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Research Article

Integrated Fault Detection and Robust Control for Linear Uncertain Switched Systems with Mode-Dependent Time-Varying State Delay

Sayyed Hossein Ejtahed[1], Naser Pariz[*1], Ali Karimpour[1]

[1]Department of Electrical Engineering, Faculty of Engineering, Ferdowsi University of Mashhad, P.O. Box. 91779-48974, Mashhad, Iran.

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Abstract. Switched linear systems are noted as a major category of control systems. Fault detection of these systems is affected by switching phenomena and therefore their integrated fault detection and robust control (IFDRC) are the central issues of recent studies. Existing studies on IFDRC do not consider the effects of all of the parameter uncertainties, input disturbance, and mode-dependent time-varying state delay in the presence of mode-dependent average dwell time (MDADT) switching together in these systems. To address the issue based on output feedback, in this paper, the IFDRC design problem is formulated as a multi-objective or mixed H_∞/H_- optimization problem. H_∞ performance indicator guarantees the robustness of residual to disturbance, and H_- performance represents the sensitivity index of residual to the fault. A piecewise Lyapunov-Krasovskii function is employed together with the MDADT scheme and therefore, sufficient conditions are derived in terms of linear matrix inequalities (LMIs) to deal with the problem. Then to clarify the design procedure, we also present an algorithm in light of the proposed approach. Eventually, to illustrate the efficiency of the suggested approach, the designed IFDRC framework is simulated for a case study of an Electrical Circuit system.

Keywords. Integrated fault detection and control, Switched systems, Uncertainty, Variable state delay.

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* Corresponding author
ejtahed@stu.um.ac.ir, n-pariz@um.ac.ir, karimpor@um.ac.ir
http://mathco.journals.pnu.ac.ir

1 Introduction

As an important class of hybrid systems, switched systems are a combination of multiple subsystems and a switching law. The switching law determines an active subsystem at the particular switching time instant. Many practical applications are modeled as switched systems, such as power electronics [36], Buck-Boost converter [10], Ball-and-Beam systems [12], flight control systems, etc. [28].

For real-world processes, fault detection (FD) has become more significant due to the increasing demand for the efficiency of supervision, safety, and reliability. Model-based FD methods have been widely used and developed over the past decades. This technique is to construct residuals based on some measured output signals of the system. The occurrence of faults is determined by comparing residuals in fault-free and faulty situations [39]. Many results have emerged from this topic for switched systems [23, 29, 37]. On the other hand, it is possible, and more importantly, desirable to consider a framework for integrating the design of fault diagnosis filters and feedback controllers. This simultaneous design unifies both control and diagnosis modules into an integrated unit. Therefore, it is unavoidable and certain that an integrated fault detection and control (IFDC) design technique should result in a far less general difficulty as compared to an approach where the two modules are designed separately [7]. Some techniques in the IFDC area are as follows: a method subject to a dwell time constraint [38], an approach based on Dynamic Observer [5], A Linear Matrix Inequality Approach [2], Average Dwell Time constraint [2, 10], and IFDC schemes under mode-dependent average dwell time constraint [33].

As a common phenomenon in many dynamic physical processes, the delay and parameter uncertainties may weaken the fault detection sensitivity and disturbance attenuation capability. Therefore, it is important to take into consideration the effect of state delay and parameter uncertainties for designing fault detection and control units under the presence of unknown inputs. Meanwhile, only a few papers have taken the state delay into account. Some of them have supposed it constant [27, 35, 39] and others have assumed it time-varying [19, 24]. Due to the complexity caused by the presence of parameter uncertainties, a few results on FD of switched delay systems with parameter uncertainties have been reported [21, 24]. As far as we know, there is a very limited number of research considering both the variable state delay and parameter uncertainties [24].

Innovation and the main contribution

In this paper, we investigate the problems of fault detection and robust control for switched linear systems in a general framework. Some documents in this field employ one of the below-mentioned five concepts separately, or at most a combination of two or three cases of them. To the best of our knowledge, the IFDRC design with a variety of these five items is not tackled yet for the switched systems. The main contribution of our work is to propose a general framework for designing IFDRC for the switched systems considering these concepts:

- MDADT: mode-dependent average dwell time,
- MDTVD: mode-dependent time-varying state delay,
- Parameter Uncertainty,
- Input disturbance,
- Mixed H_∞/H_-.

In this paper, the mode-dependent average dwell time (MDADT), which will release the restrictions of ADT, is used with mode-dependent time-varying (MDTV) state delay, and norm-bounded parameter uncertainties, and unknown input disturbances. Further, in an output feedback framework, sufficient conditions are derived and formulated for weighted H_∞ performance in terms of a set of linear matrix inequalities with MDADT switching to attenuate the disturbance of the corresponding switched linear systems. Sufficient conditions for weighted H_- performance to amplify fault sensitivity are also derived and developed in terms of a set of matrix inequalities. Based on the proposed scheme, the IFDRC problem is solved by the convex optimization technique, and the dynamic controller/detectors associated with the designed switching law are obtained such that the system with the mentioned constraint satisfies the indices.

The remainder of this paper is organized as follows: Section 2 presents the problem statement, necessary definitions, and preliminaries. It recalls the corresponding criterion and lemmas for the switched systems' fault detection and control with MDADT switching. In Section 3, the main results for mixed weighted H_∞/H_- integrated fault detection and robust control unit (IFDRCU) design for linear uncertain continuous-time switched systems with MDTV state delay and input disturbance under MDADT constraint design approaches are illustrated in detail by two theorems. The residual evaluation function and the threshold are provided. To demonstrate the effectiveness of the proposed method, an Electrical Circuit system is given as a numerical example in Section 4, followed by a conclusion in the last section.

Notations

In this paper, some standard notations are used. For a matrix A, A^T denotes its transpose. Here, $A > 0 (A >= 0)$ and $A < 0 (A <= 0)$ mean that the matrix is positive and negative (semi-)definite, respectively. The symbol * used in a matrix denotes the terms which are readily inferred from symmetry. The Hermitian part of a square matrix A is denoted by $He(A) := A + A^T$. The values $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ are the maximum and minimum eigenvalues of A, respectively. $\mathbb{R}^n$ stands for the n-dimensional real vector space; where $\mathbb{R}^{n\times m}$ indicates the space of $n \times m$ matrices with real entries; $\|x\|^2 = x^T x = x_1^2 + \cdots + x_n^2$, where x_i is the i-th element of the vector, $x \in \mathbb{R}^n$; let $\underline{l} = \{1, \ldots, l\}$, where l is an arbitrary positive integer; $\mathbb{Z}^+$ implies the set of positive integers. l_2 stands for the 2-norm; 0 and I represent the zero and identity matrices with appropriate dimensions, respectively. $A^\perp$ is defined as an orthogonal basis for the null space of A while satisfying $A^\perp A = 0$.

2 Problem Statement and Preliminaries

In this section, problem formulation, necessary assumptions, definitions, lemmas, and IFDRC concepts are presented.

2.1 The main system model

Consider the following switched linear system with mode-dependent time-varying state delays and parameter uncertainty.

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}(t)x(t) + A_{d\,\sigma(t)}(t)x(t - d_{\sigma(t)}(t)) + B_{\sigma(t)}u(t) + B_{\omega\,\sigma(t)}(t)\omega(t) + B_{f\,\sigma(t)}(t)f(t), \\ y(t) = C_{\sigma(t)}(t)x(t) + D_{\omega\sigma(t)}(t)\omega(t) + D_{f\,\sigma(t)}(t)f(t), \\ x(\theta) = \phi(\theta), \quad \theta \in [-d, 0]. \end{cases} \tag{1}$$

Here, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ denotes the control input vector, $\omega(t) \in \mathbb{R}^r$ represents the bounded disturbance input, $f(t) \in \mathbb{R}^s$ is the fault signal, and $y(t) \in \mathbb{R}^q$ signifies the measured output vector. It is assumed that $\omega(t)$ and $f(t)$ belong to $L_2[0, \infty)$ and $\|\omega(t)\|_2 \le \delta_\omega$, $\|f(t)\|_2 \le \delta_f$, where δ_ω , δ_f are represented as known constants. $\phi(\theta)$ is the continuous vector-valued initial function on $[-d, 0]$. $\sigma(t) : [0, \infty) \to \underline{l}$ is a right continuous piecewise constant function that denotes the switching law and $l > 1$ is the number of subsystems. $\sigma(t) = i$ means that the i-th subsystem is activated at time t. If

$t \in [t_k, t_{k+1})$, then $\sigma(t) = \sigma(t_k)$. The duration time $[t_k, t_{k+1})$ is called the dwell time of the currently enabled subsystem. The value t_k represent the switching time instants and $t_0 < t_1 < \cdots < t_k$, $(k \in \mathbb{Z}^+)$ represents the switching time sequence of the switching signal. $A_i, A_{di}, B_i, B_{\omega i}, B_{fi}, C_i, D_{\omega i}$, and D_{fi} represent known real constant system matrices with appropriate dimensions. $d_i(t)$ stands for the mode-dependent time-varying delay in state variables, which is a continuous function satisfying $0 < d_i(t) < d_i < d$ and $\dot{d}_i(t) < \rho_i$, and d_i, d, ρ_i are known positive scalars.

Assumption 1. ([25]): For input matrices $B_i \in \mathbb{R}^{n\times m}$ with $(B_i) = m$, there exist non-singular matrices T_i such that

$$T_i B_i = \begin{bmatrix} I \\ 0 \end{bmatrix}. \tag{2}$$

In general, for a specified B_i, the corresponding T_i is not unique. One of the matrices T_i is

$$T_i = \begin{bmatrix} (B_i^T B_i)^{-1} B_i^T \\ B_i^{\perp} \end{bmatrix}. \tag{3}$$

Also, the model uncertainties are as in (4) and $\Delta A_i, \Delta A_{di}, \Delta B_{\omega i}, \Delta B_{fi}, \Delta C_i, \Delta D_{\omega i}$, and ΔD_{fi} are norm-bounded matrices, and therefore, we obtain

$$\begin{cases} A_{\sigma(t)}(t) = A_{\sigma(t)} + \Delta A_{\sigma(t)}(t), \\ A_{d\sigma(t)}(t) = A_{d\sigma(t)} + \Delta A_{d\sigma(t)}(t), \\ B_{\sigma(t)} = B_{\sigma(t)}, \\ B_{\omega\sigma(t)}(t) = B_{\omega\sigma(t)} + \Delta B_{\omega\sigma(t)}(t), \\ B_{f\sigma(t)}(t) = B_{f\sigma(t)} + \Delta B_{f\sigma(t)}(t), \\ C_{\sigma(t)}(t) = C_{\sigma(t)} + \Delta C_{\sigma(t)}(t), \\ D_{\omega\sigma(t)}(t) = D_{\omega\sigma(t)} + \Delta D_{\omega\sigma(t)}(t), \\ D_{f\sigma(t)}(t) = D_{f\sigma(t)} + \Delta D_{f\sigma(t)}(t). \end{cases} \tag{4}$$

Assumption 2. ([11]): The parameter uncertainties are assumed to satisfy the following norm-bounded conditions:

$$\begin{bmatrix} \Delta A_i(t) & \Delta A_{di}(t) & \Delta B_{\omega i}(t) & \Delta B_{fi}(t) \\ \Delta C_i(t) & \Delta C_{di}(t) & \Delta D_{\omega i}(t) & \Delta D_{fi}(t) \end{bmatrix} = \begin{bmatrix} M_{i1} \\ M_{i2} \end{bmatrix} Q(t) \begin{bmatrix} N_{i1} & N_{i2} & N_{i3} & N_{i4} \end{bmatrix}, \tag{5}$$

where $M_{ij}(j = 1,2)$ and $N_{ik}(k = 1,2,3,4)$ are known real constant matrices and $Q(t) \in \mathbb{R}^{k\times k}$ is an unknown Lebesque-measurable real time-varying matrix subject to the following condition.

$$Q^T(t)Q(t) \le I, \tag{6}$$

for each t.

Remark 1. It is worth to be mentioned that a regulated output can also be considered for the main system in which both effects of fault and disturbance should be minimized on it to achieve a robust control objective. But for the sake of simplicity, it is ignored in this work [33, 40].

2.2 Integrated fault detection and robust control unit

To generate the control and the residual signal simultaneously, the integrated fault detection and robust control unit (IFDRCU) is employed, which integrates a fault detector and an output feedback controller within a switched linear system, as follows:

$$\begin{cases} \dot{x}_m(t) = A_{m\sigma(t)}x_m(t) + B_{m\sigma(t)}y(t), \\ r(t) = C_{m\sigma(t)}x_m(t) + D_{m\sigma(t)}y(t), \\ u(t) = K_{m\sigma(t)}x_m(t) + L_{m\sigma(t)}y(t), \end{cases} \tag{7}$$

where $x_m(t) \in \mathbb{R}^n$ represents the controller state vector and $r(t) \in \mathbb{R}^q$ is the residual signal. The matrices $A_{mi}, B_{mi}, C_{mi}, D_{mi}, K_{mi}$, and L_{mi} are the IFDRCU gains with appropriate dimensions, which should be determined.

Assumption 3. ([28]): The switching signal is not known beforehand, but it is assumed that it is determined instantaneously and IFDRCU switches synchronously with the main system. This is a common assumption in the literature. It is also considered that faults will not occur in the switching signal.

2.3 Closed-loop system description

Combining the aforementioned structures of the main system and the IFDRCU and defining the augmented state vector as $\varsigma^T(t) = [x^T(t)\; x_m^T(t)]$ to include filters state, the following augmented switched system is obtained:

$$\begin{cases} \dot{\varsigma}(t) = \bar{A}_{\sigma(t)}(t)\varsigma(t) + \bar{A}_{d\,\sigma(t)}(t)\varsigma(t - d_{\sigma(t)}(t)) + \bar{B}_{\omega\,\sigma(t)}(t)\omega(t) + \bar{B}_{f\,\sigma(t)}(t)f(t), \\ r(t) = \bar{C}_{\sigma(t)}(t)\varsigma(t) + \bar{D}_{\omega\sigma(t)}(t)\omega(t) + \bar{D}_{f\,\sigma(t)}(t)f(t), \end{cases} \tag{8}$$

where

$$\bar{A}_{\sigma(t)}(t) = \begin{bmatrix} A_{\sigma(t)}(t) + B_{\sigma(t)}L_{m\sigma(t)}C_{\sigma(t)}(t) & B_{\sigma(t)}K_{m\sigma(t)} \\ B_{m\sigma(t)}C_{\sigma(t)}(t) & A_{m\sigma(t)} \end{bmatrix},$$

$$\bar{A}_{d\,\sigma(t)}(t) = \begin{bmatrix} A_{d\,\sigma(t)}(t) & 0 \\ 0 & 0 \end{bmatrix},$$

$$
\begin{aligned}
\bar{B}_{\omega\sigma(t)}(t) &= \begin{bmatrix} B_{\omega\sigma(t)}(t) + B_{\sigma(t)}L_{m\sigma(t)}D_{\omega\sigma(t)}(t) \\ B_{m\sigma(t)}D_{\omega\sigma(t)}(t) \end{bmatrix}, \\
\bar{B}_{f\sigma(t)}(t) &= \begin{bmatrix} B_{f\sigma(t)}(t) + B_{\sigma(t)}L_{m\sigma(t)}D_{f\sigma(t)}(t) \\ B_{m\sigma(t)}D_{f\sigma(t)}(t) \end{bmatrix}, \\
\bar{C}_{\sigma(t)}(t) &= \begin{bmatrix} D_{m\sigma(t)}C_{\sigma(t)}(t) & C_{m\sigma(t)} \end{bmatrix}, \\
\bar{D}_{\omega\sigma(t)}(t) &= D_{m\sigma(t)}D_{\omega\sigma(t)}(t), \\
\bar{D}_{f\sigma(t)}(t) &= D_{m\sigma(t)}D_{f\sigma(t)}(t).
\end{aligned}
$$

2.4 The IFDRC design problem

In this section, the main problem is formulated as a multi-objective or mixed H_∞/H_- optimization problem. Therefore, our objective here is to design a switching law, a control signal, and a fault detection filter (see Figure 1) such that the exponential stability of the augmented switched system (8) is guaranteed with the specified mode-dependent average dwell time (MDADT). By setting the zero initial conditions, the effect of fault on the residual signal is maximized while the impact of disturbance is minimized on it considering the parameter uncertainties of the main system.

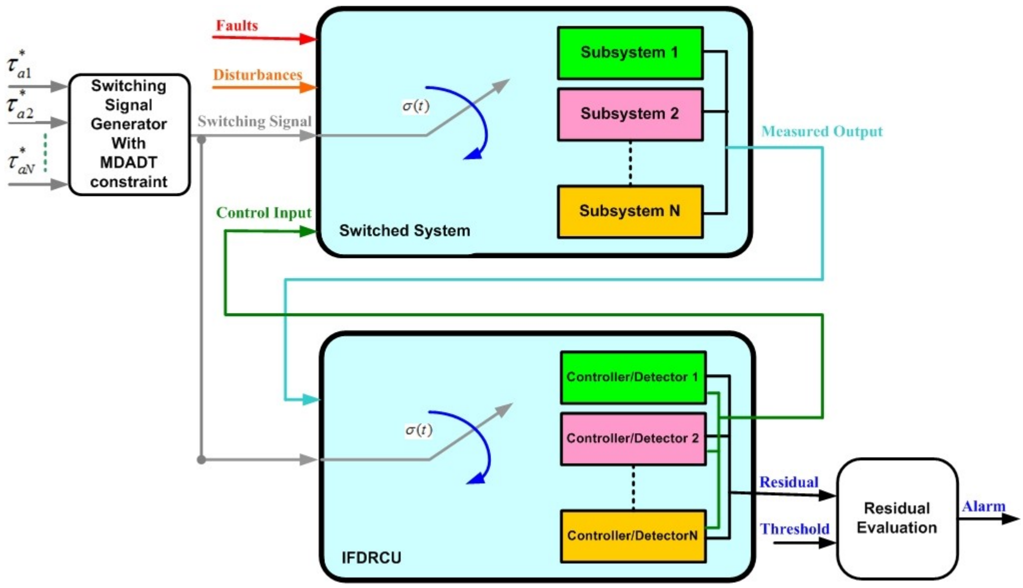

Figure 1: Switched system and Integrated Fault Detection & Robust Control Unit (IFDRCU).

2.4.1 Performance indices

For a given $\alpha_M > 0$, disturbance attenuation is characterized by the following weighted l_2 gain, which is called the weighted H_∞ performance index (α_M, γ_1) problem. It also ensures that the undetected faults are not disastrous.

$$\int_0^\infty e^{-\alpha_M t} r^T(t) r(t) dt \leq \gamma_1^2 \int_0^\infty \omega^T(t)\omega(t) dt. \tag{9}$$

Here, γ_1 is a prescribed level of disturbance attenuation. The smaller γ_1, the less affected the residual signal by disturbance.

Given $\alpha_m > 0$, fault sensitivity amplification is characterized by the following weighted l_2 gain, which is called the weighted H_- performance index (α_m, γ_2) problem.

$$\int_0^\infty r^T(t) r(t) dt \geq \gamma_2^2 \int_0^\infty e^{-\alpha_m t} f^T(t) f(t) dt. \tag{10}$$

Here, γ_2 is a prescribed level of fault sensitivity. The greater γ_2, the more sensitive to fault the residual signal.

Note that in general, α_M can be different from α_m.

Remark 2. The parameters α_M and α_m present the weighted l_2 gain index owing to the MDADT switching strategy. If $\alpha_M = \alpha_m$ is small enough which means that τ_a is selected sufficiently large, then the weighted l_2 gain approaches obviously the normal H_∞ problem. In fact, H_∞ performance is an unsolved problem for switched systems with the constraint of ADT, and therefore, a weighted H_∞ performance index should be utilized [4, 13]. Some claims in this area, such as those in [25], are not meaningful. In this work, we used a weighted H_∞/H_- performance index.

Remark 3. Some authors use a standard H_∞ model matching problem to change the H_- optimization problem into an H_∞ optimization problem by defining $r_e(t) = r(t) - f_w(t)$. This means that the residual signal, $r(t)$, robustly tracks a filtered version of the fault signal, i.e., $f_w(t)$. The filter $W(s)$ should be chosen appropriately as a stable transfer function. Since there is no straightforward method to determine this transfer function [8], the complexity is increased, and compared to those methods, our approach is more direct [33]. In some works, such as [24], $W(s)$ is defined as the filter, but the augmented system is not affected by the filter dynamics. In some other studies, like [2, 10], the use H_∞ problem is used instead of H_-, without defining $r_e(t) = r(t) - f_w(t)$.

2.4.2 Problem formulation

In this paper, the whole problem of IFDRC is transformed into the following mixed H_∞/H_- optimization problem. It is called a multi-objective or mixed optimization problem in the literature because it has two different objects and involves different norms [16, 20, 30].

$$\min_{\substack{\text{s.t.} \\ (9),(10)}} c_1\gamma_1 - c_2\gamma_2. \tag{11}$$

In practice, the two scalars $c_1, c_2 \geq 0$ are used for a trade-off between the fault detection and control requirements. For example, if the H_∞ performance index is given, the relevant scalar $c_1 = 0$ [33].

2.5 Mathematical preliminaries

This section provides definitions and lemmas corresponding to the switched systems' fault detection and control with MDADT switching.

Definition 1. ([34]): For a switching signal $\sigma(t)$ and $\forall\, T \geq t \geq 0$, let $N_{\sigma i}(t,T)$ be the number of times that the i-th subsystem is activated on the interval $[t,T)$, and $T_i(t,T)$ present the total running time of the ith subsystem on the interval $[t,T)$, $i \in \underline{l}$. If there exist positive numbers $N_{0i} \geq 0$ and $\tau_{ai} > 0$ such that

$$N_{\sigma i}(t,T) \leq N_{0i} + \frac{T_i(t,T)}{\tau_{ai}}, \tag{12}$$

for each $T \geq t \geq 0$, then we say that $\sigma(t)$ has a mode-dependent average dwell time, (MDADT),τ_{ai}, and the constant N_{0i} is called the mode-dependent chatter bound.

Remark 4. Although the constant N_{0i} should not be less than 2 in the case of average dwell time switching, it is usual in the literature to be assumed as zero for the sake of mathematical simplification [18]. In the sequel, we considered it not necessarily zero.

Definition 2. ([31]): Given scalars $\alpha > 0$ and $\gamma > 0$ the augmented system in (8) is said to be exponentially stable with weighted H_∞ performance (α,γ), if under $\sigma(t)$, it is exponentially stable with $\omega(t) = 0$, and under zero initial condition, that is, $\phi(\theta) = 0, \theta \in [-d,0]$, for any non-zero $\omega(t) \in L_2[0,\infty)$, it holds that.

$$\int_0^\infty e^{-\alpha s} r^T(s) r(s) ds \leq \gamma^2 \int_0^\infty \omega^T(s)\omega(s) ds. \tag{13}$$

Lemma 1. ([3]): (Schur complement lemma) Let Y be a symmetric matrix of real numbers partitioned as follows and D be invertible. Then Y is positive definite if and only if D and its Schur complement, (Y/D), are both positive definite.

$$Y = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \succ 0 \Leftrightarrow D \succ 0 \quad \text{and} \quad Y/D = A - BD^{-1}B^T \succ 0. \tag{14}$$

Lemma 2. For two given symmetric matrices, $\Phi, \tilde{\Phi} \in \mathbb{R}^{n\times n}$, where for each $i \neq j$

$$\Phi_{ii} \leq \tilde{\Phi}_{ii} \quad \text{and} \quad \Phi_{ij} = \tilde{\Phi}_{ij},$$

we have

$$\tilde{\Phi} \prec 0 \Rightarrow \Phi \prec 0. \tag{15}$$

Proof. Defining $\Lambda = (\lambda_1, \cdots, \lambda_n)$ *s.t.* $\lambda_i = \Phi_{ii} - \tilde{\Phi}_{ii} \leq 0$, from the assumption we have $\Phi - \tilde{\Phi} = \Lambda I_{n\times n} \prec 0$, therefore,

$$x^T \tilde{\Phi} x \prec 0 \Rightarrow x^T(\Phi - \Lambda I_{n\times n})x \prec 0 \Rightarrow x^T \Phi x - x^T \Lambda I_{n\times n} x \prec 0 \Rightarrow x^T \Phi x \prec 0.$$

□

Lemma 3. For a positive definite matrix $\Gamma \in \mathbb{R}^{n\times n}$, and any arbitrary symmetric matrix $\Lambda \in \mathbb{R}^{n\times n}$, we have

$$\Lambda \Gamma^{-1} \Lambda \geq 2\Lambda - \Gamma. \tag{16}$$

Proof. From the positive definiteness of Γ, it is clear that $x^T \Gamma^{-1} x \succ 0$. One can choose $x = (\Gamma - \Lambda)y$, therefore, $y^T(\Gamma - \Lambda)\Gamma^{-1}(\Gamma - \Lambda)y \succ 0$ which will result in

$$\begin{aligned} y^T(I_{n\times n} - \Lambda\Gamma^{-1})(\Gamma - \Lambda)y \succ 0 &\Rightarrow y^T(\Gamma - \Lambda - \Lambda\Gamma^{-1}\Gamma + \Lambda\Gamma^{-1}\Lambda)y \succ 0 \\ &\Rightarrow \Gamma - 2\Lambda + \Lambda\Gamma^{-1}\Lambda \succ 0. \end{aligned}$$

□

Lemma 4. ([41]): (Generalized square inequality lemma) If $X \in \mathbb{R}^{m\times n}, Y \in \mathbb{R}^{n\times m}, F \in \mathbb{R}^{n\times n}$, and F can be time-varying, then for arbitrary $\delta > 0$,

$$FF^T \leq I \Rightarrow He(XFY) \leq \delta XX^T + \delta^{-1} Y^T Y. \tag{17}$$

Lemma 5. ([2]): For two arbitrary scalars λ, κ, and two functions $\phi(t)$, and $\vartheta(t)$ satisfying

$$\dot{\phi}(t) \leq -\lambda\phi(t) + \kappa\vartheta(t), \tag{18}$$

we have

$$\phi(t) \leq e^{-\lambda(t-t_0)}\phi(t_0) + \kappa \int_{t_0}^{t} e^{-\lambda(t-\nu)}\vartheta(\nu)d\nu. \tag{19}$$

This inequality is a special case of the comparison lemma for integrals.

Lemma 6. ([15]): (Finsler's lemma) If $\Psi \in \mathbb{R}^{n\times n}, Z \in \mathbb{R}^{p\times n}$, where $rank(Z) < n$, then the inequality

$$Z^{\perp T} \Psi Z^{\perp} \prec 0, \tag{20}$$

is satisfied, if and only if there exists $X \in \mathbb{R}^{n\times p}$ such that

$$\Psi + He(XZ) \prec 0. \tag{21}$$

3 The Main Results

As stated in the previous section, the problem of IFDRC design for switched linear systems with mode-dependent time-varying state delay and parameter uncertainty can be formulated as a multi-objective or mixed H_∞/H_- optimization problem. In this section, we will drive sufficient conditions for analyzing the stability of the augmented system as well as obtaining fault detection and robust control objectives. These conditions will be addressed in LMIs forms.

3.1 The weighted H_∞ performance problem

In the following theorem, based on Definition 2 and the weighted H_∞ performance index (α_M, γ_1) defined in (9), sufficient conditions for the exponential stability of the augmented system in the presence of parameter uncertainties and input disturbances are derived. These conditions are in the form of LMIs. Then, an estimate of the state decay ratio is calculated. In addition, the minimum allowable average time for each subsystem to be active is calculated to satisfy the weighted H_∞ performance index (α_M, γ_1). Finally, the IFDRCU gains are determined.

Theorem 1. For given scalars $\alpha_M > 0$, $\mu_i^M \geq 1$, assume that there exist positive definite matrices $P_i \succ 0$, $R_i \succ 0$, $S_i \succ 0$ and appropriately-dimensioned real matrices $\widehat{A}_{mi}$, $\widehat{B}_{mi}$, C_{mi}, D_{mi}, $\widehat{K}_{mi}$, $\widehat{L}_{mi}$, $G_i, H_i = H_i^T$, as well as constant scalars $\gamma_{10} > 0$ and $\delta_{1i} > 0$ such that the following inequalities hold:

$$P_i \leq \mu_i^M P_j, R_i \leq \mu_i^M R_j, S_i \leq \mu_i^M S_j, \qquad i, j \in \underline{l}, \tag{22}$$

$$\Omega_{Mi} = \begin{bmatrix} \Phi_{Mi} & \Lambda_{Mi} \\ * & -\delta_{1i} I \end{bmatrix} \prec 0, \tag{23}$$

$$\Sigma_{Mi} = \begin{bmatrix} H_i & G_i \\ * & e^{-\alpha_M d_i} S_i \end{bmatrix} \succ 0, \tag{24}$$

where

$$\Phi_{Mi} = \begin{bmatrix} \Phi_{Mi11} & \Phi_{Mi12} & \Phi_{Mi13} & \Phi_{Mi14} & \Phi_{Mi15} \\ * & \Phi_{Mi22} & 0 & 0 & \Phi_{Mi25} \\ * & 0 & \Phi_{Mi33} & \Phi_{Mi34} & \Phi_{Mi35} \\ * & 0 & * & -I & 0 \\ * & * & * & 0 & S_i - 2P_i \end{bmatrix}, \tag{25}$$

$$\Lambda_{Mi} = \begin{bmatrix} P_{i1}M_{i1} + T_i^T \begin{bmatrix} \widehat{L}_{mi} \\ 0 \end{bmatrix} M_{i2} & P_{i1}M_{i1} & 0 \\ \widehat{B}_{mi}\, M_{i2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ D_{mi}M_{i2} & 0 & 0 \\ \sqrt{d_i} P_{i1}M_{i1} & \sqrt{d_i} P_{i1}M_{i1} & \sqrt{d_i}\, T_i^T \begin{bmatrix} \widehat{L}_{mi} \\ 0 \end{bmatrix} M_{i2} \\ 0 & 0 & \sqrt{d_i}\, \widehat{B}_{mi}\, M_{i2} \end{bmatrix} \tag{26}$$

$$\begin{aligned} \Phi_{Mi11} &= He\left(\begin{bmatrix} P_{i1}A_i + T_i^T \begin{bmatrix} \widehat{L}_{mi} \\ 0 \end{bmatrix} C_i + \delta_{1i} N_{i1}^T N_{i1} & T_i^T \begin{bmatrix} \widehat{K}_{mi} \\ 0 \end{bmatrix} \\ \widehat{B}_{mi}\, C_i & \widehat{A}_{mi} \end{bmatrix}\right) \\ &\quad + \alpha_M P_i + R_i + d_i H_{i1} + G_{i1} + G_{i1}^T, \qquad (27) \\ \Phi_{Mi12} &= \begin{bmatrix} P_{i1}A_{di} & 0 \\ 0 & 0 \end{bmatrix} + d_i H_{i2} - G_{i1} + G_{i2}^T, \\ \Phi_{Mi13} &= \begin{bmatrix} P_{i1}B_{\omega i} + T_i^T \begin{bmatrix} \widehat{L}_{mi} \\ 0 \end{bmatrix} D_{\omega i} + 2\delta_{1i} N_{i1}^T N_{i3} \\ \widehat{B}_{mi}\, D_{\omega i} \end{bmatrix}, \\ \Phi_{Mi14} &= \begin{bmatrix} C_i^T D_{mi}^T \\ C_{mi}^T \end{bmatrix}, \\ \Phi_{Mi15} &= \sqrt{d_i} \begin{bmatrix} A_i^T P_{i1} + C_i^T \begin{bmatrix} \widehat{L}_{mi}^T & 0 \end{bmatrix} T_i & C_i^T\, \widehat{B}_{mi}^T \\ \begin{bmatrix} \widehat{K}_{mi}^T & 0 \end{bmatrix} T_i & \widehat{A}_{mi}^T \end{bmatrix}, \\ \Phi_{Mi22} &= -(1-\rho_i)e^{-\alpha_M d_i} R_i + d_i H_{i3} - G_{i2} - G_{i2}^T + \delta_{1i} \begin{bmatrix} N_{i2}^T N_{i2} & 0 \\ 0 & 0 \end{bmatrix}, \\ \Phi_{Mi25} &= \sqrt{d_i} \begin{bmatrix} A_{di}^T P_{i1} & 0 \\ 0 & 0 \end{bmatrix}, \\ \Phi_{Mi33} &= -\gamma_{10}^2 I + 2\delta_{1i} N_{i3}^T N_{i3}, \\ \Phi_{Mi34} &= D_{\omega i}^T D_{mi}^T, \\ \Phi_{Mi35} &= \sqrt{d_i} \begin{bmatrix} B_{\omega i}^T P_{i1} + D_{\omega i}^T \begin{bmatrix} \widehat{L}_{mi}^T & 0 \end{bmatrix} T_i & D_{\omega i}^T\, \widehat{B}_{mi}^T \end{bmatrix}. \end{aligned}$$

In these relations, M_{ij}, N_{ij} are defined in (5) and T_i's are defined in (3) and

$$G_i \triangleq \begin{bmatrix} G_{i1}^T & G_{i2}^T \end{bmatrix}^T, \tag{28}$$

$$H_i \triangleq \begin{bmatrix} H_{i1} & H_{i2} \\ * & H_{i3} \end{bmatrix}, \tag{29}$$

$$P_i = \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i2} \end{bmatrix}, \quad P_{i1} = T_i^T \begin{bmatrix} \widehat{P}_{i1} & 0 \\ 0 & \widehat{P}_{i2} \end{bmatrix} T_i. \tag{30}$$

Then the augmented system (8) is exponentially stable and satisfies the weighted H_∞ performance index (α_M, γ_1) in (9) for any switching signal with MDADT met by (31)

$$\tau_{ai}^M > \tau_{ai}^{M*} = \frac{\ln \mu_i^M}{\alpha_M}. \tag{31}$$

Finally, the IFDRCU matrices will be calculated as

$$A_{mi} = P_{i2}^{-1} \widehat{A}_{mi}, \qquad B_{mi} = P_{i2}^{-1} \widehat{B}_{mi}, \qquad K_{mi} = \widehat{P}_{i1}^{-1} \widehat{K}_{mi}, \qquad L_{mi} = \widehat{P}_{i1}^{-1} \widehat{L}_{mi}. \tag{32}$$

Proof. Construct the following Lyapunov-Krasovskii functional (LKF) candidate:

$$\begin{aligned} V(\varsigma_t, \sigma) &\triangleq V_1(\varsigma_t, \sigma) + V_2(\varsigma_t, \sigma) + V_3(\varsigma_t, \sigma), \\ V_1(\varsigma_t, \sigma) &\triangleq \varsigma^T(t) P_\sigma \varsigma(t), \\ V_2(\varsigma_t, \sigma) &\triangleq \int_{t-d_\sigma(t)}^{t} e^{\alpha_M(s-t)} \varsigma^T(s) R_\sigma \varsigma(s)\, ds, \\ V_3(\varsigma_t, \sigma) &\triangleq \int_{-d_\sigma}^{0} \int_{t+\theta}^{t} e^{\alpha_M(s-t)} \dot{\varsigma}^T(s) S_\sigma \dot{\varsigma}(s)\, ds\, d\theta, \end{aligned} \tag{33}$$

where real matrices $P_\sigma > 0, R_\sigma > 0$, and $S_\sigma > 0$ should be determined.

By calculating the derivative of LKF along with the solution of the augmented system and using the Leibniz integral rule for differentiation under the integral sign, we have:

$$\begin{aligned} \dot{V}(\varsigma_t, \sigma) + \alpha_M V(\varsigma_t, \sigma) = {} & 2\varsigma^T(t) P_\sigma \dot{\varsigma}(t) \\ & - (1 - \dot{d}_\sigma(t)) e^{-\alpha_M d_\sigma(t)} \varsigma^T(t - d_\sigma(t)) R_\sigma \varsigma(t - d_\sigma(t)) \\ & + \varsigma^T(t)(\alpha_M P_\sigma + R_\sigma)\varsigma(t) + d_\sigma \dot{\varsigma}^T(t) S_\sigma \dot{\varsigma}(t) \\ & - \int_{t-d_\sigma}^{t} e^{\alpha_M(s-t)} \dot{\varsigma}^T(s) S_\sigma \dot{\varsigma}(s)\, ds, \end{aligned} \tag{34}$$

and note that

$$- \int_{t-d_\sigma}^{t} e^{\alpha_M(s-t)} \dot{\varsigma}^T(s) S_\sigma \dot{\varsigma}(s)\, ds \le - \int_{t-d_\sigma(t)}^{t} e^{-\alpha_M d_\sigma} \dot{\varsigma}^T(s) S_\sigma \dot{\varsigma}(s)\, ds, \tag{35}$$

$$-(1 - \dot{d}_\sigma(t)) e^{-\alpha_M d_\sigma(t)} \le -(1 - \rho_\sigma) e^{-\alpha_M d_\sigma} \le -(1 - \rho) e^{-\alpha_M d}. \tag{36}$$

It is obvious that

$$\begin{aligned}\dot{V}(\varsigma_t,\sigma)+\alpha_M V(\varsigma_t,\sigma) \leq\ & 2\varsigma^T(t)P_\sigma\,\dot{\varsigma}(t)\\ &-(1-\rho_\sigma)e^{-\alpha_M d_\sigma}\,\varsigma^T(t-d_\sigma(t))R_\sigma\,\varsigma(t-d_\sigma(t))\\ &+\varsigma^T(t)(\alpha_M P_\sigma+R_\sigma)\varsigma(t)+d_\sigma\,\dot{\varsigma}^T(t)S_\sigma\,\dot{\varsigma}(t)\\ &-\int_{t-d_\sigma(t)}^{t} e^{-\alpha_M d_\sigma}\dot{\varsigma}^T(s)S_\sigma\,\dot{\varsigma}(s)\,ds. \end{aligned}\tag{37}$$

Regarding (9), we define $I_\infty(t) \triangleq r^T(t)r(t)-\gamma_{10}^2\omega^T(t)\omega(t)$, and

$$\begin{aligned}\dot{V}(\varsigma_t,\sigma)+\alpha_M V(\varsigma_t,\sigma)+\mathrm{I}_\infty(t) \leq\ & 2\varsigma^T(t)P_\sigma\,\dot{\varsigma}(t)\\ &-(1-\rho_\sigma)e^{-\alpha_M d_\sigma}\,\varsigma^T(t-d_\sigma(t))R_\sigma\,\varsigma(t-d_\sigma(t))\\ &+\varsigma^T(t)(\alpha_M P_\sigma+R_\sigma)\varsigma(t)+d_\sigma\dot{\varsigma}^T(t)S_\sigma\,\dot{\varsigma}(t)\\ &-\int_{t-d_\sigma(t)}^{t} e^{-\alpha_M d_\sigma}\dot{\varsigma}^T(s)S_\sigma\,\dot{\varsigma}(s)\,ds\\ &+r^T(t)r(t)-\gamma_{10}^2\omega^T(t)\omega(t). \end{aligned}\tag{38}$$

Substituting the derivative of the state vector from equation (8) for $f(t)=0$ we find that

$$\begin{aligned}\dot{V}(\varsigma_t,\sigma)+\alpha_M V(\varsigma_t,\sigma)+\mathrm{I}_\infty(t) \leq\ & \varsigma_1^T(t,\sigma)\Theta_\sigma(t)\,\varsigma_1(t,\sigma)\\ &-\int_{t-d_\sigma(t)}^{t} e^{-\alpha_M d_\sigma}\dot{\varsigma}^T(s)S_\sigma\,\dot{\varsigma}(s)\,ds, \end{aligned}\tag{39}$$

where

$$\begin{aligned} \varsigma_1(t,\sigma) &\triangleq \begin{bmatrix} \varsigma^T(t) & \varsigma^T(t-d_\sigma(t)) & \omega^T(t) \end{bmatrix}^T,\\ \Theta_\sigma(t) &= \begin{bmatrix} \Theta_{\sigma11}(t) & \Theta_{\sigma12}(t) & \Theta_{\sigma13}(t)\\ * & \Theta_{\sigma22}(t) & \Theta_{\sigma23}(t)\\ * & * & \Theta_{\sigma33}(t) \end{bmatrix},\\ \Theta_{\sigma11}(t) &= He(P_\sigma\bar{A}_\sigma(t))+\alpha_M P_\sigma+R_\sigma+d_\sigma\bar{A}_\sigma^T(t)S_\sigma\bar{A}_\sigma(t)+\bar{C}_\sigma^T(t)\bar{C}_\sigma(t),\\ \Theta_{\sigma12}(t) &= P_\sigma\bar{A}_{d\sigma}(t)+d_\sigma\bar{A}_\sigma^T(t)S_\sigma\bar{A}_{d\sigma}(t),\\ \Theta_{\sigma13}(t) &= P_\sigma\bar{B}_{\omega\sigma}(t)+d_\sigma\bar{A}_\sigma^T(t)S_\sigma\bar{B}_{\omega\sigma}(t)+\bar{C}_\sigma^T(t)\bar{D}_{\omega\sigma}(t),\\ \Theta_{\sigma22}(t) &= -(1-\rho_\sigma)e^{-\alpha_M d_\sigma}R_\sigma+d_\sigma\bar{A}_{d\sigma}^T(t)S_\sigma\bar{A}_{d\sigma}(t),\\ \Theta_{\sigma23}(t) &= d_\sigma\bar{A}_{d\sigma}^T(t)S_\sigma\bar{B}_{\omega\sigma}(t),\\ \Theta_{\sigma33}(t) &= d_\sigma\bar{B}_{\omega\sigma}^T(t)S_\sigma\bar{B}_{\omega\sigma}(t)+\bar{D}_{\omega\sigma}^T(t)\bar{D}_{\omega\sigma}(t)-\gamma_{10}^2 I. \end{aligned}\tag{40}$$

Defining $\varsigma_2(t,\sigma) \triangleq \begin{bmatrix}\varsigma^T(t) & \varsigma^T(t-d_\sigma(t))\end{bmatrix}^T$ and $H_\sigma \triangleq \begin{bmatrix} H_{\sigma1} & H_{\sigma2}\\ * & H_{\sigma3}\end{bmatrix}$, we obtain

$$\int_{t-d_\sigma(t)}^{t}\varsigma_2^T(t,\sigma)H_\sigma\,\varsigma_2(t,\sigma)\,ds \leq d_\sigma\varsigma_2^T(t,\sigma)H_\sigma\,\varsigma_2(t,\sigma).\tag{41}$$

By the Newton-Leibniz formula, for any arbitrary matrices $G_\sigma \triangleq \begin{bmatrix} G_{\sigma 1}^T & G_{\sigma 2}^T \end{bmatrix}^T$, we have

$$\varsigma_2^T(t,\sigma)G_\sigma \left[\varsigma(t) - \varsigma(t-d_\sigma(t)) - \int_{t-d_\sigma(t)}^{t} \dot{\varsigma}(s)\,ds\right] = 0. \tag{42}$$

Suppose that the Lyapunov matrix P_σ can be considered as a block-diagonal matrix such that in (30), by T_σ as defined in (3) we obtain

$$\begin{aligned}
&P_{\sigma 1}B_\sigma = T_\sigma^T \begin{bmatrix} \widehat{P}_{\sigma 1} & 0 \\ 0 & \widehat{P}_{\sigma 2} \end{bmatrix} T_\sigma B_\sigma = T_\sigma^T \begin{bmatrix} \widehat{P}_{\sigma 1} & 0 \\ 0 & \widehat{P}_{\sigma 2} \end{bmatrix}\begin{bmatrix} I \\ 0 \end{bmatrix} = T_\sigma^T \begin{bmatrix} \widehat{P}_{\sigma 1} \\ 0 \end{bmatrix}, \\
&P_{\sigma 1}B_\sigma L_{m\sigma} = T_\sigma^T \begin{bmatrix} \widehat{P}_{\sigma 1} \\ 0 \end{bmatrix} L_{m\sigma} = T_\sigma^T \begin{bmatrix} \widehat{P}_{\sigma 1} L_{m\sigma} \\ 0 \end{bmatrix} \triangleq T_\sigma^T \begin{bmatrix} \widehat{L}_{m\sigma} \\ 0 \end{bmatrix}, \\
&P_{\sigma 1}B_\sigma K_{m\sigma} = T_\sigma^T \begin{bmatrix} \widehat{P}_{\sigma 1} \\ 0 \end{bmatrix} K_{m\sigma} = T_\sigma^T \begin{bmatrix} \widehat{P}_{\sigma 1} K_{m\sigma} \\ 0 \end{bmatrix} \triangleq T_\sigma^T \begin{bmatrix} \widehat{K}_{m\sigma} \\ 0 \end{bmatrix}, \\
&\widehat{A}_{m\sigma} \triangleq P_{\sigma 2}A_{m\sigma}, \qquad \widehat{B}_{m\sigma} \triangleq P_{\sigma 2}B_{m\sigma}.
\end{aligned} \tag{43}$$

By combining (39), (41) and (42), we can write

$$\begin{aligned}
\dot{V}(\varsigma_t,\sigma) + \alpha_M V(\varsigma_t,\sigma) + I_\infty(t) \le{}& \varsigma_1^T(t,\sigma)\Pi_\sigma(t)\varsigma_1(t,\sigma) \\
&- \int_{t-d_\sigma(t)}^{t} \varsigma_3^T(t,s,\sigma)\Sigma_{M\sigma}\,\varsigma_3(t,s,\sigma)\,ds,
\end{aligned} \tag{44}$$

where $\Sigma_{M\sigma}$ is defined in (24) and

$$\varsigma_3(t,s,\sigma) \triangleq \begin{bmatrix} \varsigma^T(t) & \varsigma^T(t-d_\sigma(t)) & \dot{\varsigma}^T(s) \end{bmatrix}^T, \tag{45}$$

$$\Pi_\sigma(t) \triangleq \begin{bmatrix} \Pi_{\sigma 11}(t) & \Pi_{\sigma 12}(t) & \Pi_{\sigma 13}(t) \\ * & \Pi_{\sigma 22}(t) & \Pi_{\sigma 23}(t) \\ * & * & \Pi_{\sigma 33}(t) \end{bmatrix}, \tag{46}$$

$$\begin{aligned}
\Pi_{\sigma 11}(t) ={}& He\left(\begin{bmatrix} P_{\sigma 1}A_\sigma(t) + T_\sigma^T \begin{bmatrix} \widehat{L}_{m\sigma} \\ 0 \end{bmatrix} C_\sigma(t) & T_\sigma^T \begin{bmatrix} \widehat{K}_{m\sigma} \\ 0 \end{bmatrix} \\ \widehat{B}_{m\sigma}\, C_\sigma(t) & \widehat{A}_{m\sigma} \end{bmatrix}\right) \\
&+ \alpha_M P_\sigma + R_\sigma + d_\sigma H_{\sigma 1} + G_{\sigma 1} + G_{\sigma 1}^T + d_\sigma \bar{A}_\sigma^T(t)S_\sigma \bar{A}_\sigma(t) + \bar{C}_\sigma^T(t)\bar{C}_\sigma(t), \\
\Pi_{\sigma 12}(t) ={}& \begin{bmatrix} P_{\sigma 1}A_{d\sigma}(t) & 0 \\ 0 & 0 \end{bmatrix} + d_\sigma \bar{A}_\sigma^T(t)S_\sigma \bar{A}_{d\sigma}(t) + d_\sigma H_{\sigma 2} - G_{\sigma 1} + G_{\sigma 2}^T, \\
\Pi_{\sigma 13}(t) ={}& \begin{bmatrix} P_{\sigma 1}B_{\omega\sigma}(t) + T_\sigma^T \begin{bmatrix} \widehat{L}_{m\sigma} \\ 0 \end{bmatrix} D_{\omega\sigma}(t) \\ \widehat{B}_{m\sigma}\, D_{\omega\sigma}(t) \end{bmatrix} + d_\sigma \bar{A}_\sigma^T(t)S_\sigma \bar{B}_{\omega\sigma}(t) + \bar{C}_\sigma^T(t)\bar{D}_{\omega\sigma}(t),
\end{aligned}$$

$$
\begin{aligned}
&\Pi_{\sigma 22}(t) = -(1-\rho_\sigma)e^{-\alpha_M d_\sigma} R_\sigma + d_\sigma \bar{A}_{d\sigma}^T(t) S_\sigma \bar{A}_{d\sigma}(t) + d_\sigma H_{\sigma 3} - G_{\sigma 2} - G_{\sigma 2}^T,\\
&\Pi_{\sigma 23}(t) = d_\sigma \bar{A}_{d\sigma}^T(t) S_\sigma \bar{B}_{\omega\sigma}(t),\\
&\Pi_{\sigma 33}(t) = d_\sigma \bar{B}_{\omega\sigma}^T(t) S_\sigma \bar{B}_{\omega\sigma}(t) + \bar{D}_{\omega\sigma}^T(t)\bar{D}_{\omega\sigma}(t) - \gamma_{10}^2 I.
\end{aligned} \tag{47}
$$

From (44), it is clear that $\dot{V}(\varsigma_t,\sigma)+\alpha_M V(\varsigma_t,\sigma)+\mathrm{I}_\infty(t) \le 0$ if $\Pi_\sigma(t) \prec 0$ and $\Sigma_{M\sigma} \succ 0$. By applying the Schur complement lemma 1, i.e., (14) to the inequality $\Pi_\sigma(t) \prec 0$, and using Lemmas 2, 28 with $\Lambda = (0,0,0,2P_\sigma - S_\sigma - P_\sigma S_\sigma^{-1} P_\sigma)$, this inequality can be substituted by $\Xi_\sigma(t) \prec 0$. Then, by considering uncertainties in system parameters defined in (4), which cause system matrices to be time-dependent, we can separate $\Xi_\sigma(t)$ to

$$
\Xi_\sigma(t) = \Xi_\sigma + \Delta\Xi_\sigma(t) \prec 0, \tag{48}
$$

where

$$
\Xi_\sigma \triangleq \begin{bmatrix} \Xi_{\sigma 11} & \Xi_{\sigma 12} & \Xi_{\sigma 13} & \Xi_{\sigma 14} & \Xi_{\sigma 15} \\ * & \Xi_{\sigma 22} & 0 & 0 & \Xi_{\sigma 25} \\ * & 0 & -\gamma_{10}^2 I & \Xi_{\sigma 34} & \Xi_{\sigma 35} \\ * & 0 & * & -I & 0 \\ * & * & * & 0 & S_\sigma - 2P_\sigma \end{bmatrix},
$$

$$
\begin{aligned}
\Xi_{\sigma 11} &= He\left(\begin{bmatrix} P_{\sigma 1} A_\sigma + T_\sigma^T \begin{bmatrix} \widehat{L}_{m\sigma} \\ 0 \end{bmatrix} C_\sigma & T_\sigma^T \begin{bmatrix} \widehat{K}_{m\sigma} \\ 0 \end{bmatrix} \\ \widehat{B}_{m\sigma}\, C_\sigma & \widehat{A}_{m\sigma} \end{bmatrix}\right)\\
&\quad + \alpha_M P_\sigma + R_\sigma + d_\sigma H_{\sigma 1} + G_{\sigma 1} + G_{\sigma 1}^T,\\
\Xi_{\sigma 12} &= \begin{bmatrix} P_{\sigma 1} A_{d\sigma} & 0 \\ 0 & 0 \end{bmatrix} + d_\sigma H_{\sigma 2} - G_{\sigma 1} + G_{\sigma 2}^T,\\
\Xi_{\sigma 13} &= \begin{bmatrix} P_{\sigma 1} B_{\omega\sigma} + T_\sigma^T \begin{bmatrix} \widehat{L}_{m\sigma} \\ 0 \end{bmatrix} D_{\omega\sigma} \\ \widehat{B}_{m\sigma}\, D_{\omega\sigma} \end{bmatrix},\\
\Xi_{\sigma 14} &= \begin{bmatrix} C_\sigma^T D_{m\sigma}^T \\ C_{m\sigma}^T \end{bmatrix},\\
\Xi_{\sigma 15} &= \sqrt{d_\sigma} \begin{bmatrix} A_\sigma^T P_{\sigma 1} + C_\sigma^T \begin{bmatrix} \widehat{L}_{m\sigma}^T & 0 \end{bmatrix} T_\sigma & C_\sigma^T\, \widehat{B}_{m\sigma}^T \\ \begin{bmatrix} \widehat{K}_{m\sigma}^T & 0 \end{bmatrix} T_\sigma & \widehat{A}_{m\sigma}^T \end{bmatrix},\\
\Xi_{\sigma 22} &= -(1-\rho_\sigma)e^{-\alpha_M d_\sigma} R_\sigma + dH_{\sigma 3} - G_{\sigma 2} - G_{\sigma 2}^T,\\
\Xi_{\sigma 25} &= \sqrt{d_\sigma} \begin{bmatrix} A_{d\sigma}^T P_{\sigma 1} & 0 \\ 0 & 0 \end{bmatrix},\\
\Xi_{\sigma 34} &= D_{\omega\sigma}^T D_{m\sigma}^T,
\end{aligned}
$$

$$\Xi_{\sigma 35} = \sqrt{d_\sigma}\begin{bmatrix} B_{\omega\sigma}^T P_{\sigma 1} + D_{\omega\sigma}^T \begin{bmatrix} \widehat{L}_{m\sigma}^T & 0 \end{bmatrix} T_\sigma & D_{\omega\sigma}^T \widehat{B}_{m\sigma}^T \end{bmatrix},$$

$$\Delta\Xi_{\sigma 11}(t) = He(\begin{bmatrix} P_{\sigma 1}\Delta A_\sigma(t) + T_\sigma^T \begin{bmatrix} \widehat{L}_{m\sigma} \\ 0 \end{bmatrix} \Delta C_\sigma(t) & 0 \\ \widehat{B}_{m\sigma}\,\Delta C_\sigma(t) & 0 \end{bmatrix}),$$

$$\Delta\Xi_{\sigma 12}(t) = \begin{bmatrix} P_{\sigma 1}\Delta A_{d\sigma}(t) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Delta\Xi_{\sigma 13}(t) = \begin{bmatrix} P_{\sigma 1}\Delta B_{\omega\sigma}(t) + T_\sigma^T \begin{bmatrix} \widehat{L}_{m\sigma} \\ 0 \end{bmatrix} \Delta D_{\omega\sigma}(t) \\ \widehat{B}_{m\sigma}\,\Delta D_{\omega\sigma}(t) \end{bmatrix},$$

$$\Delta\Xi_{\sigma 14}(t) = \begin{bmatrix} \Delta C_\sigma^T(t) D_{m\sigma}^T \\ 0 \end{bmatrix},$$

$$\Delta\Xi_{\sigma 15}(t) = \sqrt{d_\sigma}\begin{bmatrix} \Delta A_\sigma^T(t) P_{\sigma 1} + \Delta C_\sigma^T(t) \begin{bmatrix} \widehat{L}_{m\sigma}^T & 0 \end{bmatrix} T_\sigma & \Delta C_\sigma^T(t)\, \widehat{B}_{m\sigma}^T \\ 0 & 0 \end{bmatrix},$$

$$\Delta\Xi_{\sigma 25}(t) = \sqrt{d_\sigma}\begin{bmatrix} \Delta A_{d\sigma}^T(t) P_{\sigma 1} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Delta\Xi_{\sigma 34}(t) = \Delta D_{\omega\sigma}^T(t) D_{m\sigma}^T,$$

$$\Delta\Xi_{\sigma 35}(t) = \sqrt{d_\sigma}\begin{bmatrix} \Delta B_{\omega\sigma}^T(t) P_{\sigma 1} + \Delta D_{\omega\sigma}^T(t) \begin{bmatrix} \widehat{L}_{m\sigma}^T & 0 \end{bmatrix} T_\sigma & \Delta D_{\omega\sigma}^T(t)\, \widehat{B}_{m\sigma}^T \end{bmatrix}.$$

Referring to Assumption 2, we get

$$\Delta\Xi_\sigma(t) = He(\Lambda_{M\sigma}(Q(t), Q(t), Q(t))\Gamma_{M\sigma}), \tag{49}$$

where $\Lambda_{M\sigma}$ is defined in (26), and

$$\Gamma_{M\sigma} \triangleq \begin{bmatrix} N_{\sigma 1} & 0 & 0 & 0 & N_{\sigma 3} & 0 & 0 & 0 \\ 0 & 0 & N_{\sigma 2} & 0 & 0 & 0 & 0 & 0 \\ N_{\sigma 1} & 0 & 0 & 0 & N_{\sigma 3} & 0 & 0 & 0 \end{bmatrix}, \tag{50}$$

and by using the generalized square inequality in Lemma 4, that is (17), we get

$$\Delta\Xi_\sigma(t) \le \delta_{1\sigma}^{-1}\Lambda_{M\sigma}\Lambda_{M\sigma}^T + \delta_{1\sigma}\Gamma_{M\sigma}^T\Gamma_{M\sigma}. \tag{51}$$

Now according to (51), inequality (48) can be rearranged to

$$(\Xi_\sigma + \delta_{1\sigma}\Gamma_{M\sigma}^T\Gamma_{M\sigma}) + \delta_{1\sigma}^{-1}\Lambda_{M\sigma}\Lambda_{M\sigma}^T \le 0, \tag{52}$$

and by the new variable $\Phi_{M\sigma} \triangleq \Xi_\sigma + \delta_{1\sigma}\Gamma_{M\sigma}^T\Gamma_{M\sigma}$, we have

$$\Phi_{M\sigma} + \delta_{1\sigma}^{-1}\Lambda_{M\sigma}\Lambda_{M\sigma}^T \le 0, \tag{53}$$

where $\Phi_{M\sigma}$ is defined in (25). Finally, by using the Schur complement in Lemma 1, that is (14), inequality (53) turns to (23).

Also, from (43), it is apparent that we can calculate IFDRCU parameters and get (32).

At this point, we will prove the exponential stability of the augmented system (8) with $\omega(t) = 0, f(t) = 0$ and without parameter uncertainties. If (23) and (24) are held, then from (44), we have

$$\dot{V}(\varsigma_t, \sigma) < -\alpha_M V(\varsigma_t, \sigma) - r^T(t) r(t) < -\alpha_M V(\varsigma_t, \sigma). \tag{54}$$

Using Lemma 5, and by integrating (54) from t_k to t we get:

$$V(\varsigma_{t_k}, \sigma) \leq e^{-\alpha_M (t-t_k)} V(\varsigma_{t_k}, \sigma(t_k)), \tag{55}$$

where t_k is the switching time instant. Using (22) at instant t_k, we have

$$V(\varsigma_{t_k}, \sigma(t_k)) \leq \mu_{t_k}^M V(\varsigma_{t_k^-}, \sigma(t_k^-)). \tag{56}$$

It follows from (55), (56), and (12) that

$$\begin{aligned} V(\varsigma_t, \sigma) &\leq \mu_{t_k}^M e^{-\alpha_M (t-t_k)} V(\varsigma_{t_k^-}, \sigma(t_k^-)) \leq \cdots \\ &\leq \prod_{j=1}^{N_\sigma(t_0,t)} \mu_{\sigma(t_j)}^M e^{-\alpha_M (t-t_0)} V(\varsigma_{t_0}, \sigma(t_0)) \\ &\leq e^{\sum_{p=1}^{l} N_{0p} \ln \mu_p^M} e^{\max\limits_{p \in \underline{l}} (\frac{\ln \mu_p^M}{\tau_{ap}^M} - \alpha_M)(t-t_0)} V(\varsigma_{t_0}, \sigma(t_0)). \end{aligned} \tag{57}$$

On the other hand, using Rayleigh's inequality [22], one can easily find from (33) that

$$a \|\varsigma(t)\|^2 \leq V(\varsigma_t, \sigma) \leq b \|\varsigma(t)\|^2, \tag{58}$$

where

$$\begin{aligned} a &= \min\{\lambda_{\min}(P_\sigma) \ \ |\sigma \in \underline{l}\}, \\ b &= \max\{\lambda_{\max}(P_\sigma) \ \ |\sigma \in \underline{l}\} + d.\max\{\lambda_{\max}(R_\sigma) \, |\sigma \in \underline{l}\}, \\ &\quad + \frac{d^2}{2} \max\{\lambda_{\max}(S_\sigma) \, |\sigma \in \underline{l}\}. \end{aligned}$$

Notice from (58) that

$$V(\varsigma_t, \sigma) \geq a \|\varsigma(t)\|^2,$$

$$V(\varsigma_{t_0},\sigma(t_0)) \leq b\|\varsigma(t_0)\|^2. \tag{59}$$

Combining (57) and (59) results in

$$\|\varsigma(t)\|^2 \leq \frac{b}{a} e^{\sum_{p=1}^{l} N_{0p}\ln\mu_p^M} e^{\max_{p\in\underline{l}}(\frac{\ln\mu_p^M}{\tau_{ap}^M}-\alpha_M)(t-t_0)}\|\varsigma(t_0)\|^2, \tag{60}$$

$$\|\varsigma(t)\| \leq \sqrt{\frac{b}{a} e^{\sum_{p=1}^{l} N_{0p}\ln\mu_p^M}}\, e^{-\frac{1}{2}\max_{p\in\underline{l}}(\alpha_M-\frac{\ln\mu_p^M}{\tau_{ap}^M})(t-t_0)}\|\varsigma(t_0)\|. \tag{61}$$

This means that the switched system (8) is exponentially stable with the estimated state decay ratio given by (61).

Remark 5. For $\mu_i^M = 1$ in $\tau_{ai}^M > \tau_{ai}^{M*} = \frac{\ln\mu_i^M}{\alpha_M}$ we have $\tau_a > \tau_a^* = 0$ which means that the switching signal is arbitrary, and the only possible case for (22) is the equality instead of inequality which imposes a common Lyapunov function for all subsystems.

Now, we will establish the weighted H_∞ performance (α_M,γ_1) for the augmented system without fault and parameter uncertainties. If (23) and (24) are held, from (44), we have

$$\dot{V}(\varsigma_t,\sigma) < -\alpha_M V(\varsigma_t,\sigma) - I_\infty(t). \tag{62}$$

For any $t>0$ and for any arbitrary piecewise constant switching signal $\sigma(t)$, we let $t_0 = 0 < t_1 < t_2 < \cdots < t_k < \cdots < t_{N_\sigma(0,t)}$ denote the switching points of the $\sigma(t)$ over the interval $[0,t]$, where $N_\sigma(0,t) = \sum_{k=1}^{l} N_k(0,t)$. For any $t\in[t_k,t_{k+1})$, the $\sigma(t_k)$th subsystem is active. Using Lemma 5, by integrating (62) from t_k to t, it follows from (55), (56) and (12) that

$$\begin{aligned} V(\varsigma_t,\sigma) &\leq e^{-\alpha_M(t-t_k)} V(\varsigma_{t_k},\sigma(t_k)) - \int_{t_k}^{t} e^{-\alpha_M(t-\nu)} \mathrm{I}_\infty(\nu)\,d\nu \\ &= \prod_{p=1}^{l} \mu_p^{N_{\sigma p}(t_0,t)} e^{-\alpha_M(t-t_0)}\, V(\varsigma_{t_0},\sigma(t_0)) \\ &\quad - \int_{t_0}^{t}\prod_{p=1}^{l}\mu_p^{N_{\sigma p}(\nu,t)} e^{-\alpha_M(t-\nu)} I_\infty(\nu)\,d\nu. \end{aligned} \tag{63}$$

Notice that for the time between two consequence switching instants, we have from (12):

$$\forall\, t_{j-1} < \nu < t_j \Rightarrow N_{\sigma p}(\nu,t) \leq N_{0p} + \frac{T_p(\nu,t)}{\tau_{ap}} = N_{0p} + \frac{T_p(t_{j-1},t)}{\tau_{ap}} = N_{\sigma p}(t_{j-1},t). \tag{64}$$

Since $V(\varsigma_t,\sigma)$ is positive, for zero initial condition, (63) results in

$$\int_{t_0}^{t} e^{-\alpha_M(t-\nu)+\sum_{p=1}^{l} N_{\sigma p}(\nu,t)\ln\mu_p^M} r^T(\nu)r(\nu)\,d\nu$$

$$\leq \gamma_{10}^2 \int_{t_0}^{t} e^{-\alpha_M (t-\nu)+\sum_{p=1}^{l} N_{\sigma p}(\nu,t)\ln \mu_p^M} \omega^T(\nu)\omega(\nu)\,d\nu. \tag{65}$$

Multiplying the both sides of (65) by $e^{-\sum_{p=1}^{l} N_{\sigma p}(0,t)\ln \mu_p^M}$ yields:

$$\int_{t_0}^{t} e^{-\alpha_M (t-\nu)+(\sum_{p=1}^{l}(N_{\sigma p}(\nu,t)-N_{\sigma p}(0,t))\ln \mu_p^M)} r^T(\nu)r(\nu)\,d\nu$$
$$\leq \gamma_{10}^2 \int_{t_0}^{t} e^{-\alpha_M (t-\nu)-\sum_{p=1}^{l} N_{\sigma p}(0,\nu)\ln \mu_p^M} \omega^T(\nu)\omega(\nu)\,d\nu. \tag{66}$$

From (12) and (31), we know that

$$-\sum_{p=1}^{l} N_{\sigma p}(0,\nu)\ln \mu_p^M \geq -\alpha_M \nu - \alpha_M \sum_{p=1}^{l} \tau_{ap}^M N_{0p}. \tag{67}$$

Therefore

$$\int_{t_0}^{t} e^{-\alpha_M t} r^T(\nu)r(\nu)\,d\nu \leq e^{\alpha_M \sum_{p=1}^{l} \tau_{ap}^M N_{0p}} \gamma_{10}^2 \int_{t_0}^{t} e^{-\alpha_M (t-\nu)} \omega^T(\nu)\omega(\nu)\,d\nu. \tag{68}$$

And we get

$$\int_{t_0}^{t} e^{-\alpha_M t} r^T(\nu)r(\nu)\,d\nu \leq \gamma_1^2 \int_{t_0}^{t} e^{-\alpha_M (t-\nu)} \omega^T(\nu)\omega(\nu)\,d\nu. \tag{69}$$

Integrating the both sides of (69) from t_0 to ∞ will result in

$$\int_{t_0}^{\infty} e^{-\alpha_M \nu} r^T(\nu)r(\nu)\,d\nu \leq \gamma_1^2 \int_{t_0}^{\infty} \omega^T(\nu)\omega(\nu)\,d\nu. \tag{70}$$

This means that the switched system (8) satisfies the weighted H_∞ performance (α_M, γ_1) with $\gamma_1 = \gamma_{10} \exp\left(0.5\,\alpha_M \sum_{p=1}^{l} \tau_{ap}^M N_{0p}\right)$ in (9). This completes the proof. □

3.2 The weighted H_- performance problem

In the following theorem, given the IFDRCU gain matrices and based on the weighted H_- performance index (α_m, γ_2) defined in (10), sufficient conditions in the form of matrix inequalities are derived for the exponential stability of the augmented system in the presence of parameter uncertainties and input disturbances. In addition, the minimum allowable average time per each subsystem activity is calculated to satisfy the weighted H_- performance index (α_m, γ_2).

Theorem 2. For given scalars $\alpha_m > 0, \mu_i^m \geq 1$ and appropriately-dimensioned real matrices $\widehat{A}_{mi}, \widehat{B}_{mi}, C_{mi}, D_{mi}, \widehat{K}_{mi}, \widehat{L}_{mi}$, if there exist positive definite matrices $P_i \succ 0$, $R_i \succ 0, S_i \succ 0$, appropriately-dimensioned real matrices $G_i, H_i = H_i^T, Y_{ki}$ $(i = 1,2,3)$, and constant scalars $\gamma_{20} > 0, \delta_{2i} > 0$ such that the following inequalities hold, then we have

$$P_i \leq \mu_i^m P_j, R_i \leq \mu_i^m R_j, S_i \leq \mu_i^m S_j \qquad i,j \in \underline{l}, \tag{71}$$

$$\Omega_{mi} = \begin{bmatrix} \Phi_{mi} & \Lambda_{mi} \\ * & -\delta_{2i} I \end{bmatrix} \prec 0, \tag{72}$$

$$\Sigma_{mi} = \begin{bmatrix} H_i & G_i \\ * & e^{-\alpha_m d_i} S_i \end{bmatrix} \succ 0, \tag{73}$$

where

$$\Phi_{mi} = \begin{bmatrix} \Phi_{mi11} & \Phi_{mi12} & \Phi_{mi13} & \Phi_{mi14} & \Phi_{mi15} \\ * & \Phi_{mi22} & 0 & 0 & \Phi_{mi25} \\ * & 0 & \Phi_{mi33} & \Phi_{mi34} & \Phi_{mi35} \\ * & 0 & * & -3I & 0 \\ * & * & * & 0 & S_i - 2P_i \end{bmatrix}, \tag{74}$$

$$\Lambda_{mi} = \begin{bmatrix} P_{i1} M_{i1} + (T_i^T \begin{bmatrix} \widehat{L}_{mi} \\ 0 \end{bmatrix} - Y_{1i}^T D_{mi}) M_{i2} & P_{i1} M_{i1} & 0 \\ (\widehat{B}_{mi} - Y_{2i}^T D_{mi}) M_{i2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -Y_{3i}^T D_{mi} M_{i2} & 0 & 0 \\ D_{mi} M_{i2} & 0 & 0 \\ \sqrt{d_i} P_{i1} M_{i1} & \sqrt{d_i} P_{i1} M_{i1} & \sqrt{d_i} T_i^T \begin{bmatrix} \widehat{L}_{mi} \\ 0 \end{bmatrix} M_{i2} \\ 0 & 0 & \sqrt{d_i}\, \widehat{B}_{mi} M_{i2} \end{bmatrix}, \tag{75}$$

$$\Phi_{mi11} = He\left(\begin{bmatrix} P_{i1} A_i + (T_i^T \begin{bmatrix} \widehat{L}_{mi} \\ 0 \end{bmatrix} - Y_{1i}^T D_{mi}) C_i + \delta_{2i} N_{i1}^T N_{i1} & T_i^T \begin{bmatrix} \widehat{K}_{mi} \\ 0 \end{bmatrix} - Y_{1i}^T C_{mi} \\ (\widehat{B}_{mi} - Y_{2i}^T D_{mi}) C_i & \widehat{A}_{mi} - Y_{2i}^T C_{mi} \end{bmatrix}\right),$$

$$+ \alpha_m P_i + R_i + d_i H_{i1} + G_{i1} + G_{i1}^T,$$

$$\Phi_{mi12} = \begin{bmatrix} P_{i1} A_{di} & 0 \\ 0 & 0 \end{bmatrix} + d_i H_{i2} - G_{i1} + G_{i2}^T,$$

$$
\begin{aligned}
\Phi_{mi13} &= \begin{bmatrix} P_{i1}B_{fi} + (T_i^T \begin{bmatrix} \widehat{L}_{mi} \\ 0 \end{bmatrix} - Y_{1i}^T D_{mi})D_{fi} - C_i^T D_{mi}^T Y_{3i} + 2\delta_{2i} N_{i1}^T N_{i4} \\ (\widehat{B}_{mi} - Y_{2i}^T D_{mi})D_{fi} - C_{mi}^T Y_{3i} \end{bmatrix},\\
\Phi_{mi14} &= \begin{bmatrix} Y_{1i}^T + C_i^T D_{mi}^T \\ Y_{2i}^T + C_{mi}^T \end{bmatrix},\\
\Phi_{mi15} &= \sqrt{d_i}\begin{bmatrix} A_i^T P_{i1} + C_i^T \begin{bmatrix} \widehat{L}_{mi}^T & 0 \end{bmatrix} T_i & C_i^T \widehat{B}_{mi}^T \\ \begin{bmatrix} \widehat{K}_{mi}^T & 0 \end{bmatrix} T_i & \widehat{A}_{mi}^T \end{bmatrix},\\
\Phi_{mi22} &= -(1-\rho_i)e^{-\alpha_m d_i} R_i + d_i H_{i3} - G_{i2} - G_{i2}^T + \delta_{2i}\begin{bmatrix} N_{i2}^T N_{i2} & 0 \\ 0 & 0 \end{bmatrix},\\
\Phi_{mi25} &= \sqrt{d_i}\begin{bmatrix} A_{di}^T P_{i1} & 0 \\ 0 & 0 \end{bmatrix},\\
\Phi_{mi33} &= \gamma_{20}^2 I - He(Y_{3i}^T D_{mi} D_{fi}) + 2\delta_{2i} N_{i4}^T N_{i4},\\
\Phi_{mi34} &= D_{fi}^T D_{mi}^T + Y_{3i}^T,\\
\Phi_{mi35} &= \sqrt{d_i}\begin{bmatrix} B_{fi}^T P_{i1} + D_{fi}^T \begin{bmatrix} \widehat{L}_{mi}^T & 0 \end{bmatrix} T_i & D_{fi}^T \widehat{B}_{mi}^T \end{bmatrix},
\end{aligned} \tag{76}
$$

where M_{ij}, N_{ij} are defined in (5), T_i is defined in (3), and

$$
G_i \triangleq \begin{bmatrix} G_{i1}^T & G_{i2}^T \end{bmatrix}^T, \tag{77}
$$

$$
H_i \triangleq \begin{bmatrix} H_{i1} & H_{i2} \\ * & H_{i3} \end{bmatrix}, \tag{78}
$$

$$
P_i = \begin{bmatrix} P_{i1} & 0 \\ 0 & P_{i2} \end{bmatrix}, \qquad P_{i1} = T_i^T \begin{bmatrix} \widehat{P}_{i1} & 0 \\ 0 & \widehat{P}_{i2} \end{bmatrix} T_i, \tag{79}
$$

$$
\widehat{A}_{mi} = P_{i2}A_{mi}, \qquad \widehat{B}_{mi} = P_{i2}B_{mi}, \qquad \widehat{K}_{mi} = \widehat{P}_{i1} K_{mi}, \qquad \widehat{L}_{mi} = \widehat{P}_{i1} L_{mi}. \tag{80}
$$

Then the augmented system (8) is exponentially stable and satisfies the weighted H_- performance index (α_m, γ_2) in (10) for any switching signal with MDADT met by (81)

$$
\tau_{ai}^m > \tau_{ai}^{m*} = \frac{\ln \mu_i^m}{\alpha_m}. \tag{81}
$$

Proof. By defining $I_-(t) \triangleq \gamma_{20}^2 f^T(t)f(t) - r^T(t)r(t)$, this theorem can be proved by employing similar techniques as in the proof of Theorem 1.

For $\omega(t) = 0$, the inequality $\dot{V}(\varsigma_t, \sigma) + \alpha_m V(\varsigma_t, \sigma) + I_-(t) \le 0$ holds if both (73) and the inequality (82) hold.

$$\begin{bmatrix} \Psi_{\sigma 11}(t) & \Psi_{\sigma 12}(t) & \Psi_{\sigma 13}(t) & \Psi_{\sigma 14}(t) \\ * & \Psi_{\sigma 22}(t) & 0 & \Psi_{\sigma 24}(t) \\ * & 0 & \Psi_{\sigma 33}(t) & \Psi_{\sigma 34}(t) \\ * & * & * & -P_\sigma S_\sigma^{-1} P_\sigma \end{bmatrix} - \begin{bmatrix} \bar{C}_\sigma^T(t)\bar{C}_\sigma(t) & 0 & \bar{C}_\sigma^T(t)\bar{D}_{f\sigma}(t) & 0 \\ 0 & 0 & 0 & 0 \\ * & 0 & \bar{D}_{f\sigma}^T(t)\bar{D}_{f\sigma}(t) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \prec 0. \quad (82)$$

Applying Lemmas 2 and 28, (82) can be rewritten as

$$\begin{bmatrix} \Psi_{\sigma 11}(t) & \Psi_{\sigma 12}(t) & \Psi_{\sigma 13}(t) & \Psi_{\sigma 14}(t) \\ * & \Psi_{\sigma 22}(t) & 0 & \Psi_{\sigma 24}(t) \\ * & 0 & \Psi_{\sigma 33}(t) & \Psi_{\sigma 34}(t) \\ * & * & * & S_\sigma - 2P_\sigma \end{bmatrix} - \begin{bmatrix} \bar{C}_\sigma^T(t) \\ 0 \\ \bar{D}_{f\sigma}^T(t) \\ 0 \end{bmatrix} \begin{bmatrix} \bar{C}_\sigma(t) & 0 & \bar{D}_{f\sigma}(t) & 0 \end{bmatrix} \prec 0. \quad (83)$$

This is apparently equal to

$$E_\sigma^{\perp T} \Delta_\sigma(t) E_\sigma^{\perp} \prec 0, \quad (84)$$

with

$$E_\sigma^{\perp} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ \bar{C}_\sigma(t) & 0 & \bar{D}_{f\sigma}(t) & 0 \\ 0 & 0 & 0 & I \end{bmatrix},$$

$$\Delta_\sigma(t) \triangleq \begin{bmatrix} \Psi_{\sigma 11}(t) & \Psi_{\sigma 12}(t) & \Psi_{\sigma 13}(t) & 0 & \Psi_{\sigma 14}(t) \\ * & \Psi_{\sigma 22}(t) & 0 & 0 & \Psi_{\sigma 24}(t) \\ * & 0 & \Psi_{\sigma 33}(t) & 0 & \Psi_{\sigma 34}(t) \\ 0 & 0 & 0 & -I & 0 \\ * & * & * & 0 & S_\sigma - 2P_\sigma \end{bmatrix}. \quad (85)$$

Then, if we choose $E_\sigma = \begin{bmatrix} -\bar{C}_\sigma(t) & 0 & -\bar{D}_{f\sigma}(t) & I & 0 \end{bmatrix}$ as one annihilator for $E_\sigma^{\perp}$ and an arbitrary matrix $Y_\sigma = \begin{bmatrix} \begin{bmatrix} Y_{1\sigma} & Y_{2\sigma} \end{bmatrix} & 0 & Y_{3\sigma} & -I & 0 \end{bmatrix}^T$, using the similar technique as in [40, 32], and by Finsler's Lemma 6, we deduce that (84) holds if

$$T_\sigma(t) = \Delta_\sigma(t) + He(Y_\sigma E_\sigma) \prec 0, \quad (86)$$

in which $Y_{i\sigma}$ $(i = 1, 2, 3)$ are some arbitrary tuning matrices.

Remark 6. Since a particular structure is chosen for the tuning matrix in Finsler's lemma in (86), it becomes a sufficient condition for satisfying (84). A more general form for the tuning matrix can also be chosen, but the complexity of the resulting LMIs will be increased.

Therefore, it is evident that $\dot{V}(\varsigma_t,\sigma)+\alpha_m V(\varsigma_t,\sigma)+I_{-}(t)\leq 0$ holds, if both (73) and the following inequality hold.

$$T_\sigma(t)=\begin{bmatrix} \Psi_{\sigma 11}(t)-He(\begin{bmatrix} Y_{1\sigma} & Y_{2\sigma}\end{bmatrix}^T\bar{C}_\sigma(t)) & \Psi_{\sigma 12}(t) & \Psi_{\sigma 13}(t)-\begin{bmatrix} Y_{1\sigma} & Y_{2\sigma}\end{bmatrix}^T\bar{D}_{f\sigma}(t)-\bar{C}_\sigma^T(t)Y_{3\sigma} & \begin{bmatrix} Y_{1\sigma} & Y_{2\sigma}\end{bmatrix}^T+\bar{C}_\sigma^T(t) & \Psi_{\sigma 14}(t) \\ * & \Psi_{\sigma 22}(t) & 0 & 0 & \Psi_{\sigma 24}(t) \\ * & 0 & \Psi_{\sigma 33}(t)-He(Y_{3\sigma}^T\bar{D}_{f\sigma}(t)) & \bar{D}_{f\sigma}^T(t)+Y_{3\sigma}^T & \Psi_{\sigma 34}(t) \\ * & 0 & * & -3I & 0 \\ * & * & * & 0 & -S_\sigma \end{bmatrix}\prec 0. \tag{87}$$

Referring to Assumption 2, and pursuing the same line as in Theorem 1, we have

$$\Delta T_\sigma(t)=He(\Lambda_{m\sigma}(Q(t),Q(t),Q(t))\Gamma_{m\sigma}), \tag{88}$$

where $\Lambda_{m\sigma}$ is defined in (75) and

$$\Gamma_{m\sigma}\triangleq\begin{bmatrix} N_{\sigma 1} & 0 & 0 & 0 & N_{\sigma 4} & 0 & 0 & 0 \\ 0 & 0 & N_{\sigma 2} & 0 & 0 & 0 & 0 & 0 \\ N_{\sigma 1} & 0 & 0 & 0 & N_{\sigma 4} & 0 & 0 & 0 \end{bmatrix}, \tag{89}$$

and by using the generalized square inequality Lemma 4, i.e., (17), we obtain

$$\Delta T_\sigma(t)\leq\delta_{2\sigma}^{-1}\Lambda_{m\sigma}\Lambda_{m\sigma}^T+\delta_{2\sigma}\Gamma_{m\sigma}^T\Gamma_{m\sigma}. \tag{90}$$

And we can get

$$\Phi_{m\sigma}+\delta_{2\sigma}^{-1}\Lambda_{m\sigma}\Lambda_{m\sigma}^T\leq 0, \tag{91}$$

where $\Phi_{m\sigma}$ is defined in (74). Finally, using the Schur complement lemma (14), inequality (91) turns to (72).

The rest of the proof is omitted because it is similar to that of Theorem 1. This means that the switched system (8) satisfies the weighted H_{-} performance (α_m,γ_2) with

$$\gamma_2=\gamma_{20}\exp\left(-0.5\,\alpha_m\sum_{p=1}^{l}\tau_{ap}^m N_{0p}\right)$$

in (10). This completes the proof. $\square$

3.3 The mixed weighted H_∞/H_{-} problem

In this section, the combination of both the problems of disturbance attenuation and fault sensitivity amplification is described by the following corollary as a mixed weighted H_∞/H_{-} problem. To solve this problem, an algorithm is also presented.

Corollary 1. By combining the results of Theorems 1 and 2 referring to the optimization problem defined in (11), the proposed IFDRC scheme can be summarized as follows:

Under the switching law $\sigma(t)$ with the defined MDADT in (92), if conditions (22)-(24) and (71)-(73) are satisfied, then the augmented system (8) is exponentially stable with the estimated state decay ratio in (61), and also satisfies mixed weighted H_∞/H_- performance indices (9) and (10).

$$\tau_{ai} \geq \max(\tau_{ai}^{m*}, \tau_{ai}^{M*}). \tag{92}$$

Moreover, the IFDRCU matrices can be constructed by (32).

Since (22)-(24) are in the LMI form, and (71)-(73) are BMI, the IFDRCU design problem yields the following two-step optimization algorithm [17].

Algorithm 1

0. Select the scalars $\alpha_M > 0$, $\mu_i^M \geq 1$, $\alpha_m > 0$, $\mu_i^m \geq 1$.
1. Solve (22)-(24) to obtain the minimum permitted level of disturbance attenuation, γ_1, which will lead to the appropriate robust controller to satisfy the H_∞ performance index (9).
2. Substitute the resulted controller gains from the first step into (71)-(73) and check the feasibility of these inequalities to find the maximum permitted level of fault sensitivity, γ_2, that satisfies the H_- performance index (10).

Also, compromising between the desired γ_1, γ_2 can be done by repeating the two aforementioned steps.

3.4 Residual signal evaluation

For successful fault detection and generating fault occurrence alarm, the last step after designing the residual generator is to evaluate the residual signal (Figure 1). This step includes two tasks:

- Producing an evaluation function ($J_{RMS}(L)$)

- Specifying a threshold (J_{th}).

By employing a similar method to the other fault detection literature [6, 14], which relaxes the necessity to estimate the fault signal, the following residual evaluation function is used:

$$J_{RMS}(L) = \|r(t)\|_2 = \left(\frac{1}{L}\int_{t_0}^{t_0+L} r^T(\tau)r(\tau)d\tau\right)^{\frac{1}{2}}, \tag{93}$$

where L is the evaluation time step and t_0 is the initial evaluation time instant.

To identify when a fault has occurred, this evaluation function can be compared to the threshold by the following rule:

$$J_{RMS}(L) - J_{th} = \begin{cases} > 0, & \text{fault occured} \Rightarrow \text{Alarm}, \\ < 0, & \text{No fault}. \end{cases} \tag{94}$$

As indicated in [9], the threshold can be chosen as

$$J_{th} = \sup_{\|\omega(t)\|_2 \le \delta_\omega, f=0} J_{RMS}(L). \tag{95}$$

4 A Numerical Example

In this section, a numerical example is considered as a case study for simulating the proposed framework for the IFDRCU design technique to illustrate the effectiveness and applicability of the theoretical results.

The realization of this numerical example can be given by the Electrical Circuit system, which is shown in Figure 2.

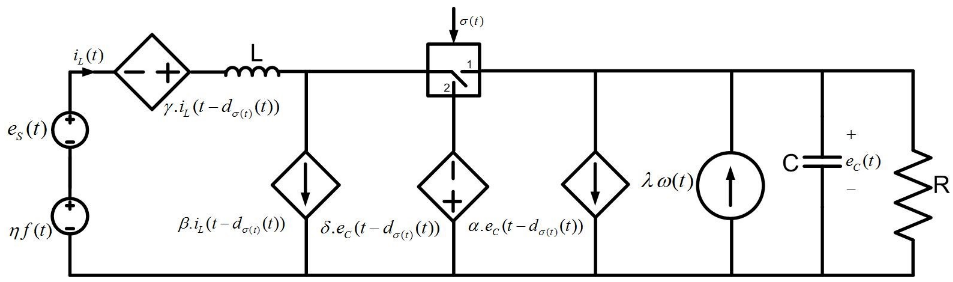

Figure 2: A sample Electrical Circuit switched system.

According to Kirchhoff's Circuit Law, for two switching modes of this Electrical Circuit, we have

$$\begin{aligned} KCL: C\frac{de_C}{dt} &+ \frac{e_C(t)}{R} + i_L(t)\cdot(\sigma(t)-2) + \alpha\cdot e_C(t-d_{\sigma(t)}(t)) \\ &- \beta\cdot i_L(t-d_{\sigma(t)}(t)).(\sigma(t)-2) - \lambda\cdot\omega(t) = 0, \\ KVL: L\frac{di_L}{dt} &- e_C(t)\cdot(\sigma(t)-2) - \delta\cdot e_C(t-d_{\sigma(t)}(t))\cdot(\sigma(t)-1) \\ &- \gamma\cdot i_L(t-d_{\sigma(t)}(t)) - e_s(t) - \eta\cdot f(t) = 0. \end{aligned} \tag{96}$$

The state-space representations of this circuit are given by

$$\dot{x}(t) = \begin{bmatrix} -\frac{1}{RC} & \frac{(2-\sigma(t))}{C} \\ \frac{(\sigma(t)-2)}{L} & 0 \end{bmatrix} x(t) + \begin{bmatrix} -\frac{\alpha}{C} & \frac{\beta.(\sigma(t)-2)}{C} \\ \frac{\delta.(\sigma(t)-1)}{L} & \frac{\gamma}{L} \end{bmatrix} x(t-d_{\sigma(t)}(t))$$

$$+\begin{bmatrix}0\\\frac{1}{L}\end{bmatrix}u(t)+\begin{bmatrix}\frac{\lambda}{C}\\0\end{bmatrix}\omega(t)+\begin{bmatrix}0\\\frac{\eta}{L}\end{bmatrix}f(t), \tag{97}$$

where $\begin{bmatrix}x_1(t) & x_2(t)\end{bmatrix}^T = \begin{bmatrix}e_C(t) & i_L(t)\end{bmatrix}^T$ and $u(t) = e_S(t)$ are the state vector and input signal, respectively.

When the parameters $\alpha, \beta, \delta, \gamma, \eta, \lambda$ are set to zero in this electrical circuit, this is equivalent to the Boost Converter switched system. As a typical circuit system, the Boost Converter is used to transform the source voltage into a higher voltage. This class of power converters has been modeled as switched systems. In recent years, the fault detection and control problems for such power converters have been widely studied in the literature [10, 26]. More details of this system are given in [36].

For $\alpha = -0.2, \beta = 0.3, \delta = 0.4, \gamma = -0.5, \lambda = -0.1, \eta = 0.4$ and $R = 1\,\Omega, L = 1\,H, C = 1\,F$ the following state-space matrices are obtained.

$$A_1 = \begin{bmatrix}-1 & 1\\-1 & 0\end{bmatrix}, A_{d1} = \begin{bmatrix}0.2 & -0.3\\0 & -0.5\end{bmatrix}, B_1 = \begin{bmatrix}0\\1\end{bmatrix}, B_{\omega 1} = \begin{bmatrix}0.1\\0\end{bmatrix},$$
$$B_{f1} = \begin{bmatrix}0\\0.4\end{bmatrix}, C_1 = \begin{bmatrix}0.1 & 0.1\end{bmatrix}, D_{\omega 1} = [0], D_{f1} = [0.1],$$
$$A_2 = \begin{bmatrix}-1 & 0\\0 & 0\end{bmatrix}, A_{d2} = \begin{bmatrix}0.2 & 0\\0.4 & -0.5\end{bmatrix}, B_2 = \begin{bmatrix}0\\1\end{bmatrix}, B_{\omega 2} = \begin{bmatrix}0.1\\0\end{bmatrix},$$
$$B_{f2} = \begin{bmatrix}0\\0.4\end{bmatrix}, C_2 = \begin{bmatrix}0.3 & 0.4\end{bmatrix}, D_{\omega 2} = [0], D_{f2} = [0.1], \tag{98}$$

which are similar to the Boost Converter matrices in [10], except that it does not have state delay. Also, output matrices are considered the same as in [10].

For parameter uncertainties, the following real constant matrices and $Q(t) = \sin(3t)$ are considered:

$$M_1 = \begin{bmatrix}0.1\\-0.2\end{bmatrix}, M_2 = [0.1], \tag{99}$$
$$N_1 = \begin{bmatrix}-0.2 & 0.1\end{bmatrix}, N_2 = \begin{bmatrix}0.1 & -0.1\end{bmatrix}, N_3 = [0.2], N_4 = [-0.1].$$

Time-varying state delays for two subsystems are supposed to be $d_1(t) = 0.2 + 0.1\cos(t)$ and $d_2(t) = 0.3 - 0.2\sin(t)$. Therefore, the upper bound of delay and its derivative for two modes will be $d_1 = 0.3$, $\rho_1 = 0.1$ and $d_2 = 0.5$, $\rho_2 = 0.2$, respectively.

Given $\alpha_M = 0.1, \alpha_m = 0.3, \mu_{M1} = 1.01, \mu_{m1} = 1.1, \mu_{M2} = 1.02, \mu_{m2} = 1.5$, the allowed minimum MDADT for each subsystem could be obtained from (31) and (81), and (92) as $\tau^*_{a1} = \max(0.3177, 0.0995) = 0.3177$, $\tau^*_{a2} = \max(1.3516, 0.1980) = 1.3516$. By MDADT constraints $\tau_{a1} = 0.53, \tau_{a2} = 1.39$, the switching signal in Figure 3.a is chosen.

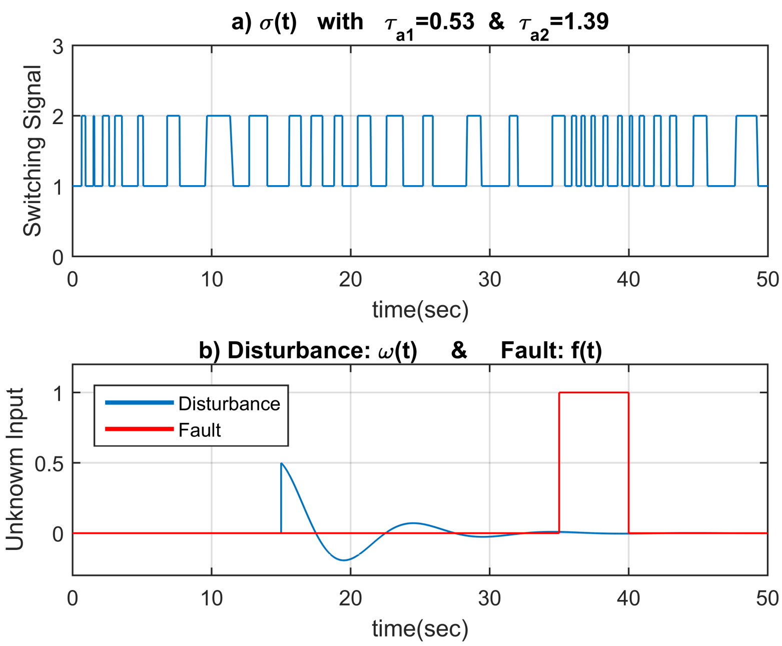

Figure 3: (a). Switching signal (b). Fault and disturbance signals.

Solving the LMIs in (22)-(24) by MOSEK solver [1] in MATLAB/YALMIP, results in the following controller/detector gains, and minimum disturbance attenuation level,$\gamma_1 = 0.187$.

$$\begin{bmatrix} A_{m1} & B_{m1} \\ C_{m1} & D_{m1} \\ K_{m1} & L_{m1} \end{bmatrix} = \begin{bmatrix} -0.7175 & 0.3226 & -0.0786 \\ 0.3226 & -0.7175 & -0.0786 \\ -0.0113 & -0.0113 & -0.4972 \\ -0.1252 & -0.1252 & -1.6310 \end{bmatrix},$$
$$\begin{bmatrix} A_{m2} & B_{m2} \\ C_{m2} & D_{m2} \\ K_{m2} & L_{m2} \end{bmatrix} = \begin{bmatrix} -0.7378 & 0.3134 & -0.0284 \\ 0.3134 & -0.7378 & -0.0284 \\ -0.0208 & -0.0208 & -0.2519 \\ 0.1272 & 0.1272 & -1.6611 \end{bmatrix}, \tag{100}$$

Then, solving the LMIs in (71)-(73) results in the fault sensitivity level, $\gamma_2 = 0.016$.

For simulation, we assume that the unknown bounded input, called disturbance, is given by $\omega(t) = 0.5\exp(-2(t-15))\cos(0.2\pi(t-15))u(t-15)$ with $\delta_\omega = 0.5$, and the fault occurs as a step in $t = 35s$ and remains for 5 seconds, while disturbance is present from $t = 15\ s$ as shown in Figure 3.b.

Choosing the initial state $x_0 = \begin{bmatrix} 0.6 & -0.4 \end{bmatrix}^T$, Figure 4.a and Figure 4.b show trajectories of the state responses of the system ($x(t)$) and its control input ($u(t)$), respectively, from which we can see that the closed-loop system is exponentially stable under the initial state and unknown disturbances.

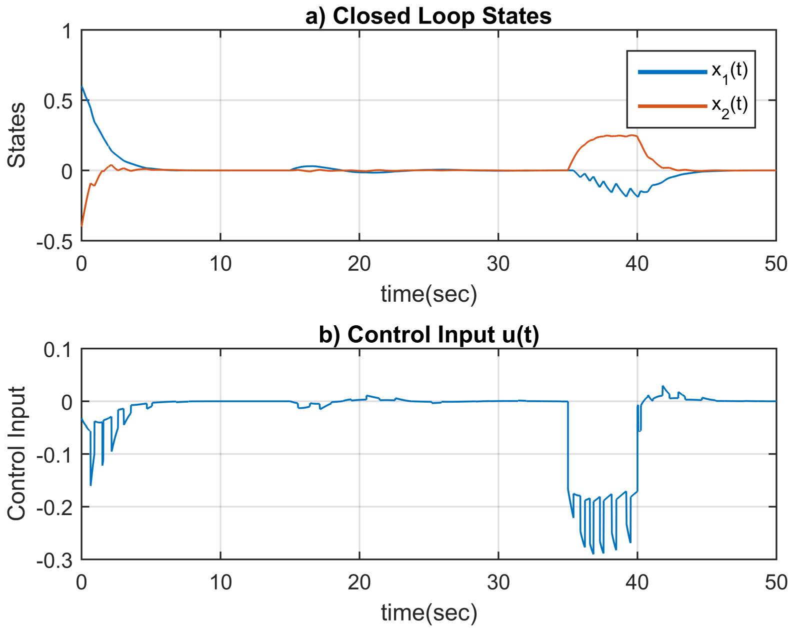

Figure 4: (a). State responses of the closed-loop system. (b). Control input.

The generated residual signal and the evolution of the residual evaluation function are shown in Figure 5.a and Figure 5.b.

Simulating the system in a fault-free case, the threshold can be determined as $J_{th} = 0.004$. It can be seen from Figure 5.b that fault is detected at $t = 35.2\ s$

Simulation results show that the early detection of fault can be achieved by the controller/detector immediately and effectively when faults occur, although disturbance input, mode-dependent time-varying state delay, and parameter uncertainties are present and the control loop is closed. The benefit of integrated fault detection and control design of the system is that fault occurrence cannot be hidden by the control action.

To illustrate the excellence of the proposed technique, it is compared with the existing method [10] in two cases; with and without state delay and parameter uncertainty. Comparing the disturbance attenuation level values (γ_1), as shown in Table 1, show that the proposed approach is less conservative. It has a better disturbance rejection capacity because the residual signal is less affected by the unknown input.

By comparing the minimum allowed average dwell time values in Table 1, the proposed approach has more flexibility in the switching times, since it admits different average dwell times for each subsystem. Note that since the compared paper did not

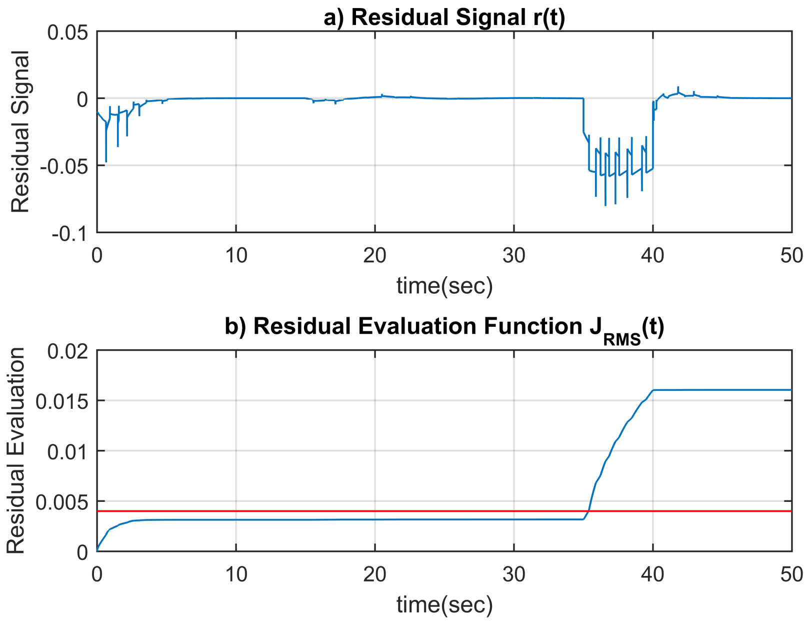

Figure 5: (a). Generated residual signal. (b). Residual evaluation function.

Table 1: Comparison with the existing results

Method	Delay	Uncertainty	Disturbance attenuation level (γ_1)	Average Dwell time#1	Average Dwell time#2
[10]	No	No	0.91	12.307	12.307
This paper	Yes	Yes	0.1871	0.3176	1.3516
This paper	No	No	0.1644	0.3176	1.3516

consider state-space delay and parameter uncertainties, our results were reported with and without state delay and parameter uncertainties.

5 Conclusion

The proposed MDADT switching strategy was less conservative and allowed lower and as well different ADTs for each subsystem compared with the general ADT switching method. The main objective of this paper was to propose a general framework for IFDRC of linear continuous-time switching systems suffering from mode-dependent time-varying state delay, parameter uncertainties, and input disturbance. Sufficient conditions for IFDRC design were derived based on the MDADT technique. Multiple Lyapunov-Krasovskii functions under the framework of mixed H_∞/H_-, and the fault detection filters and controllers were developed together. Finally, the proposed scheme was applied to a switched model of an Electrical Circuit system, and the simulation results indicated the effectiveness of the proposed technique.

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