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Control and Synchronization of the Genesio-Tesi Chaotic System: A Contraction Analysis-Based Graphical Method

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Abstract. In this paper, we use a graphical algorithm to control and synchronization of a chaotic system. Most of the controllers designed for synchronizing chaotic systems are complex, but the controllers designed using contraction and graphical methods are often simple and linear. Therefore, we explain the relationship between contraction analysis and the graphical method for controlling and synchronizing chaotic systems. We apply this approach to control and synchronize the chaotic Genesio-Tesi system. The stability of the error system in synchronization is investigated using the contraction method. Finally, we provide numerical simulations to demonstrate the effectiveness of the proposed method.

Keywords. Contraction analysis, Graphical method, Chaotic systems, Control, Synchronization.

MSC. 90C34; 90C40.

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1 Introduction

Henri Poincare is considered the founder of chaos theory, which he developed in 1908 in an attempt to solve the three-body problem. However, his theory did not receive much attention at the time. The first numerical research that led to the widespread introduction of chaos was presented in 1960 by Lorenz, a meteorologist [21]. In recent years, deterministic chaos has been used as a model in electronic circuits, lasertechnology, cardiology, chemical reactions, turbulence, population dynamics, and weather forecasting. Control is one of the most essential interdisciplinary fields of science, which is almost ubiquitous in all aspects of human life. Due to the different applications of controlling chaotic systems, numerous researchers have become interested in it. Since the seminal paper about chaos by Ott et al. in 1990 [25], many methods have been developed to control the chaos phenomenon. Basic chaos control techniques can be found in the excellent textbooks of Kapitaniak [13, 14]. Different researchers have studied control of the Chen turbulence system, Lorenz system, Duffing system, and Genesio-Tesi system using different methods [2, 3, 10, 24, 25, 34, 40]. In most of the mentioned models, the design of the controller is based on Lyapunov stability theory. However, in [12], contraction analysis is used to achieve the exponential stability of the Lorenz system. Chaotic synchronization refers to the possibility of two or more chaotic systems oscillating in a synchronized way. Pecora and Carroll first explored the synchronization of chaotic systems in 1990 [26]. Since then, Due to its widespread use, especially in secure communications, it has attracted the attention of many researchers. Various synchronization methods have been used in the literature, such as variable structure control, adaptive parameter control, step-by-step design technique, OGY method, active control, and observer-based control [1, 7, 9, 11, 16, 17, 36, 39, 41]. The majority of these methods are based on analysis using the Lyapunov technique.

Contraction analysis is a recent method for the convergence analysis of nonlinear systems [18, 19, 20, 37]. Contraction analysis has been treated as an incremental form of stability. The main idea of this method is to establish conditions in some regions of phase space that ensure exponential convergence of neighboring trajectories. There is a long history of contraction analysis. The concept of contraction in metric functional spaces is due to the work of Banach and Caccioppoli [6], and in the field of dynamic systems, it dates back to [8] and even [15]. The basic concepts, definitions, and notions of the dynamic contracting system were presented by Slotine and his co-authors [19, 27, 38]. In particular, it can be considered an effective tool in designing communication protocols in network control systems [22, 23, 31]. Given the importance of controlling dynamical systems, especially chaotic systems, and the advantages of the contraction method as an efficient and useful method, our motivation is to use the contraction method to control and synchronize the chaotic Genesio-Tesi system.

The paper is outlined as follows: In Section 2, we discuss the concept of contraction analysis and graphical method of contraction are discussed. The control and synchronization of the chaotic Genesio-Tesi system, based on a graphical approach, are presented in Sections 3 and 4, respectively. Finally, our concluding remarks are provided in Section 5.

2 Contraction and Graphical Analysis

In this section, we review the required concepts of the contraction analysis.

2.1 Concept of contraction

Contraction analysis is a relatively new theory for the analysis of nonlinear systems [19]. This theory attempts to answer the following fundamental questions: Is the limit behavior of a given dynamical system independent of its initial conditions? Is stability examined based on the incremental between two arbitrary trajectories in the contraction analysis? This theory is used to determine the convergence of closely related trajectories as well as the global convergence of trajectories. In this subsection, we summarize the basic contents of contraction analysis. For more details, refer to [19]. Contraction analysis, a standard nonlinear stability analysis method, nicely complements the Lyapunov theory. In continuous autonomous systems associated with constant metrics, the nonlinear contraction result reduces to Krasowski's theorem [35]. Consider a nonlinear system given by:

$$\dot{x} = f(x,t), \tag{1}$$

where $x \in \mathbb{R}^{n\times 1}$ is a state vector and f is an $n \times 1$ vector function. Let $f(x,t)$ be a continuously differentiable function. δx indicates the virtual displacement in the state x, which represents infinitesimal displacements at a fixed time. Due to the introduction of the concept of virtual dynamics, the first variation of the system in (1) can be expressed as

$$\delta\dot{x} = \frac{\partial f(x,t)}{\partial x}\,\delta x. \tag{2}$$

Thus, we can conclude that

$$\frac{d}{dt}(\delta x^T \delta x) = 2\,\delta x^T \delta\dot{x} = 2\,\delta x^T \frac{\partial f}{\partial x}\,\delta x \le 2\,\lambda_m(x,t)\,\delta x^T \delta x, \tag{3}$$

where $\frac{\partial f}{\partial x}$ is the Jacobian matrix and $\lambda_m(x,t)$ denotes the largest eigenvalue of the symmetrical part of the Jacobin. If $\lambda_m(x,t)$ is strictly uniformly negative, then time derivative of $(\delta x^T \delta x)$ negative. Here, $(\delta x^T \delta x)$ represents the squared distance between the neighboring trajectories and it is positive by nature. Thus, any infinitesimal length $\|\delta x\|$ tends to zero exponentially. Therefore, (3) guarantees that all response trajectories of the system (1) converge exponentially to a single trajectory, regardless of the initial conditions.

Definition 1. Let $\dot{x} = f(x,t)$ represent the desired system. A contraction region is defined as a state space region in which the Jacobian $\frac{\partial f}{\partial x}$ is uniformly negative definite (UND) within that region [19].

The Jacobian $\frac{\partial f(x,t)}{\partial x}$ is UND if there exists a scalar $\alpha > 0$, such that, $\forall x, \forall t \ge 0$, $\frac{\partial f}{\partial x} \le -\alpha I < 0$. By the symmetry of the square matrix and its symmetric part, it can be written as follows: $\frac{1}{2}(\frac{\partial f}{\partial x} + \frac{\partial f^T}{\partial x}) \le -\alpha I < 0$. The following theorem [28] provides convergence of two different paths in a contraction region.

Theorem 1. Suppose that C is a convex subset of $\mathbb{R}^n$ and that $f(t,x)$ is infinitesimally contracting with contraction rate α. Then, for any two solutions $x_1(t) = \phi(t,0,\xi)$ and $x_2(t) = \phi(t,0,\zeta)$ it holds that

$$|x_1(t) - x_2(t)| \le Ke^{-\alpha t}|\xi - \zeta|, \quad \forall t > 0. \tag{4}$$

The extension of this theorem to N arbitrary paths in a contraction region is straightforward. According to Definition 1, the necessary conditions for exponential convergence of the trajectories can be expressed as follows [18, 19, 20].

Lemma 1. Given the system equations $\dot{x} = f(x,t)$, a contraction region of the system, and a ball in the region with a constant radius centered about a given trajectory that is contained at all times in the contraction region, every trajectory that starts inside the ball not only stays in that ball but also converges exponentially to the given trajectory. Furthermore, if the whole state space is the contraction region, then global exponential convergence to the given trajectory is guaranteed.

The above results can be extended to a more general setting with the following coordinate transformation.

$$\delta z = \theta \delta x, \tag{5}$$

where the matrix $\theta(x,t)$ is uniformly invertible. Concepts, definitions, and theorems after generalization can be found in references [18, 19, 20]. For some systems that have the form (1), the Jacobian matrix $\frac{\partial f}{\partial x}$ can turn out to be a negative semi-definite matrix. By applying Definition 1, such systems are called semi-contracting system, and asymptotic stability can be guaranteed using contraction analysis results.

2.2 A graphical approach to prove contraction

In this subsection, we present an approach to proving the contraction of nonlinear dynamical systems using non-Euclidean norms and their related matrix measures. A graphical method is used to investigate the system's contraction, which, can be applied to design a control law for a contracting system or the synchronization strategy in a network of nonlinear oscillators. The algorithmic method to investigate the system's contraction was developed by G. Russo and M. di Bernardo in collaboration with Slotine, as described in [29, 32, 33]. A prior prescription of the algorithm can be found in [30]. One of the main advantages of this method the procedure is that it provides the proof of the system's contraction in all trajectories convergence, without the need to identify an appropriate metric explicitly. Simply, The purpose of the graphical approach is to provide sufficient conditions to guarantee the existence of such a metric. The relation between the contraction condition for a system and graph theory is expressed in the following theorem.

Theorem 2. A continuous-time n-dimensional dynamical system $\dot{x} = f(x,t)$ is contracting if its Jacobian matrix, J, satisfies the following conditions:

- $J_{i,i}(t,x) < 0, \quad \forall i = 1, \cdots, n,$
- the graph Gd(A) constructed from J as detailed above does not contain (directed) loops and $\alpha_{ij}(t,x)\alpha_{ji}(t,x) \leq 1$.

An outline of Theorem 2 for the continuous-time system is provided below.

Outline. In this procedure, the Jacobin matrix of the system is obtained, which is generally dependent on time and state. It is obtained by differentiating the given system follows:

$$J = \begin{pmatrix} J_{1,1}(t,x) & J_{1,2}(t,x) & \cdots & J_{1,m}(t,x) \\ J_{2,1}(t,x) & J_{2,2}(t,x) & \cdots & J_{2,m}(t,x) \\ \vdots & \vdots & \vdots & \vdots \\ J_{m,1}(t,x) & J_{m,2}(t,x) & \cdots & J_{m,m}(t,x) \end{pmatrix}, \tag{6}$$

where $J_{i,j}(t,x)$ is the $\frac{\partial f_i}{\partial x_j}$. In the next step, a directed graph is constructed from the Jacobian matrix of the system. To do this, an adjacency matrix A is created from J using the following rules:

- Initialize A such that $A(i,j) = 0, \quad \forall i,j,$
- For all $i \neq j$, set $A(i,j) = A(j,i) = 1$ if either $J_{i,j}(t,x) \neq 0$, or $J_{j,i}(t,x) \neq 0$.

According to graph theory, this matrix represents a simple graph (not directed graph) denoted by $G(A)$ [5]. The next step is to direct the edges of $G(A)$ to create a directed graph, denoted by $G_d(A)$. It will occur This is achieved by computing the value of

$$\alpha_{i,j}(t,x) = \frac{|J_{i,j}(t,x)|}{|J_{i,i}(t,x)|}(m - n_{0i} - 1), \tag{7}$$

where n_{0i} is the number of zero elements on the i-th row of A.

It is possible that $J_{i,i}(t,x)$ becomes zero for some i. Therefore, the system structure or parameters of controllers should be adjusted to ensure that $J_{i,i}(t,x) \neq 0$ for all i. This can be achieved by changing the order of equations in the system or modifying the parameters of controllers.

For any arbitrary nodes i and j of the graph $G_d(A)$, the direction of the edge is determined as follows:

- The edge is from i to j, if $\alpha_{i,j}(t,x) < 1$;
- The edge is from j to i, if $\alpha_{i,j}(t,x) \geq 1$.

Note that the values of $\alpha_{i,j}(t,x)$ will generally depend on time. Therefore, the directions of the graph may vary with time, and it is assumed that the edge between nodes i and j indicates a bidirectional relationship $\alpha_{i,j}(t,x)$ depends on the states.

After completing the aforementioned steps, contraction will be ensured under the following conditions:

- All of the diagonal elements of the Jacobian matrix must be uniformly negative, i.e., $J_{i,i}(t,x) < 0$ for all i;
- There should be no loops in the directed graph $G_d(A)$ for all t and $\alpha_{i,j}(t,x)\alpha_{j,i}(t,x) \leq 1$ for any $i \neq j$.

If the above conditions for contraction are not satisfied, then the contraction of systems is investigated by applying the main approach following these rules:

1. If the uniform negativeness of $J_{i,i}(t,x)$ for some i is not satisfied, then, if possible, control input will be used to satisfy the elements of $J_{i,i}(t,x)$ that does not fulfill it.
2. If certain directions of edges of $G_d(A)$ cause a loop in the digraph, then the parameters of the system or appropriate control input will be tuned to make a loopless condition in $G_d(A)$.
3. Each returned edge (e.g., the edge between nodes i and j) is associated with one of the following inequalities:
    - $\alpha_{i,j}(t,x) < 1$, if the edge is returned from i to j;
    - $\alpha_{i,j}(t,x) \geq 1$, if the edge is returned from j to i;
    - $\alpha_{i,j}(t,x)\alpha_{j,i}(t,x) \leq 1$ should be guaranteed.

To apply the algorithm to discrete-time systems and understand how it works, the reader can refer to [33, 28].

3 Control of Chaotic Genesio-Tesi System

Chaos is a long-term non-periodic behavior in a definite system that indicates sensitivity to initial conditions. Chaos occurs when the system is so sensitive to the disturbance that even the slightest disturbance can quickly lead to significant changes. The Lyapunov exponent and bifurcation are the most essential criteria for recognizing chaotic behavior.

The Genesio-Tesi system is one example proposed in [4]. This system is described as follows:

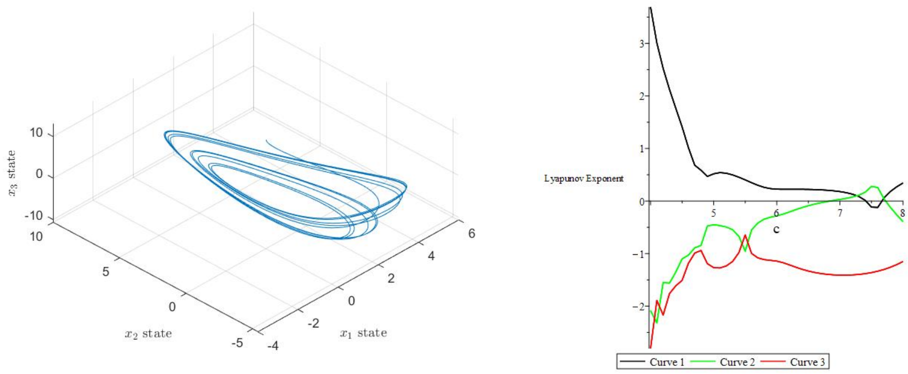
$$\begin{cases} \dot{x_1} = x_2, \\ \dot{x_2} = x_3, \\ \dot{x_3} = -cx_1 - bx_2 - ax_3 + x_1^2, \end{cases} \tag{8}$$

where x_1, x_2 and x_3 are state variables, and a, b, and c are constants that satisfy $ab < c$ and a, b, $c \in \mathbb{R}^+$. For instance, the system is chaotic for the parameters values $a = 1.2$, $b = 2.92$, and, $c = 6$, which is confirmed by its corresponding Lyapunov exponent diagrams. The phase portrait and Lyapunov exponent diagrams are given in Figures 1a and 1b, respectively.

To control the chaos in system (8), we add the controllers as follow:

$$\begin{cases} \dot{x_1} = x_2 + u_1, \\ \dot{x_2} = x_3 + u_2, \\ \dot{x_3} = -cx_1 - bx_2 - ax_3 + x_1^2 + u_3, \end{cases} \tag{9}$$

where $u = (u_1, u_2, u_3)^T$ is controller vector.



(a) Phase portrait. **(b)** Lyapunov exponent.

Figure 1: Phase portrait and Lyapunov exponent of system (8)

Theorem 3. The actual Genesio-Tesi system given in (8) can be controlled by the controller designed as follows:

$$u = (-kx_1, -mx_2, 0)^T \text{ s.t. } 0 < k \leq 1 \text{ and } m \geq 2.4\overline{3}.$$

Proof. The conditions for the contraction algorithm cannot be established because the system is chaotic. Therefore, we will attempt to control chaos by using the appropriate control function $u = (u_1, u_2, u_3)^T$.

By applying the proposed algorithm, the Jacobian matrix of system (8) with chaotic parameters $a = 1.2$, $b = 2.92$ and, $c = 6$ is obtained as follows:

$$J = \frac{\partial f}{\partial x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2x_1 - 6 & -2.92 & -1.2 \end{pmatrix}.$$

We obtain the adjacency matrix, named A from J (using the algorithm), and the related simple graph $G(A)$ from A (Figure 2a). Note that for each state of the system corresponds to one vertex in the graph $G(A)$: Vertices 1, 2 and 3 in $G(A)$ represent the representation of x_1, x_2, and x_3 states, respectively.

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

The coefficients $\alpha_{i,j}(t,x)$ are now computed to provide the directions of the edges of $G(A)$ where $i \neq j$. According to (7), we get

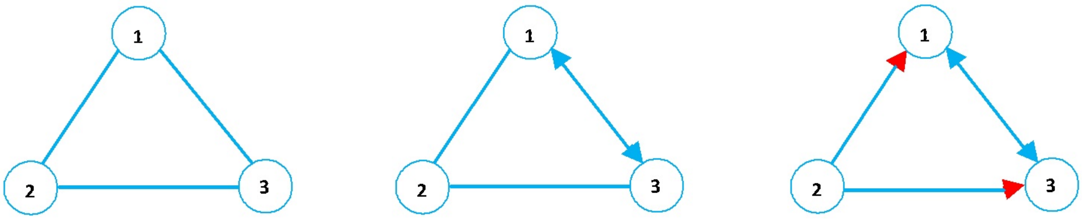
$$\begin{aligned}
\alpha_{1,2}(t,x) &= \frac{|J_{1,2}(t,x)|}{|J_{1,1}(t,x)|}(m - n_{01} - 1) = \frac{|1|}{|0|}(3-1-1), \\
\alpha_{2,1}(t,x) &= \frac{|J_{2,1}(t,x)|}{|J_{2,2}(t,x)|}(m - n_{02} - 1) = \frac{|0|}{|0|}(3-1-1), \\
\alpha_{1,3}(t,x) &= \frac{|J_{1,3}(t,x)|}{|J_{1,1}(t,x)|}(m - n_{01} - 1) = \frac{|0|}{|0|}(3-1-1), \\
\alpha_{3,1}(t,x) &= \frac{|J_{3,1}(t,x)|}{|J_{3,3}(t,x)|}(m - n_{03} - 1) = \frac{|2x_1 - 6|}{|-1.2|}(3-1-1) = \frac{|2x_1 - 6|}{1.2}, \\
\alpha_{2,3}(t,x) &= \frac{|J_{2,3}(t,x)|}{|J_{2,2}(t,x)|}(m - n_{02} - 1) = \frac{|1|}{|0|}(3-1-1), \\
\alpha_{3,2}(t,x) &= \frac{|J_{3,2}(t,x)|}{|J_{3,3}(t,x)|}(m - n_{03} - 1) = \frac{|-2.92|}{|-1.2|}(3-1-1) \simeq 2.4\overline{3}.
\end{aligned}$$

Note that $\alpha_{3,1}$ depends on the state x_1. This implies that the direction of the corresponding edge in $G_d(A)$ can be time-varying, as it is associated with conditions that are a function of the state. Therefore, it is denoted with a double arrow (between vertices 1 and 3) as the direction of this link might vary in time (Figure 2b).

Now, we will attempt to direct the remaining edges of $G(A)$ such that no loop is created. For this, there are two choices: both remaining edges can be either incoming links to vertex 2 or both can be outgoing links to vertex 2. We choose the second option, i.e., both remaining edges are outgoing links to vertex 2 (Figure 2c).

Now, we will try to direct the remaining edges of $G(A)$ such that no loop is created. For this, there are two choices: The direction of both remaining edges must be either incoming links to vertex 2 or both outgoing links to vertex 2. we choose the second one i.e. , both remaining edges are outgoing links to vertex 2 (Figure 2c).

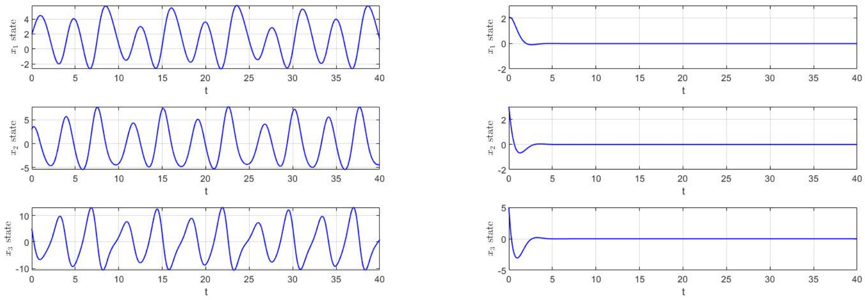
In order for the coefficients to be defined, we need the nonzero elements of the diagonal arrays of the Jacobian. Furthermore, according to the algorithm, we need the diagonal array of the Jacobian matrix to be negative. Additionally, we have the constraint that $\alpha_{i,j}(t,x)\alpha_{j,i}(t,x) \leq 1$ for any $i \neq j$. An



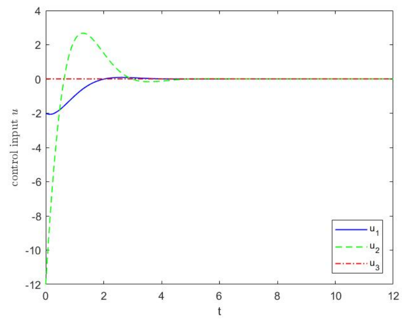
(a) $G(A)$. **(b)** Partially directed of $G(A)$. **(c)** $G_d(A)$.

Figure 2: The procedure of constructing a loopless directed graph $G_d(A)$

analysis of the algorithm reveals that the controller $u_1 = -kx_1$ for $0 < k \leq 1$ satisfies our requirements for $J_{11} < 0$, direction from vertex 2 to vertex 1 for loopless condition, and $\alpha_{i,j}(t,x)\alpha_{j,i}(t,x) \leq 1$ for related i and j. Moreover, choosing $u_2 = -mx_2$ for $m \geq 2.4\overline{3}$ satisfies our requirements for $J_{22} < 0$, directions from vertex 2 to vertex 1 and from vertex 2 to vertex 3 for loopless condition, and $\alpha_{i,j}(t,x)\alpha_{j,i}(t,x) \leq 1$ for related i and j. The success of the presented controllers in controlling chaos is shown in Figure 3. The initial conditions for the system are taken as $x(0) = (2, 3, 5)^T$.



(a) State variables without controllers. **(b)** State variables with controllers.



(c) Controller variables.

Figure 3: Chaos control of system (8)

□

4 Synchronization

Simply put, synchronizing two dynamical systems means making changes to the two systems or one of them so that they both behave the same way. This can be defined for more systems similarly. The main idea of synchronization is to use the master system's output for controlling the slave system, such that the slave system states asymptotically follow the master system states. In this paper, nonlinear contraction analysis has been proposed [19] as an effective tool to study this property for synchronization.

For synchronization problems, let the slave system dynamics be given by:

$$\begin{cases} \dot{y_1} = y_2 + u_1, \\ \dot{y_2} = y_3 + u_2, \\ \dot{y_3} = -cy_1 - by_2 - ay_3 + y_1^2 + u_3, \end{cases} \tag{10}$$

where, $u = (u_1, u_2, u_3)^T$ is the controller input applied to attain the synchronization operation. The error states variable between the states of master and slave systems are defined as:

$$e_i = y_i - x_i; \quad i = 1, 2, 3. \tag{11}$$

Thus, the error dynamics are obtained as follows:

$$\begin{cases} \dot{e_1} = e_2 + u_1, \\ \dot{e_2} = e_3 + u_2, \\ \dot{e_3} = -ce_1 - be_2 - ae_3 + y_1^2 - x_1^2 + u_3. \end{cases} \tag{12}$$

Theorem 4. The slave system given in (10) synchronizes exponentially with the master Genesio-Tesi system (8) if controller is designed as follows:

$$u = (-ky_1 + kx_1, -my_2 + mx_2, 0), \quad 0 < k \leq 1 \quad \text{and} \quad m \geq 2.4\overline{3}.$$

Proof. Similar to the previous theorem, the proof process will be based on the algorithm introduced in subsection 2.2. We obtain the Jacobian matrix of the system (12) with chaotic parameters $a = 1.2$, $b = 2.92$ and $c = 6$ as:

$$J = \frac{\partial f}{\partial e} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ H(e_1) & -2.92 & -1.2 \end{pmatrix},$$

where $H(e_1)$, a function of e_1, is equal to $\frac{\partial f_3}{\partial e_1}$. We now derive the adjacency matrix A from J and the related simple graph $G(A)$. Note that for each state of the system, there corresponds one vertex in the graph $G(A)$: vertices 1, 2 and 3 represent e_1, e_2 and e_3 states of error system, respectively.

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

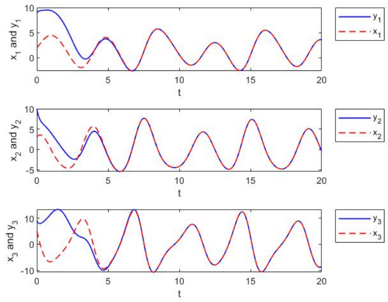
Due to the similarity of the Jacobian matrices of the Genesio-Tesi main system in the previous section and the corresponding error system in this section, all coefficients $\alpha_{i,j}(t,x)$; $\forall i, j (i \neq j)$ except $\alpha_{3,1}(t,x)$ will be the same. Therefore, only $\alpha_{3,1}(t,x)$ needs to be computed

$$\alpha_{3,1}(t,x) = \frac{|J_{3,1}(t,x)|}{|J_{3,3}(t,x)|}(m - n_{03} - 1) = \frac{|H(e_1)|}{|-1.2|}(3 - 1 - 1) = \frac{|H(e_1)|}{1.2}. \tag{13}$$

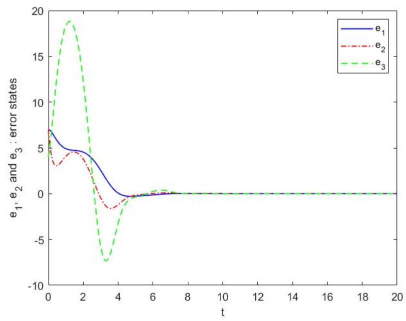
Relation (13) shows dependence of $\alpha_{3,1}(t,x)$ on $H(e_1)$. Therefore, the corresponding edge in $G_d(A)$ will be bidirectional, similar to Figure 2b. The continuation of the proof process is the same as the proof process of Theorem 3. The algorithm is followed to orient the rest of the edges of $G(A)$, and the controllers are obtained as follows:

$$u = (-ky_1 + kx_1, -my_2 + mx_2, 0), \qquad 0 < k \leq 1 \quad \text{and} \quad m \geq 2.4\overline{3}.$$

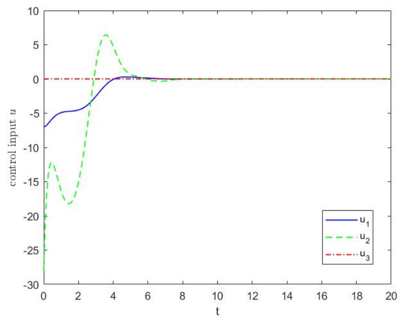
For the master and slave systems, the initial conditions are taken as $x(0) = (2,3,5)^T$ and $y(0) = (9,10,9)^T$, respectively. A numerical simulation of synchronization is given in Figure 4.



(a) Synchronization.



(b) Synchronization error.



(c) Controller.

Figure 4: Identical synchronization of the Genesio-Tesi chaotic system

□

5 Conclusion

This paper explained the relationship between contraction analysis and the graphical method for controlling and synchronizing chaotic systems. We applied this approach to control and synchronize the chaotic

Genesio-Tesi system and demonstrated the power and efficiency of the proposed method through numerical simulations.

Declarations

Availability of Supporting Data

All data generated or analyzed during this study are included in this published paper.

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Competing Interests

The authors declare that they have no competing interests relevant to the content of this paper.

Authors' Contributions

The main text of manuscript is collectively written by the authors.

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