using of simple homogeneous stationary finite states of Markov chains. We hope that the results of this paper will be used in general case of steganography systems of [14]. In Section 2, we recall the main definitions and results of [6] and [13] for many hypotheses testing. In Section 3, we present the problem of identification of distributions for two independent objects via Markov chains. In Section 4, we show the numerical reliability matrix of distributions and its related figures.

2 On Many Hypotheses Testing for Markov Chains

We recall the main definitions and results [6] and [13] for further use.

Let $\mathbf{x} = (x_0, x_1, x_2, \ldots, x_N), x_n \in \mathcal{X} = \{1, 2, \ldots, I\}, \mathbf{x} \in \mathcal{X}^{N+1}$, $N = 0, 1, 2, \ldots,$ be a vector of observations of a simple homogeneous irreducible stationary Markov chain with finite number I of states. The L hypotheses H_l concern the matrix of the transition probabilities

$$P_l = \{P_l(j|i),\ i = \overline{1,I},\ j = \overline{1,I}\}, \quad l = \overline{1,L}.$$

The stationarity of the chain provides existence for each $l = \overline{1,L}$ of the unique stationary distribution $Q_l = \{Q_l(i),\ i = \overline{1,I}\}$, such that

$$\sum_i Q_l(i) P_l(j|i) = Q_l(j), \quad \sum_i Q_l(i) = 1, \quad i = \overline{1,I}, \quad j = \overline{1,I}.$$

The joint distributions of pairs $(i, j) \in I^2$ are

$$Q_l \circ P_l = \{Q_l(i) P_l(j|i), i = \overline{1,I}, j = \overline{1,I}\}, \quad l = \overline{1,L}.$$

We denote by $D(Q \circ P \| Q_l \circ P_l)$ the Kullback-Leibler divergence

$$D(Q \circ P \| Q_l \circ P_l) = \sum_{i,j} Q(i) P(j|i) [\log Q(i) P(j|i) - \log Q_l(i) P_l(j|i)]$$

$$= D(Q\|Q_l) + D(Q \circ P\|Q \circ P_l),$$

of a joint distribution

$$Q \circ P = \{Q(i)P(j|i), i = \overline{1,I}, j = \overline{1,I}\},$$

from joint to distribution $Q_l \circ P_l$, where the divergence for marginal distributions is

$$D(Q\|Q_l) = \sum_i Q(i)[\log Q(i) - \log Q_l(i)], \quad l = \overline{1,L}.$$

The second order type of Markov the vector $\mathbf{x}$ (see [7]) is the square matrix of I^2 relative frequencies $\{N(i,j)N^{-1}, \quad i = \overline{1,I},\ j = \overline{1,I}\}$ of the simultaneous appearance in $\mathbf{x}$ of the states i and j on the pairs of neighbor places. It is clear that $\sum_{ij} N(i,j) = N$. Denote by $\mathcal{T}^N_{Q\circ P}$ the set of vectors $\mathbf{x}$ from $\mathcal{X}^{N+1}$ which have the second order type such that for some joint PD $Q \circ P$

$$N(i,j) = NQ(i)P(j|i), \quad i = \overline{1,I}, \quad j = \overline{1,I}.$$

The set of joint PD $Q \circ P$ on I^2 is denoted by $\mathcal{Q} \circ \mathcal{P}$. Non-randomized test $\phi_N(\mathbf{x})$ accepts one of the hypotheses H_l, $l = \overline{1,L}$ on the basis of the trajectory $\mathbf{x} = (x_0, x_1, \ldots, x_N)$ of the $N+1$ observations. We denote by $\alpha^{(N)}_{l|m}(\phi_N)$ the probability to accept the hypothesis H_l under the condition that H_m, $m \neq l$, is true. For $l = m$ we denote by $\alpha^{(N)}_{m|m}(\phi_N)$ the probability to reject the hypothesis H_m. It is clear that

$$\alpha^{(N)}_{m|m}(\phi_N) = \sum_{l\neq m} \alpha^{(N)}_{l|m}(\phi_N), \quad m = \overline{1,L}. \tag{1}$$

To each trajectory $\mathbf{x}$ the test ϕ_N puts in correspondence one from L hypotheses. The space $\mathcal{X}^{N+1}$ will be divided into L parts,

$$\mathcal{G}^N_l = \{\mathbf{x},\ \phi_N(\mathbf{x}) = l\}, \quad l = \overline{1,L},$$

and

$$\alpha^{(N)}_{l|m}(\phi_N) = Q_m \circ P_m(\mathcal{G}^N_l), \quad m,l = \overline{1,L}.$$

We consider the matrix of “reliabilities”,

$$E = \{E_{l|m}(\phi) = \lim_{N\to\infty} -\frac{1}{N} \log \alpha^N_{l|m}(\phi_N), \quad m,l = \overline{1,L}\}. \tag{2}$$

It follows from relations (1) and (2) that

$$E_{m|m} = \min_{l\neq m} E_{l|m}. \tag{3}$$

Let P be a matrix of transition probabilities of some stationary Markov chain, and Q be the corresponding stationary PD. For given family of positive numbers $E_{1|1}, E_{2|2}, \ldots, E_{L-1|L-1}$, we consider the decision rule ϕ^* by the sets of distributions:

$$\mathcal{R}_l \triangleq \{Q \circ P : D(Q \circ P\|Q \circ P_l) \leq E_{l|l}, \quad D(Q\|Q_l) < \infty\}, \quad l = \overline{1,L-1},$$

$$\mathcal{R}_L \triangleq \{Q \circ P : D(Q \circ P \| Q \circ P_l) > E_{l|l}, \quad l = \overline{1, L-1}\}, \tag{4}$$

and the functions:

$$\begin{aligned}
&E^*_{l|l}(E_{l|l}) \triangleq E_{l|l}, \quad l = \overline{1, L-1}, \\
&E^*_{l|m}(E_{l|l}) = \inf_{Q \circ P \in \mathcal{R}_l} D(Q \circ P \| Q \circ P_m), \quad m = \overline{1, L}, \ l \neq m, \ l = \overline{1, L-1}, \\
&E^*_{L|m}(E_{1|1}, \ldots, E_{L-1|L-1}) \triangleq \inf_{Q \circ P \in \mathcal{R}_L} D(Q \circ P \| Q \circ P_m), \quad m = \overline{1, L-1}, \\
&E^*_{L|L}(E_{1|1}, \ldots, E_{L-1|L-1}) \triangleq \min_{l=\overline{1,L-1}} E^*_{l|L}.
\end{aligned} \tag{5}$$

In Figure 1, we show the reigns rule for the case of $l = 1, 2, 3$ hypotheses testing.

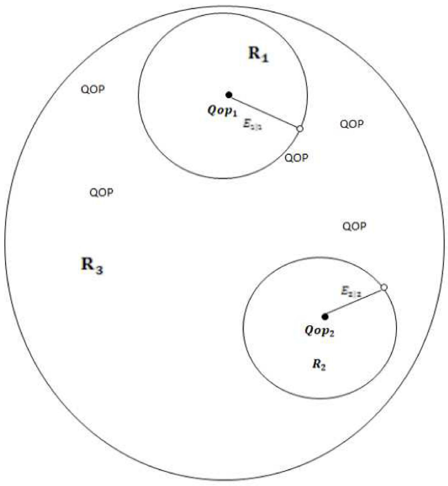

Figure 1: The reigns rule.

The following theorems are the main results of papers [6] and [13], respectively.

Theorem 1. Let $\mathcal{X} = \{1, 2, \ldots, I\}$ be a finite set of the states of the stationary Markov chain possessing an irreducible transition matrix P and $\mathcal{A}$ be a nonempty and open subset or convex subset of joint distributions $Q \circ P$ and Q_m is stationary distribution for P_m, then for the type $Q \circ P(\mathbf{x})$ of a vector $\mathbf{x}$ from $Q_m \circ P_m$ on $\mathcal{X}$:

$$\lim_{N \to \infty} -\frac{1}{N} \log Q_m \circ P_m^N \{\mathbf{x} : Q \circ P(\mathbf{x}) \in \mathcal{A}\} = \inf_{Q \circ P \in \mathcal{A}} D(Q \circ P \| Q \circ P_m).$$

Theorem 2. Let $\mathcal{X}$ be a fixed finite set, for a family of distinct distributions $P_1, \cdots, P_L$ the following two statements hold. If the positive finite numbers $E_{1|1}, \cdots, E_{L-1|L-1}$ satisfy conditions:

$$\begin{gathered}
0 < E_{1|1} < \min[D(Q_m \circ P_m \| Q_m \circ P_1), \ m = \overline{2, L}], \\
\vdots \\
0 < E_{l|l} < \min[E^*_{l|m}(E_{m|m}), \ m = \overline{1, l-1}, D(Q_m \circ P_m \| Q_m \circ P_l), \ m = \overline{l+1, L}], \\
l = \overline{2, L-1},
\end{gathered} \tag{6}$$

then

a. there exists a *LAO* (logaritmically asymptotically optimal) sequence of tests ϕ^*, the reliability matrix of which $\left\{E^*_{l|m}(\phi^*)\right\}$ is defined in (5), and all elements of it are positive,

b. even if one of conditions (6) is violated, then the reliability matrix of an arbitrary test necessarily has an element equal to zero (the corresponding error probability does not tend exponentially to zero).

3 The Problem of Identification of Distributions for Two Independent Markov Chains and Formulation of Results

In this section we expand the concept of Section 2 for two independent homogeneous stationary finite Markov chain. Let x_1 and x_2 be independent RV taking values in the same finite state of Markov cvhains of set $\mathcal{X}$ with one of L PDs, they are characteristics of corresponding independent objects, the random vector (X_1, X_2) assumes values $(x^1, x^2) \in \mathcal{X} \times \mathcal{X}$.

Let

$$(\mathbf{x_1}, \mathbf{x_2}) = \Big((x_0^1, x_0^2), \ldots, (x_n^1, x_n^2), \ldots, (x_N^1, x_N^2)\Big), x^i \in \mathcal{X},$$
$$i = 1, 2, \quad n = \overline{1, N},$$

be a sequence of results of $N+1$ independent observations of a simple homogeneous stationary Markov chain with finite number I of states . The statistication must define unknown PDs of the objects on the base of observed data. Select for each object and denote it by Φ_N. The objects independence test Φ_N may be considered as the pair of the tests φ_N^1 and φ_N^2 for the respective separate objects. We will show the whole compound test sequence by Φ. The test φ_N^i is defined by a partition of the space $^{N+1}$ on the L sets and to every trajectory $\mathbf{x}$ the test ϕ_N puts in correspondence one from L hypotheses. So the space $\mathcal{X}^{N+1}$ will be divided into L parts,

$$\mathcal{G}_{l,i}^N = \{\mathbf{x_i},\ \phi_N(\mathbf{x_i}) = l\}, \quad l = \overline{1, L}, i = 1, 2.$$

We define

$$\alpha_{l_1,l_2|m_1,m_2}^{(N)}(\Phi_N) = Q_{m_1} \circ P_{m_1}(\mathcal{G}_{l_1,1}^N) Q_{m_2} \circ P_{m_2}(\mathcal{G}_{l_2,2}^N),$$

be the probability of the erroneous acceptance by the test Φ_N of the hypotheses pair (H_{l_1}, H_{l_2}) provided that (H_{m_1}, H_{m_2}) is true, where $(m_1, m_2) \neq (l_1, l_2),\ m_i, l_i = \overline{1, L},\ i = 1, 2$. The probability to reject a true pair of hypotheses (H_{m_1}, H_{m_2}) by analogy with (1) is the following:

$$\alpha_{m_1,m_2|m_1,m_2}^{(N)}(\Phi_N) \triangleq \sum_{(l_1,l_2) \neq (m_1,m_2)} \alpha_{l_1,l_2|m_1,m_2}^N(\Phi_N). \tag{7}$$

We also study corresponding limits $E_{l_1,l_2|m_1,m_2}(\Phi_N)$ of error probability exponents of the sequence of tests Φ, called reliabilities :

$$E_{l_1,l_2|m_1,m_2}(\Phi) \triangleq \overline{\lim_{N\to\infty}} - \frac{1}{N} \log \alpha_{l_1,l_2|m_1,m_2}(\Phi_N),\ m_i, l_i = \overline{1, L},\ i = 1, 2. \tag{8}$$

We denote by $E(\varphi^i)$, the reliability matrices of the sequences of tests $\varphi^i, \quad i=1,2,$ for each of the objects.

By using (7) and (8), it follows

$$E_{m_1,m_2|m_1,m_2}(\Phi)=\min_{(l_1,l_2)\neq(m_1,m_2)} E_{l_1,l_2|m_1,m_2}(\Phi). \tag{9}$$

In this section we use the following lemma.

Lemma 1. [7], [8] If elements $E_{l|m}(\varphi^i), \quad m,l=\overline{1,L}, i=1,2,$ are strictly positive, then the following equalities hold for $\Phi=(\varphi^1,\varphi^2)$:

$$E_{l_1,l_2|m_1,m_2}(\Phi)=E_{l_1|m_1}(\varphi^1)+E_{l_2|m_2}(\varphi^2), \quad if \quad m_1\neq l_1, \quad m_2\neq l_2, \tag{10}$$

$$E_{l_1,l_2|m_1,m_2}(\Phi)=E_{l_i|m_i}(\varphi^i), \quad if \quad m_{3-i}=l_{3-i} \quad m_i\neq l_i, \quad i=1,2. \tag{11}$$

Consider for given positive elements $E_{m,m|m,L}$ and $E_{m,m|L,m}, \quad m=\overline{1,L-1},$ the family of regions:

$$\mathcal{R}_m^{(1)} \triangleq \{Q\circ P: D(Q\circ P\|Q\circ P_m)\leq E_{m,m|L,m}\}, \quad m=\overline{1,L-1},$$

$$\mathcal{R}_m^{(2)} \triangleq \{Q\circ P: D(Q\circ P\|Q\circ P_m)\leq E_{m,m|m,L}\}, \quad m=\overline{1,L-1},$$

$$\mathcal{R}_L^{(1)} \triangleq \{Q\circ P: D(Q\circ P\|Q\circ P_m)> E_{m,m|L,m}, \quad m=\overline{1,L-1}\},$$

$$\mathcal{R}_L^{(2)} \triangleq \{Q\circ P: D(Q\circ P\|Q\circ P_m)> E_{m,m|m,L}, \quad m=\overline{1,L-1}\}.$$

There are two error probabilities for each (r_1,r_2), $r_i=\overline{1,L}$, $i=1,2,$ the probability $\alpha^{(N)}_{(l_1,l_2)\neq(r_1,r_2)|(m_1,m_2)=(r_1,r_2)}$ to accept (l_1,l_2) different from (r_1,r_2), when (r_1,r_2) is in reality, and the probability $\alpha^{(N)}_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)}$ that (r_1,r_2) is accepted, when it is not correct. The probability $\alpha^{(N)}_{(l_1,l_2)\neq(r_1,r_2)|(m_1,m_2)=(r_1,r_2)}$ is already known, it coincides with the probability $\alpha^{(N)}_{(r_1,r_2)|(r_1,r_2)}$. Our aim is to determine the dependence of $\alpha^{(N)}_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)}$ on given $\alpha^{(N)}_{(r_1,r_2)|(r_1,r_2)}$.

We need to use the probabilities of different hypotheses. Let us assume that the hypotheses $H_l: l=\overline{1,L}$ have, say, probabilities $P_{\mathbf{r}}(r), \quad r=\overline{1,L}$. The only assumption we shall use is that $P_{\mathbf{r}}(r)>0, \quad r=\overline{1,L}$. We will see, that the result formulated in the following theorem does not depend on values of $P_{\mathbf{r}}(r), \quad r=\overline{1,L},$ if they all are strictly positive. Now we can make the following reasoning for each $r_i=\overline{1,L}, i=1,2:$

$$\alpha^{(N)}_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)}=\frac{P^{(N)}_{\mathbf{r}}((l_1,l_2)=(r_1,r_2),(m_1,m_2)\neq(r_1,r_2))}{P_{\mathbf{r}}(m_1,m_2)\neq(r_1,r_2))},$$

$$\alpha^{(N)}_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)}= \frac{1}{\sum\limits_{m:(m_1,m_2)\neq(r_1,r_2)} P_{\mathbf{r}}(m_1,m_2)} \sum_{m:(m_1,m_2)\neq(r_1,r_2)} \alpha_{(m_1,m_2)|(r_1,r_2)} P^{(N)}_{\mathbf{r}}(m_1,m_2).$$

Finally for $r=\overline{1,L},$ we can write:

$$E_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)}=\min_{(m_1,m_2):(m_1,m_2)\neq(r_1,r_2)} E^*_{(r_1,r_2)|(m_1,m_2)}. \tag{12}$$

For each *LAO* test Φ^* from(9), (10), (11) and (11) we obtain that

$$E_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)} = \min_{m_1\neq r_1, m_2\neq r_2}(E^1_{r_1|m_1}, E^2_{r_2|m_2}), \tag{13}$$

where $E^1_{r_1|m_1}, E^2_{r_2|m_2}$ are determined by (5) for, correspondingly, the first and the second objects. For each *LAO* test Φ^* from (9), (10) and (11) we deduce that

$$E_{(r_1,r_2)|(r_1,r_2)} = \min_{m_1\neq r_1, m_2\neq r_2}(E^1_{r_1|m_1}, E^2_{r_2|m_2}) = \min(E^1_{r_1|r_1}, E^2_{r_2|r_2}), \tag{14}$$

and each of $E^1_{r_1|r_1}, E^2_{r_2|r_2}$ satisfies the following conditions (see Theorem 2, condition (6)).

$$0 < E^1_{r_1|r_1} < \min\left[\min_{l=\overline{1,r_1-1}} E^*_{l|m}(E^1_{l|l}), \min_{l=\overline{r_1+1,L}} D(Q_l\circ P_l\|Q_l\circ P_{r_1})\right], \tag{15}$$

$$0 < E^2_{r_2|r_2} < \min\left[\min_{l=\overline{1,r_2-1}} E^*_{l|m}(E^2_{l|l}), \min_{l=\overline{r_2+1,L}} D(Q_l\circ P_l\|Q_l\circ P_{r_2})\right]. \tag{16}$$

From (5) we see that the elements $E^*_{l|m}(E^1_{l|l})$, $r_1 = \overline{1, r_1-1}$ and $E^*_{l|m}(E^2_{l|l})$, $r_2 = \overline{1, r_2-1}$ are determined only by $E^1_{l|l}$ and $E^2_{l|l}$. But we are considering only elements $E^1_{r_1|r_1}$ and $E^2_{r_2|r_2}$. By using Theorem 1, (15) and (16) we obtain

$$0 < E^1_{r_1|r_1} < \min\left[\min_{l=\overline{1,r_1-1}} D(Q_l\circ P_l\|Q_l\circ P_{r_1}), \min_{l=\overline{r_1+1,L}} D(Q_l\circ P_l\|Q_l\circ P_{r_1})\right], \tag{17}$$

$$0 < E^2_{r_2|r_2} < \min\left[\min_{l=\overline{1,r_2-1}} D(Q_l\circ P_l\|Q_l\circ P_{r_2}), \min_{l=\overline{r_2+1,L}} D(Q_l\circ P_l\|Q_l\circ P_{r_2})\right]. \tag{18}$$

Let us denote $r = \max(r_1, r_2)$ and $k = \min(r_1, r_2)$. From (13) we have that, when $E_{(r_1,r_2)|(r_1,r_2)} = E^1_{r_1|r_1}$, then $E^1_{r_1|r_1} \leq E^2_{r_2|r_2}$ and when $E_{(r_1,r_2)|(r_1,r_2)} = E^2_{r_2|r_2}$, then $E^1_{r_1|r_1} \geq E^2_{r_2|r_2}$. Hence, we deduce that given strictly positive elements $E_{(r_1,r_2)|(r_1,r_2)}$ must meet both inequalities (17), (18) and the combination of these restrictions gives

$$0 < E_{(r_1,r_2)|(r_1,r_2)} < \min\left[\min_{l=\overline{1,r-1}} D(Q_l\circ P_l\|Q_l\circ P_r), \min_{l=\overline{r+1,L}} D(Q_l\circ P_l\|Q_l\circ P_k)\right]. \tag{19}$$

Using (15) and (16) we can determine reliability $E_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)}$ in a function of $E_{(r_1,r_2)|(r_1,r_2)}$, namely,

$$\begin{aligned} &E_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)}(E_{(r_1,r_2)|(r_1,r_2)}) = \\ &\min_{m_1\neq r_1, m_2\neq r_2}(E_{r_1|r_1}(E_{(r_1,r_2)|(r_1,r_2)}), E_{r_2|r_2}(E_{(r_1,r_2)|(r_1,r_2)}), \end{aligned} \tag{20}$$

where $(E_{r_1|r_1}(E_{(r_1,r_2)|(r_1,r_2)})$ and $E_{r_2|r_2}(E_{(r_1,r_2)|(r_1,r_2)})$ are determined by (5). The results can be summarized in the following theorem.

Theorem 3. If the distrbutions H_m, $m = \overline{1, L}$, are different and the given strictly positive number $E_{(r_1,r_2)|(r_1,r_2)}$ satisfy condition (19), then the reliability $E_{(l_1,l_2)=(r_1,r_2)|(m_1,m_2)\neq(r_1,r_2)}$is defined in (20).

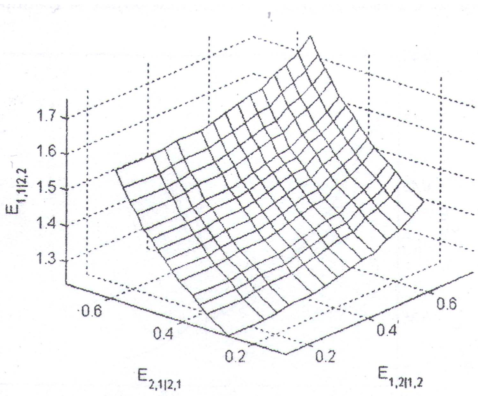

Figure 2: Diagram of $E_{(1,1)|(2,2)}, E_{(1,2)|(1,2)}$ and $E_{(2,1)|(2,1)}$ for Example 1

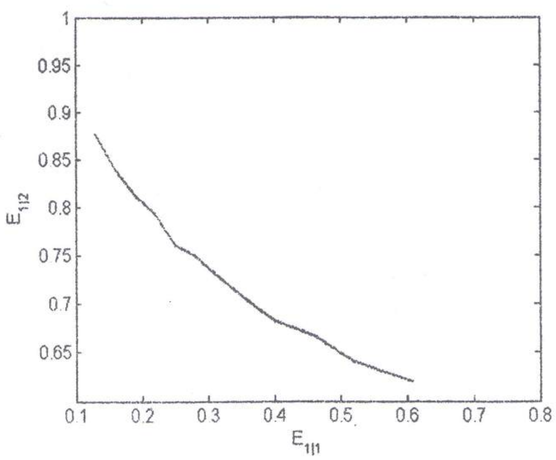

Figure 3: Diagram of $E_{1|2}(E_{1|1})$ for the first object of Example 1

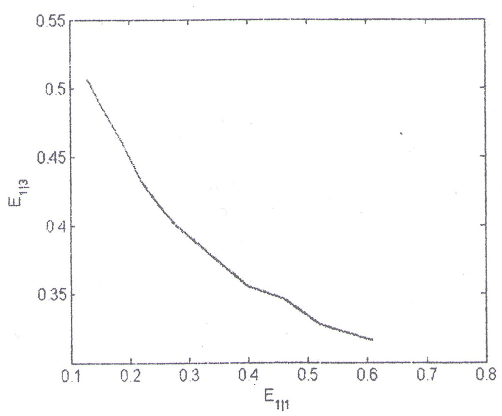

Figure 4: Diagram of $E_{1|3}(E_{1|1})$ for the second object of Example 1

4 Numerical Examples

We consider text classification on application of multiple Hypotheses testing for Markov chains. Assume that a model English text as a Markov process where the probability of observing any text word depends on the previous word.

For this example we consider a document which is comprised of an ordered sequence of word events. Suppose that the probability of each word in the document depends on of the previous word, but it is independent of its position in the document. In other words if we have vocabulary $X = \{x_1, \dots, x_L\}$ each category of the document is described by the conditional probabilities matrix $P = \{P(x|u), u, x \in \mathcal{X}\}$. Now our aim is to assign each document to the appropriate category, based on the designed rules. So, we have L hypotheses and based on sequence of words the classifier has to decide if a particular feature vector is likely to be drawn from a given category or not and try to minimize misclassification (error probabilities).

In order to better understand the hypotheses testing and text categorization theories it would be pertinent to discuss an example with the binary set $\mathcal{X} = \{0, 1\}$. In the example we assume that there are given two Markov sources with alphabet $\mathcal{X} = \{0, 1\}$.

Suppose an outcome of language research that enables a representation of different languages genres reflected in the following transition matrices as hypothesis to test for each of two texts:

Example 1.

$$H_1 : P_1 = \begin{bmatrix} 0.295 & 0.705 \\ 0.1 & 0.9 \end{bmatrix}, H_2 : P_2 = \begin{bmatrix} 0.49 & 0.51 \\ 0.92 & 0.08 \end{bmatrix}, H_3 : P_3 = \begin{bmatrix} 0.9 & 0.1 \\ 0.45 & 0.55 \end{bmatrix}.$$

Example 2.

$$H_1 : P_1 = \begin{bmatrix} 0.705 & 0.295 \\ 0.295 & 0.705 \end{bmatrix}, H_2 : P_2 = \begin{bmatrix} 0.49 & 0.51 \\ 0.92 & 0.08 \end{bmatrix}, H_3 : P_3 = \begin{bmatrix} 0.9 & 0.1 \\ 0.45 & 0.55 \end{bmatrix}.$$

In this kind of categorization problems the performance of algorithms is discussed in complexities point of view. In term of this example we would like to introduce a framework of problems where the quality of categorization of objects is considered via error exponents analysis.

For the aforementioned hypotheses, applying Theorem 3 in [7] we got values for all elements of reliability matrix, given fixed elements $E_{3,1|1,1}, E_{3,2|2,2}$ and $E_{2,3|2,2}$. For numerical experiments we generate a sequence of those reliability matrices in the following way. At first we initialize a matrix with fixed components equal to 0.01. By increasing those values by step $\delta = 0.1$, we get a sequence of reliability matrices. Based on that sequence we draw the surface of $(E_{1,1|2,2}, E_{1,2|1,2}, E_{2,1|2,1})$ in Figure 2.

Applying Lemma 1 for each object we get the planes $(E_{1|2}(E_{1|1}), E_{1|1})$ and $(E_{1|3}(E_{1|1}), E_{1|1})$ with the graphs in Figures 3 and 4, respectively.
The surface in Figure 2 illustrates the interdependence of reliabilities $E_{1,1|2,2}$, $E_{1,2|1,2}$ and $E_{2,1|2,1}$.

Note that in Figure 3 starting from the value of $E_{1|1} \approx 0.35$ the value of reliability $E_{1|2}(E_{1|1})$ decreases faster. In Figure 4 the value of reliability $E_{1|3}(E_{1|1})$ decreases faster starting from the value of $E_{1|1} \approx 0.25$.

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